**Coherence Without Symmetry: A Structural Heuristic for Catching Flawed Physics**

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Many physical theories are built on compositional structures—operations that can be assembled in sequence or in parallel. The *coherence* of those structures (associators, unitors, functorial composition, pentagon/hexagon constraints, path ordering) guarantees that all the mandated recouplings and rewritings agree. *Symmetry*, by contrast, is an extra invariant: swaps or permutations act trivially (or up to a sign/known involution). Confusing these two ideas is a common source of error. In domains where order intrinsically matters—non-Abelian holonomy, anyon braiding, chiral CFT, quantum circuits—demanding symmetry where only coherence is justified will erase real physics (monodromy, phases, noncommuting observables). Conversely, failing coherence where it is required (e.g., violating pentagon/hexagon or Jacobi identities, ignoring anomaly cancellation) yields internally inconsistent models.

This paper develops a practical diagnostic: use *coherence without symmetry* as a lens to stress-test proposals. We formalize the minimal diagrams and algebraic laws that must commute, show how to choose the correct monoidal level (plain, braided, symmetric), and compile a field-tested checklist that has surfaced problems in toy anyon models, adiabatic transport, operator product expansions, and circuit abstractions. The result is not a new no-go theorem but a sharp, portable heuristic that helps reviewers, builders, and experimentalists separate innocuous simplifications from structural mistakes before they metastasize.

Keywords: coherence, symmetry, braided monoidal categories, pentagon hexagon, path ordering, non-Abelian holonomy, anyons, CFT crossing, quantum circuits, anomalies, Jacobi identity, ordered logics, E_n algebras, coherence vs commutativity, fault detection.

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## 1. Thesis and Scope

### 1.1 Coherence vs symmetry: definitions and stakes

**Working definitions (structure-first).**

- *Coherence* = the requirement that all diagrams mandated by the given structure commute. In a monoidal or higher-categorical setting this means: associators and unitors satisfy the triangle/pentagon; braidings (if present) satisfy the hexagon; functors respect composition/identities; parallel transport composes functorially; OPE recoupling is consistent across channels; proof rewritings yield the same composite. Formally, coherence certifies that “all allowable rebracketings and rewirings agree,” with the agreement witnessed by specified isomorphisms or higher homotopies.
- *Symmetry* = an additional invariant that makes certain swaps or permutations “trivial” (or trivial up to a fixed involution/sign). In a symmetric monoidal category, the swap $A\otimes B \to B\otimes A$ squares to id and coheres with associativity; in abelian algebra, addition and convolution commute; in QFT, equal-time fields at spacelike separation commute/anticommute by symmetry constraints.

**Disentangling the notions.**

- Coherence is a *consistency contract* for the structure you already chose.
- Symmetry is an *extra freedom to forget order* (or to reduce it to a benign action).
- You can have coherence without symmetry (braided but non-symmetric, ordered logics, path-ordered holonomies). You cannot have a usable theory without coherence; lack of coherence means the composition laws do not make sense globally.

**Stakes for physics and math.**

- *Predictivity*: With coherence, composite operations yield well-defined predictions independent of arbitrary parenthesization or recoupling choices.
- *Order sensitivity*: If symmetry is falsely assumed, you erase real monodromy/phase/ordering effects (e.g., anyons, non-Abelian transport).
- *Auditability*: Coherence furnishes small, checkable equations (pentagon, hexagon, Jacobi, anomaly cancellation) that act as fast sanity checks.
- *Portability*: Correctly separating coherence from symmetry clarifies which results persist under generalization (E1 vs E2 vs E_infty; monoidal vs braided vs symmetric).

**Rule of thumb.**
First pick the *right ambient structure* (monoidal/braided/symmetric; strict/weak; E_n level). Then enforce *its* coherence. Add symmetry only when physically justified by locality, statistics, or an emergent limit.

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**1.2 Where the confusion happens in practice**

**(a) "Symmetric-by-habit" tensors in non-symmetric worlds.**

- *Anti-pattern*: Modeling anyon fusion or quantum circuits with a symmetric tensor where swap = id.
- *Why it is wrong*: In 2+1D, exchanges are braids, not swaps; in circuits, SWAP is a gate, not the identity.
- *Fix*: Use braided (or just monoidal) structure and enforce pentagon/hexagon coherence for F- and R-moves.

**(b) Dropping path ordering for non-Abelian transport.**

- *Anti-pattern*: Writing exp(integral A) instead of P exp(integral A).
- *Why it is wrong*: Non-Abelian holonomies depend on segment order; different loop orderings are physically distinct.
- *Fix*: Treat parallel transport as a functor from a path groupoid (or a 2-functor for surfaces) and keep P-ordering explicit.

**(c) Conflating "commutes up to isomorphism" with "is equal."**

- *Anti-pattern*: Replacing canonical isomorphisms (associators, braidings) by equalities in proofs or code.
- *Why it is wrong*: You silently assume symmetry/strictness that may not hold; downstream, diagrams that should only commute via witnesses are taken as identities.
- *Fix*: Track witnesses explicitly or work in a framework (proof assistant, typed IR) that enforces them.

**(d) Crossing symmetry vs commutativity in CFT/OPE.**

- *Anti-pattern*: Treating operator insertions as if they commute, rather than checking crossing/associativity constraints (Moore–Seiberg).

- *Why it is wrong*: The OPE is associative but not generally commutative; monodromy data matters.
- *Fix*: Verify the minimal crossing equations (coherence) instead of imposing a symmetric tensor structure.

**(e) Jacobi and anomaly “afterthoughts.”**

- *Anti-pattern*: Proposing commutation relations or symmetry actions without checking Jacobi identity or anomaly cancellation.
- *Why it is wrong*: These are coherence conditions for the algebra/gauge action; violations make the theory inconsistent.
- *Fix*: Make Jacobi and anomaly budgets first-class checks, not appendices.

**(f) Emergent symmetry mistaken for microscopic symmetry.**

- *Anti-pattern*: Using IR/thermodynamic effective commutativity as a UV axiom.
- *Why it is wrong*: Coarse-graining can abelianize or symmetrize; the microtheory can remain non-symmetric.
- *Fix*: State scale and regime. Treat symmetry as emergent unless a microscopic argument forces it.

**(g) Ordered logics and process calculi treated as exchange-permitting.**

- *Anti-pattern*: Allowing Exchange (reordering of hypotheses) in systems that model word order, resources, or causality.
- *Why it is wrong*: It collapses the very constraints those logics encode.
- *Fix*: Keep contexts ordered; use coherence theorems for cut elimination without Exchange.

**A 60-second checklist (diagnostic habit).**

1. What is my ambient tensor/compose structure: monoidal, braided, or symmetric?
2. Which coherence equations apply (triangle/pentagon/hexagon; Jacobi; crossing; anomaly cancellation)?
3. Do any predictions depend on order, orientation, or monodromy? If yes, avoid symmetry assumptions.
4. Where, if anywhere, does symmetry provably emerge (scale, limit, locality)? State it.

5. Can I turn the qualitative claim into a small commuting diagram or algebraic identity and actually check it?

**Bottom line.**
Confusion arises when we use symmetry as a convenience where only coherence is warranted, or when we skip coherence checks entirely. The cure is structural hygiene: choose the right categorical level, enforce its coherence, and let symmetry be earned rather than assumed.

## 2. Structural Primer (Minimal Machinery)

### 2.1 Monoidal levels: plain, braided, symmetric (and ribbon)

**Plain monoidal.**
Data: a tensor $\otimes$, unit $I$, and natural isomorphisms

- associator $\alpha_{A,B,C}: (A \otimes B) \otimes C \to A \otimes (B \otimes C)$
- unitors $\lambda_A: I \otimes A \to A$, $\rho_A: A \otimes I \to A$.
  No swap is assumed.
  *Examples:* $(\mathrm{End}(\mathcal{C}), \circ, I = \mathrm{Id})$; categories of processes/strings; bimodules with $\otimes_R$.

**Braided monoidal.**
Add a **braiding** $c_{A,B}: A \otimes B \to B \otimes A$that is natural and invertible, but not necessarily an involution. Crossing wires is meaningful.
*Examples:* braid category $\mathcal{B}$; $\mathrm{Rep}(U_q(\mathfrak{g}))$for generic $q$.

**Symmetric monoidal.**
The braiding squares to identity: $c_{B,A} \circ c_{A,B} = \mathrm{id}$. Swapping is "benign."
*Examples:* $(\mathrm{Set}, \times)$, $(\mathrm{Vect}_k, \otimes)$.

**Ribbon / balanced.**
A braided monoidal category with a **twist** $\theta_A: A \to A$satisfying balancing axioms (compatibility with $\otimes$and $c$). Supports framed knots, quantum traces.
*Examples:* modular tensor categories from Chern–Simons/CFT data.

**Choosing the right level (quick rule).**

- Order-sensitive physics/computation → plain or braided.
- Wire swaps as real gates (SWAP) → plain monoidal.
- Anyons, monodromy → braided (often ribbon).
- Classical structures, freely swappable factors → symmetric.

### 2.2 Coherence laws: triangle, pentagon, hexagon

**Triangle (unit coherence).**
$\alpha_{A,I,B} \circ (\mathrm{id}_A \otimes \lambda_B) = \rho_A \otimes \mathrm{id}_B$.
Stake: inserting/removing the unit on either side gives the same composite.

**Pentagon (associativity coherence).**
All rebracketings of $A \otimes B \otimes C \otimes D$ via $\alpha$ agree. Concretely, the two canonical ways from $((A \otimes B) \otimes C) \otimes D$ to $A \otimes (B \otimes (C \otimes D))$ coincide.
Stake: parenthesization does not matter up to the given isos.

**Hexagon (braid–associate coherence).**
Two canonical routes
$A \otimes (B \otimes C) \to (B \otimes C) \otimes A \to B \otimes (C \otimes A)$ and

$$A \otimes (B \otimes C) \to (A \otimes B) \otimes C \to (B \otimes A) \otimes C \to B \otimes (A \otimes C)$$

agree, and similarly for the "mirror" hexagon.
Stake: moving $A$ past $B \otimes C$ coheres with rebracketing.

**Why they matter.**
Triangle/pentagon/hexagon turn local structure maps into a **global discipline**: all diagrams built from $\alpha, \lambda, \rho$ (and $c$ if present) commute. You can then reason "as if" associative (and, when symmetric, "as if" commutative) without ambiguity.

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### 2.3 Algebraic backbones: Jacobi, associativity, unit laws

**Associativity (algebras).**
$(x * y) * z = x * (y * z)$. In categorical form, algebra objects in a monoidal category are monoids $m: A \otimes A \to A$, $u: I \to A$ satisfying associativity/unit diagrams with $\alpha, \lambda, \rho$.

**Unit laws.**
$u * x = x = x * u$. Categorically: $m \circ (u \otimes \mathrm{id}) = \lambda^{-1}$, $m \circ (\mathrm{id} \otimes u) = \rho^{-1}$.

**Jacobi (Lie-type backbones).**
$[x,[y,z]] + [y,[z,x]] + [z,[x,y]] = 0$. Guarantees the consistency of infinitesimal symmetries; equivalent to associativity of commutator in an associative envelope.
*Sanity check:* any proposed commutation relations or OPE brackets must satisfy Jacobi (or its homotopy/quantum analogs).

**Where these sit relative to coherence.**

- Triangle/pentagon/hexagon are **categorical coherence**.

- Associativity/unit/Jacobi are **algebraic laws** internal to the chosen monoidal ambient. Healthy theories satisfy **both layers**: the ambient wiring is coherent; the algebra living inside obeys its own backbone identities (or explicitly relaxed $A_\infty/L_\infty$versions with higher coherent homotopies).

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**2.4 Ordered vs exchange-permitting logics (Lambek, linear)**

**Exchange rule (what symmetry means in proofs).**
Given $\Gamma, A, B, \Delta \vdash C$, Exchange allows reordering to $\Gamma, B, A, \Delta \vdash C$.

- **With Exchange**: contexts are commutative up to permutation (proof theory models symmetric monoidal structure).
- **Without Exchange**: contexts are **ordered**; proofs encode word/causal/resource order (models plain or braided monoidal structure).

**Lambek calculus (ordered).**
No Exchange, often no Weakening/Contraction. Connectives \,/,·form a **residuated monoid**: $A \cdot B \le C \iff A \le C/B \iff B \le A \backslash C$.
*Use:* syntax/semantics of word order; process grammars.
*Semantics:* monoidal (not symmetric) categories of strings/processes.

**Linear logic (resource sensitivity).**

- **Classical/intuitionistic linear logic (LL/ILL)** typically **permits Exchange**, modeling symmetric monoidal closed (*-autonomous) categories; resources cannot be duplicated/erased without modalities (!, ?).
- **Non-commutative linear logics** (pomset logic, cyclic LL) **drop Exchange**, modeling braided/ordered monoidal categories; good for concurrency and causality.

**Takeaway for our heuristic.**

- If your target domain has **order-sensitive composition** (language word order, circuit wire order, causal processes), work **without Exchange** and in **non-symmetric** monoidal semantics; enforce coherence (cut-elimination, associativity of composition) but do **not** slip symmetry in.
- If the domain is **order-insensitive** (multiset-like resources), Exchange is appropriate and symmetric structure is justified.

**60-second checks (logic layer).**

1. Do proofs depend on hypothesis order? If yes, forbid Exchange.
2. Does your categorical semantics match: monoidal vs braided vs symmetric?
3. Are cut-elimination/coherence theorems available at the chosen level? Ensure they hold before adding further equations.

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**Section summary.**
Pick the **ambient monoidal level** that matches physical/semantic order-sensitivity; enforce **triangle/pentagon/hexagon**; ensure **associativity/unit/Jacobi** (or their homotopy analogs) inside. Only then consider **symmetry/Exchange**—and only where it is truly warranted.

### 3. Physics Domains Where Symmetry Fails but Coherence Must Hold

#### 3.1 Anyons / topological order: F- and R-symbols; braid vs swap

**Setting.** In 2+1D, exchanging identical excitations is governed by the braid group, not the symmetric group. The category-theoretic data are:

- fusion spaces V_{ab}^c,
- associators encoded by F-symbols: F^{abc}_d : (a*b)c -> a(b*c),
- braiding encoded by R-matrices: R^{ab}_c : a*b -> b*a (in channel c).

**Coherence (must-haves).**

- Pentagon (F moves): all recouplings of a*b*c*d coincide via products of F's.
- Hexagon (F with R): sliding a over b*c via braiding equals rebracketing-then-braiding coherently.

**Non-symmetry.** Generally R^{ba}_c o R^{ab}_c != id. Swapping is a *physical operation* with nontrivial phase or matrix, not a benign permutation. Coherence ensures consistent many-anyon recoupling; symmetry would erase braids and kill the physics.

**Practical audit.** Given candidate F, R, check: (i) pentagon equations, (ii) both hexagons, (iii) unit/duality constraints. Failure anywhere -> inconsistent theory.

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### 3.2 Chern–Simons TQFT & knot data: quantum R-matrix, ribbon structure

**Setting.** WZW/Chern–Simons theories yield modular/ribbon categories. Wilson lines carry representations; crossings and twists evaluate knots/links (Jones/RT invariants).

**Data.**

- Braiding via quantum R-matrix: R : V_a \otimes V_b -> V_b \otimes V_a from the quasi-triangular Hopf algebra (e.g., U_q(g)).
- Twist (ribbon) map theta_A : A -> A compatible with R and tensor.

**Coherence (must-haves).**

- Hexagon conditions for R with the associator.
- Ribbon axioms: theta_{A*B} = (theta_A \otimes theta_B) o (braid stuff), quantum trace well-defined.

**Non-symmetry.** Typically R^2 != id; crossings encode topological charge. Symmetric modeling (swap=id) would force link invariants to trivialize. Coherence ensures any diagrammatic reduction gives the same invariant.

**Practical audit.** Verify: (i) R solves the Yang–Baxter equation, (ii) hexagon/ribbon axioms hold, (iii) quantum traces close to scalars as required.

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### 3.3 Chiral CFT: OPE associativity and Moore–Seiberg equations

**Setting.** Chiral vertex operator algebras have operator product expansions (OPEs) that are associative but not generally commutative; analytic continuation around insertions produces monodromy.

**Data.**

- Fusion matrices F (change of channel for 4-pt blocks).
- Braiding/monodromy matrices R (analytic continuation around singularities).

**Coherence (must-haves).**

- Moore–Seiberg equations: pentagon for F and hexagon for (F,R), ensuring crossing symmetry (equivalence of s-, t-, u-channel decompositions).

**Non-symmetry.** Swapping insertion order can change the conformal block by a nontrivial R; demanding AB=BA is wrong. Coherence guarantees *crossing equality* after the correct F/R transforms.

**Practical audit.** Numerically or symbolically test 4-pt blocks: crossing equations; check that F,R solve the required consistency equations for the spectrum at hand.

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### 3.4 Non-Abelian holonomy & Berry/Wilczek–Zee phases: P exp(integral A)

**Setting.** In non-Abelian gauge transport or degenerate adiabatic subspaces, the holonomy is
U(gamma) = P exp( \int_gamma A ),
with P = path-ordering. Composition satisfies functoriality:
U(gamma2 * gamma1) = U(gamma2) U(gamma1).

**Coherence (must-haves).**

- Functoriality: concatenation -> multiplication; identity path -> id.
- Gauge covariance: U transforms by conjugation under gauge changes.

**Non-symmetry.** For two loops gamma1, gamma2, generally U(gamma1) U(gamma2) != U(gamma2) U(gamma1). Order matters; dropping P (or assuming commutation) corrupts phases and interferometry predictions.

**Practical audit.** Insist on P-ordering; verify functorial laws; test noncommuting loops (AB vs BA) in model potentials; ensure basepoint handling is explicit.

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### 3.5 Quantum circuits: SWAP as a gate; tensor not symmetric

**Setting.** Circuits form a strict (often) monoidal category: sequential composition (;) and parallel composition (tensor). Wire order is part of the type.

**Coherence (must-haves).**

- Associativity of composition and tensor; unit object I; interchange law: (f;g) ⊗ (h;k) = (f⊗h);(g⊗k).
- Structural isomorphisms (associators/unitors) treated coherently if not strict.

**Non-symmetry.**

- SWAP is a *nontrivial gate*, not the identity. Identifying A⊗B with B⊗A breaks typing and entanglement accounting.

- Controlled operations are order-sensitive: (CX on 1->2) vs (CX on 2->1) differ.

**Practical audit.** Check that any "relabeling" of wires is realized by explicit permutations/SWAP networks; forbid implicit symmetric identification in proofs and compilers; ensure monoidal functors to hardware respect wire order.

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### 3.6 Defects / bimodules: left/right actions and stacking order

**Setting.** Domain walls, interfaces, and topological defects compose by stacking (parallel) and concatenation (series). Algebraically: bimodules and their tensor products.

**Coherence (must-haves).**

- Associativity of stacking/concatenation up to specified isomorphisms.
- Unit defects behave neutrally (left/right unitors).
- Interchange between stacking and concatenation when both present (bicategorical coherence).

**Non-symmetry.**

- Left vs right action differ: M \otimes_R N depends on sidedness; generally M⊗_R N != N⊗_R M.
- Stacking order of defects changes boundary conditions and correlators; "swap for free" is invalid.

**Practical audit.** Model defects in a (bi)category of bimodules/functors; track orientations and sidedness; verify coherence diagrams (associators, unitors) commute; never identify swaps unless a symmetric structure is provably present.

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**Section takeaway.**
Across these domains, the *ambient wiring* is disciplined by coherence (pentagon/hexagon, functoriality, bicategorical laws), while *swaps remain physical operations* with content (braids, monodromy, order of holonomies, wire permutations, left/right actions). Treat swaps as identities and you erase real phenomena; enforce coherence and you get a consistent, predictive calculus without overassuming symmetry.

## 4. The Heuristic: Coherence-First Fault Detector

### 4.1 Pick the correct monoidal ambient: symptoms of a wrong choice

**Decision tree.**

- Do swaps change observables (order, phase, topology, causality)? → Use plain or braided monoidal (not symmetric).
- Are crossings physically meaningful (braids, R-matrices)? → Braided (often ribbon).
- Are factors freely permutable without gates or phases? → Symmetric.

**Symptoms you picked the wrong ambient.**

- Predicted invariants collapse (e.g., link invariants trivialize when you model braids with swap = id).
- "Exchange" silently added to a logic/type system that models order/causality (Lambek, non-commutative LL).
- Circuit models that never pay for SWAP, or that "relabel wires" by fiat.
- Holonomy/adiabatic transport written without path ordering; loop order "doesn't matter."

**Fix.** State the ambient level up front (plain/braided/symmetric; strict/weak; $E_n$). Explain why that level matches the physics.

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### 4.2 Write the smallest necessary diagrams; demand they commute

**Minimal diagrams to check (by domain).**

- **Monoidal basics:** triangle and pentagon (always).
- **Braided/ribbon:** both hexagons; twist axioms if ribbon.
- **VOA/CFT:** Moore–Seiberg (pentagon + hexagon) = crossing equations.
- **Gauge/Berry:** functoriality of parallel transport; basepoint naturality.
- **Algebras:** associativity/unit diagrams; Jacobi for brackets; interchange laws for actions.

**Micro-audit pattern.**

1. Identify the smallest nontrivial composite (3–4 objects/insertions/segments).

2. Draw the two canonical routes between the same endpoints.
3. Compute both sides explicitly (symbolic or numeric).
4. Require equality (or specified iso/homotopy) — and record it.

**Anti-pattern.** "It's obvious by symmetry." If the domain is order-sensitive, that phrase is a bug, not a proof.

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#### 4.3 Time/path ordering as non-negotiable

**Rule.** In non-Abelian settings, replace exp(∫A) by P exp(∫A); replace products by time-ordered products T{…}; replace naive loop concatenations by functorial composition from a path groupoid.

**Fast tests.**

- **Two-loop test:** compute U(γ1)U(γ2) vs U(γ2)U(γ1). If they're equal generically, you either chose an Abelian limit or dropped ordering.
- **Interferometer test:** does the prediction depend on traversal order or encirclement direction? It should, unless the sector is Abelian.
- **Circuit test:** does your compiler emit explicit SWAPs for wire permutations? If not, it's smuggling symmetry.

**Documentation norm.** Every time you remove P or T, specify the regime (Abelian, commuting subalgebra, asymptotic, mean-field) that justifies it.

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#### 4.4 Locality and microcausality: where commutators may vanish (and where not)

**Microcausality baseline (QFT).**

- Spacelike separation: [A(x), B(y)] = 0 (or { , } for fermions).
- Elsewhere: do **not** assume vanishing commutators.

**Common failure modes.**

- Imposing AB = BA "for convenience" beyond spacelike separations.
- Treating equal-time algebras as commuting across interacting sectors.

- Ignoring domain issues (distributions): products at coincident points require renormalized definitions; naive commutation can be meaningless.

**Coherence framing.**

- Locality = a *structured* vanishing of commutators in a specific region of configuration space.
- The "coherence check" is whether your algebra and propagators enforce exactly those vanishings and no more (e.g., correct support properties, correct analytic wavefront set).

**Quick audit.** For a proposed field algebra or effective action,

1. compute [A, B] on simple wavepackets with supports arranged spacelike vs timelike;
2. check the OPE reproduces the same constraints (associative, but not fully commutative);
3. verify that any additional vanishings are derived, not assumed.

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#### 4.5 Anomaly accounting as coherence of symmetry actions

**Viewpoint.** An anomaly is a **coherence failure** in implementing a symmetry across compositions (gauge transformations, gluing of patches, or mapping class actions). If the anomaly does not cancel (by inflow, counterterms, or spectrum), your symmetry action cannot be made consistent.

**Checklist.**

- **Gauge/global anomalies:** compute variation of the effective action; require exact cancellation.
- **'t Hooft anomalies:** track background gauge fields through compactifications/gluings; conserved but nontrivial anomaly = constraint on IR, not an error — document it.
- **Modular/categorical anomalies:** verify pentagon/hexagon and modular S,T representations close projectively as required; fix with appropriate central extensions/twists if physical.

**Red flags.**

- Declaring a symmetry that cannot be realized on the Hilbert space (projective obstruction not addressed).
- Topological order data (F,R) that violate pentagon/hexagon yet are used to compute braiding phases.

- Chern–Simons level choices that break large-gauge invariance on nontrivial manifolds.

**Resolution playbook.**

1. Identify the anomaly (cohomology class, index, obstruction).
2. Choose the mechanism: cancel (additional matter, counterterm), match (anomaly inflow/boundary), or admit (as a protected IR constraint).
3. Re-run minimal coherence diagrams with the fix in place.

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**Section takeaway**

Treat **coherence** as the non-negotiable backbone: pentagon/hexagon, functoriality, microcausality, Jacobi, anomaly cancellation. Treat **symmetry** as earned: include it only where physics proves swaps, permutations, or equal-time commutators are benign. Most “too-good-to-be-true” shortcuts fail one of these five checks.

## 5. Failure Modes Catalog (With Minimal Tests)

### 5.1 “Symmetric tensor by habit” in anyon toy models

**Symptom.** The model treats tensoring of charges as if $A \otimes B = B \otimes A$ by definition (swap = id), or never supplies R- and F-data.

**Why it’s bad.** In 2+1D, exchanges are braids, not swaps. Without nontrivial R and coherent F, multi-anyon recoupling is ill-defined and all braid-sensitive observables collapse.

**Minimal test (3–step).**

1. **Data check.** Demand explicit F^{abc}_d and R^{ab}_c. If absent, fail.
2. **Pentagon/hexagon.** Evaluate the pentagon and both hexagons on a smallest nontrivial fusion pattern (e.g., three Fibonacci anyons). If any equation fails numerically/symbolically, the theory is inconsistent.
3. **Braid nontriviality.** Compute R_{B,A} R_{A,B}. If it always equals id, you’ve reduced to a symmetric theory; for anyons it should be nontrivial.

**Quick remediation.** Choose a known consistent set (e.g., Fibonacci/Ising modular data), or show your sector is abelian (then symmetry is justified and R^2 = id).

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### 5.2 Dropped path ordering in interferometry/transport

**Symptom.** Holonomies written as $\exp(\int A)$ instead of $P \exp(\int A)$, or claims that loop order does not affect outcomes in a non-Abelian setting.

**Why it's bad.** For noncommuting A(s), order matters: $U(\gamma 2)U(\gamma 1) \neq U(\gamma 1)U(\gamma 2)$ in general. Omitting P erases physical phases and matrix structure.

**Minimal test (worked micro-example).**

- Pick a 2×2 toy gauge potential along piecewise-constant segments:
  $A_1 = a\, \sigma_x$ on segment 1, $A_2 = b\, \sigma_z$ on segment 2 ($\sigma_i$ Pauli).
- Compute
  $U_{12} = \exp(b\, \sigma_z L_2) \exp(a\, \sigma_x L_1)$,
  $U_{21} = \exp(a\, \sigma_x L_1) \exp(b\, \sigma_z L_2)$.
- If $U_{12} == U_{21}$ in your derivation, you implicitly forced $[\sigma_x, \sigma_z] = 0$ (false). Path ordering is required.

**Quick remediation.** Restore P; state regimes where commutation holds (abelian subgroup, adiabatic projection to 1D subspace, etc.) and verify them.

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### 5.3 Non-crossing-symmetric correlators in CFT

**Symptom.** Four-point correlators are built in one channel and asserted valid globally without checking crossing; or operator insertions are treated as commuting.

**Why it's bad.** The OPE is associative (coherence) but not generally commutative. Crossing (Moore–Seiberg) ensures s- and t-channel decompositions match after F/R transforms.

**Minimal test (scalar primary 4-pt).**

1. Choose spectrum $O_i$ and OPE coefficients $C_{ij}^k$ (or conformal blocks).
2. Compute the reduced 4-pt $G_s(z)$ in the s-channel and $G_t(1-z)$ in the t-channel with the same data.
3. **Crossing check.** Verify $G_s(z) = G_t(z)$ for z in (0,1) (within numerical tolerance) after applying the appropriate F/R change-of-basis. Failure ⇒ inconsistent (or incomplete) data.

**Quick remediation.** Fit or import consistent F, R, and $C_{ij}^k$ (e.g., minimal models, Liouville with DOZZ) and re-check.

### 5.4 Circuit models that identify SWAP with id

**Symptom.** Compiler or formal model "relabels wires" for free; proofs conflate $A \otimes B$ with $B \otimes A$; no SWAP gates appear in hardware maps.

**Why it's bad.** Wire order is part of the type. SWAP is a nontrivial gate with depth/cost; erasing it miscounts resources and can break entanglement tracking.

**Minimal test (permutation synthesis).**

- Target permutation on 3 wires: (1,2,3) -> (3,1,2).
- **Expect:** At least 2 SWAPs (or equivalent sequence) appear in the compiled circuit on hardware without native wire crossovers.
- **Check:** Does your tool produce an explicit SWAP (or nearest-neighbor swap network) and update qubit placements? If not, it's smuggling symmetry.

**Entanglement sanity check.** Prepare Bell on wires (1,2). "Relabel" to measure on (2,3). If your model claims identical statistics without routing or teleportation, it's assuming swap=id.

**Quick remediation.** Treat circuits as (strict) monoidal with explicit permutation gates; ensure back-ends implement routing (SWAP insertion, teleportation, or movement primitives).

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### 5.5 Algebraic proposals violating Jacobi/associativity

**Symptom.** New commutation relations or brackets are posited, but Jacobi is not checked; or product laws are given that fail associativity/unit diagrams.

**Why it's bad.** Jacobi is the coherence of infinitesimal symmetry composition; associativity/unit are the coherence of multiplication. Violations break the algebra's internal consistency.

**Minimal tests.**

**(A) Jacobi for structure constants.**
Given $[T_a, T_b] = f^c_{ab} T_c$ with $f^c_{ab} = - f^c_{ba}$, compute
$J^e_{abc} = f^d_{ab} f^e_{dc} + f^d_{bc} f^e_{da} + f^d_{ca} f^e_{db}$.
Require $J^e_{abc} = 0$ for all a,b,c,e. Any nonzero ⇒ not a Lie algebra.

**(B) Associativity for algebras in a monoidal ambient.**
Given $m: A \otimes A \to A$ and unit $u: I \to A$, check:
$m \circ (m \otimes id) = m \circ (id \otimes m) \circ \alpha$,
$m \circ (u \otimes id) = \lambda^{-1}$, $m \circ (id \otimes u) = \rho^{-1}$.
In a strict setting, these reduce to $(xy)z = x(yz)$ and $ex = x = xe$.

**(C) OPE/Jacobi-like constraints.**
For proposed OPE coefficients C_{ij}^k and singular terms, test associativity on 3-point expansions to 4-pt functions; inconsistent channel recombination signals a Jacobi/coherence failure in disguise.

**Quick remediation.**

- If Jacobi fails: reinterpret as an L_infty/A_infty structure and supply higher brackets with homotopy-Jacobi; or adjust f's to a known Lie algebra.
- If associativity fails unintentionally: restrict to a subalgebra, or introduce associators with explicit coherence (quasi-Hopf, Drinfeld associator) and upgrade the ambient to match.

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**One-page audit crib (use on any candidate model)**

1. **Ambient choice:** monoidal vs braided vs symmetric? Is that justified by observables?
2. **Core diagrams:** triangle, pentagon (always); hexagon (if braided/ribbon); crossing (if CFT/VOA).
3. **Ordering:** is P/T-ordering present where operators/potentials don't commute?
4. **Locality:** do commutators vanish only where microcausality allows?
5. **Algebra backbones:** associativity/unit/Jacobi checked? If not, supply higher-coherence variants with proofs.
6. **Anomalies:** computed and canceled/matched/acknowledged?
7. **Resource realism (circuits):** are swaps/permutations paid for?

If any item fails on the smallest nontrivial example, stop and repair before scaling up.

## 6. Worked Micro-Audits

### 6.1 Turning a qualitative claim into a diagram check

**Pattern (4 steps).**

1. **Claim -> typing.** Write the claim as a typed composite. Example: "Sliding A past B*C is harmless.*" *Translate to two routes*
   *r1: A*(B*C) --c_{A,B*C}--> (B*C*)A --alpha--> B(C*A)
   r2: A*(B*C) --alpha--> (A*B)*C --c_{A,B}id--> (BA)C --alpha--> B(A*C) --id*c_{A,C}--> B*(C*A).

2. **Smallest instance.** Choose the smallest nontrivial arity (usually 3 or 4 objects; for OPE, 4 insertions).
3. **Compute both sides.** Symbolically (matrix products, F/R moves) or numerically (plug concrete reps).
4. **Decision.** Pass if $r1 == r2$ (exactly, or up to the specified iso); fail otherwise. Record the diagram and equality used.

**Notes.**

- If the ambient is braided, use hexagon; if symmetric, $c^2 = id$; if plain monoidal, there is no c, so any "swap" claim is already suspect.
- Prefer the 4-object pentagon and the 3-object hexagon as your default "unit tests."

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### 6.2 A 1-page audit for braiding data (F,R; pentagon/hexagon)

**Inputs.** Fusion rules $N_{ab}^c$, F-symbols $F^{abc}_d$, R-symbols $R^{ab}_c$, unit 1, duals $a^*$.

**Goal.** Check that the modular/anyonic data define a coherent braided (ribbon, if twist given) tensor category on the nose.

**Checklist.**

- **(A) Normalizations.**
    1. Identity object: $F^{a1b}_c = F^{1ab}_c = id$, $R^{a1}_a = R^{1a}_a = id$.
    2. Duality sanity: $N_{a a^*}^1 >= 1$; evaluate unit and counit triangles if provided.
- **(B) Pentagon (associativity coherence).**
  For each admissible a,b,c,d,e, verify
  $F^{abc}_e F^{a e d}_f = (id * F^{bcd}_f) F^{a b f}_g (F^{a b c}_e * id)$
  in component form on the fusion spaces. (Write both routes explicitly as products of F's.)
- **(C) Hexagon (braid-assoc coherence).**
  For each a,b,c and channel d, verify both hexagons, e.g.
  $(id * R^{bc}_e) F^{a b c}_d (R^{a b}_e * id) = F^{b a c}_d R^{a c}_d F^{a b c}_d$,
  and its mirror (replace R by $R^{-1}$ and swap roles accordingly).
- **(D) Ribbon (if twist theta given).**
    1. Balancing: $\theta_{a*b} = (R^{ba} R^{ab}) (\theta_a * \theta_b)$.

2. Quantum trace consistency (left = right), if categorical traces are used.

- **(E) Nontriviality sanity.**
  Check R^{ba}_c R^{ab}_c != id generically (unless your theory is explicitly abelian).
  Compute topological spins from theta_a and verify expected relations with S,T if modular data are supplied.

**Pass/Fail.** All equalities hold (symbolically or within numeric tolerance). Any failure pinpoints an incoherence: fix F,R (often by solving the pentagon first, then hexagon).

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**6.3 A 1-page audit for holonomy (functoriality, basepoint, ordering)**

**Setting.** Non-Abelian parallel transport or Wilczek–Zee phases: U(gamma) = P exp( \int_gamma A ).

**Inputs.** Connection A(x) valued in a matrix Lie algebra; a set of piecewise-smooth paths {gamma_i}; a basepoint x0.

**Checklist.**

- **(A) Path ordering present.**
  All exponentials along noncommuting segments use P. If any derivation drops P, require an Abelian justification (e.g., [A(s),A(t)]=0 along gamma) and verify it.
- **(B) Functoriality (concatenation).**
  For gamma2 * gamma1 (first gamma1, then gamma2),
  U(gamma2 * gamma1) = U(gamma2) U(gamma1).
  **Test:** Choose two segments with noncommuting constant A1, A2 (e.g., a*sigma_x, b*sigma_z). Compute both sides explicitly. They must match.
- **(C) Inverses and retracing.**
  U(gamma^{-1}) = U(gamma)^{-1}; U(gamma * gamma^{-1}) = id.
  **Test:** Numerically integrate a loop and its reverse; check numerical inverse within tolerance.
- **(D) Basepoint covariance.**
  Changing basepoint via a path eta gives conjugation:
  U_{x1}(gamma) = U(eta) U_{x0}(eta^{-1} * gamma * eta) U(eta)^{-1}.
  **Test:** Evaluate both sides on a simple rectangle with two basepoints.
- **(E) Gauge covariance.**
  Under g(x): A -> gAg^{-1} - (dg)g^{-1}, check

U(gamma) -> g(x_1) U(gamma) g(x_0)^{-1}.
**Test:** Pick a nontrivial g(x) (e.g., rotation about sigma_y with angle linear in x), verify conjugation numerically.

- **(F) Noncommutation sanity.**
  For two loops L1, L2 based at x0, compute U(L1)U(L2) vs U(L2)U(L1). They should differ generically. If equal in your derivation, you likely lost ordering.

**Pass/Fail.** All functorial and covariance properties hold; order sensitivity appears when expected.

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### 6.4 A 1-page audit for circuits (tensor order, wire discipline)

**Setting.** Quantum circuits as (strict) monoidal: sequential ';' and parallel '⊗' (tensor). Wire order is part of the type.

**Inputs.** Gate set with types (domains/codomains), a target permutation of wires, and a simple entanglement experiment.

**Checklist.**

- **(A) Typing sanity.**
  Ensure the compiler/type checker rejects ill-typed identifications like A⊗B == B⊗A. SWAP must be a gate with type (q_i,q_j)->(q_j,q_i).
- **(B) Explicit permutations.**
  For a nontrivial wire permutation p on n wires:
  1. Synthesize a SWAP network (e.g., nearest-neighbor swaps) or teleportation route.
  2. Verify the compiler inserted these gates.
     **Test:** Target (1,2,3)->(3,1,2) on linear connectivity; expect at least two SWAPs (or equivalent).
- **(C) Interchange law.**
  Check (f;g) ⊗ (h;k) == (f⊗h);(g⊗k).
  **Test:** Build both sides and compare unitaries; they must match exactly.
- **(D) Entanglement routing sanity.**
  Prepare Bell on wires (1,2). Without SWAP/teleportation, measurements on (2,3) should *not* show Bell correlations.

**Test:** If your tool "relabels" wires with no gates yet preserves correlations, it has smuggled symmetry.

- **(E) Cost visibility.**
  Count depth/2-qubit gate cost before vs after required permutations. Absence of a cost delta under nontrivial p indicates an implicit identification of $A \otimes B$ with $B \otimes A$.
- **(F) Backend mapping.**
  Ensure monoidal functor to hardware preserves order: hardware constraints (coupling graph) force explicit routing.
  **Test:** Compile to a specific device; verify inserted SWAPs match graph constraints.

**Pass/Fail.** All reorderings are paid via explicit gates; interchange holds; entanglement does not teleport "for free."

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**Section takeaway.**
Each micro-audit distills a sweeping structural question ("is order benign?") into a smallest nontrivial diagram or equality you can actually compute. If it fails there, stop—your model has a coherence/symmetry bug that will only grow at scale.

## 7. Bridges to CT/HoTT and Higher Algebra

### 7.1 E_1 / E_2 / E_infty: coherence ladders vs true symmetry

**Heuristic picture.**

- **E_1 (A_infty)**: associative up to coherent homotopies; *no* commutativity. Think "plain monoidal." Multiplications m_2, m_3, ... witness associativity via higher cells.
- **E_2**: introduces a *braiding up to homotopy*. Swapping factors is meaningful and governed by coherent "half-twists." Think "braided monoidal."
- **E_infty**: fully commutative up to all higher homotopies in a contractible way; braiding squares to the identity at every coherence level. Think "symmetric monoidal."

**Why it matters.**
Choosing E_n is choosing how far "order-independence" is licensed:

- E_1: order matters; coherence ensures rebracketing is safe.
- E_2: crossings carry content (braids), but are coherently manageable.

- E_infty: order is benign; you may treat swaps as trivial (up to the unique coherent homotopy).

**Examples.**

- Loop spaces: Omega^n X are naturally E_n-algebras; for n>=2, the Eckmann–Hilton phenomenon forces higher homotopy groups pi_k (k>=2) to be abelian: emergent symmetry from higher coherence.
- Algebraic topology toolchain: E_1-ring spectra (associative), E_2 (braided-like multiplicative behavior), E_infty (commutative ring spectra). The multiplicative structure you can *push through* (Kunneth, power ops) depends on n.

**Diagnostic use.**
If physics shows genuine braid/monodromy effects, stop at **E_2** (or below). Modeling such a domain with **E_infty** structure quietly erases the phenomenon.

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### 7.2 Coherence as "all paths agree" proofs; symmetry as an extra axiom

**HoTT lens.**

- Equality is a path. Coherence means: for a specified diagram, *any two composites between fixed endpoints are connected by a higher path.*
- In practice: you encode associators/unitors/braidings as paths; the triangle/pentagon/hexagon become higher equalities between paths of paths (2-paths), etc.

**Univalence and transport.**

- Univalence turns equivalences into equalities (paths), letting you *transport* structure along equivalences. This is coherence-friendly: you can state "these two routes build equivalent objects, hence equal (by univalence) once data are transported."
- None of this forces commutativity. It only says: once you fix the structure (monoidal, braided, ...), *all required paths commute up to higher homotopy*.

**Where symmetry really enters.**

- Symmetry corresponds to an *extra inhabitant* witnessing that the braiding is an involution: c_{A,B}; c_{B,A} ; (coherence cells) = id. In HoTT terms: you assume a specific higher path that collapses braiding to a trivial swap and that this choice coheres at all higher levels (an E_infty-like contractibility).

**Reviewer's trick.**

- Ask: "Is the proof using only coherence (paths between composites) or did it sneak in a symmetry axiom (a special 2-path identifying swap with identity)?"
- If the model's predictions require monodromy, any hidden symmetry axiom will surface as a contradiction in a small hexagon or crossing square.

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**7.3 Directed homotopy / process models: when non-symmetry is the feature**

**Motivation.**
Causality, resources, and concurrency are *directional*: reordering steps is not neutral. The right semantics are **non-symmetric** and often **non-invertible**.

**Frameworks.**

- **d-Spaces / directed homotopy**: paths have orientation; concatenation is defined, reversal need not be. Coherence: functoriality of concatenation; *no* symmetry of paths.
- **Double categories / bicategories of processes**: horizontal = time, vertical = resource/channel; interchange is coherent, but swapping axes is not symmetric.
- **Profunctors / optics / lenses**: left/right actions differ; composition is coherent but order-sensitive.
- **Non-commutative linear logics (no Exchange)**: proof contexts record order/resources; cut-elimination is a coherence theorem without symmetry.

**Physics-leaning instances.**

- **Quantum circuits**: wires carry types; SWAP is a gate. Category: strict monoidal (often only planar), sometimes braided if physical crossings are meaningful.
- **Open systems / causal networks**: morphisms that forget information are not invertible; symmetric closure would misrepresent arrow-of-time constraints.
- **Non-Abelian transport**: path ordering as primitive; directed concatenation is coherent, loop swaps are not.

**Design rule.**
If the domain encodes: (i) a before/after relation, (ii) resource consumption/production, or (iii) nontrivial monodromy, then **directed/ordered models are a feature, not a defect**. Build coherence (associativity, unit, interchange), *do not* impose symmetry. Doing so converts real effects (race conditions, path-dependence, braids) into artifacts.

**Bridge back to CT/HoTT.**

- CT supplies the ambient (monoidal/braided/bicategorical) and the coherence theorems.
- HoTT supplies the proof language where "all required composites agree up to higher paths," while allowing directionality by choosing the right identity/path structure (e.g., semi-simplicial or directed variants).
- Higher algebra (E_n) tells you exactly *how much* commutation you are allowed. Pick n by physics, not by convenience.

**One-line takeaway.**
CT/HoTT let you enforce *just enough* agreement (coherence) for composition to be meaningful while withholding *extra* agreement (symmetry) unless the world earns it.

## 8. Experimental Implications

### 8.1 Designing order-sensitivity probes (AB vs BA, loop sequencing)

**Core idea.** Build paired experiments that differ only by operation order. If the domain is non-symmetric, AB != BA should be *observable*.

**Templates.**

- **Holonomy (non-Abelian transport).**
  Protocol: choose two loops L1, L2 based at x0. Measure U(L1)U(L2) vs U(L2)U(L1) via interferometry on the same sample. Observable: fringe shifts or state-tomography matrices that disagree beyond error. Control: Abelian sector where loops commute (should null the effect).
- **Circuit order.**
  Protocol: prepare |00>, run AB (e.g., A = CX(1→2), B = H on qubit 1) vs BA; tomograph output. Observable: different Pauli expectation vector; depth overhead when reordering requires SWAPs. Control: gates chosen from a commuting set (e.g., Z-rotations on distinct qubits) should match.
- **Anyon braids.**
  Protocol: implement braid word b1 b2 vs b2 b1 on the same fusion channel. Observable: distinct outcome probabilities after fusion or distinct extracted topological spins.
  Control: abelian anyon sector where R^2 = id.

**Design notes.**

- Keep everything identical (state prep, noise, readout); only swap the order.
- Use the smallest nontrivial instance (two loops; two gates; three anyons).
- Pre-register pass/fail regions from theory with error bars.

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### 8.2 Interference and monodromy signatures

**Core idea.** Monodromy = phase/matrix acquired when analytically continuing or encircling singular structure. Detect it with controlled encirclements and path homotopies.

**Signatures.**

- **Encirclement direction.**
  Clockwise vs counterclockwise loops give complex-conjugate (or inverse) holonomies. Observable: opposite fringe shifts; nonidentical tomographies.
- **Linked worldlines/defects.**
  Interference visibility or phase depends on whether paths link a defect line or not. Observable: stepwise phase jump when you thread or unthread a loop through a flux/anyon.
- **CFT-like crossing.**
  In platforms emulating chiral dynamics (cold atoms, photonics), compare "s-channel" vs "t-channel" recombination of four excitations. Observable: equal totals *after* the correct change-of-basis; disagreements until the Moore–Seiberg transform is applied indicate nontrivial monodromy, not failure.
- **Braid word reduction.**
  Same end-to-end permutation but different braid representatives produce different matrix elements unless reduced by hexagon relations. Observable: distinguishable outcome statistics map directly to R and F data.

**Practical controls.**

- Shrink loops continuously to test homotopy invariance within a topological sector.
- Insert/removing a known Abelian flux to calibrate the phase meter.
- Run "no-link" baselines to separate geometric from topological phase.

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### 8.3 Robustness checks: what would symmetry wrongly predict?

**Use symmetry as a straw-man null.** Write the predictions of a (wrong) symmetric model and check where they fail most starkly.

**Null (symmetric) predictions and decisive refuters.**

- **Commutation null.**
  Prediction: AB = BA.
  Refuter: measured distance D(AB, BA) in state space (e.g., trace distance of output density matrices) exceeds noise floor.
- **Swap-for-free null (circuits).**
  Prediction: relabeling wires has zero cost and preserves correlations.
  Refuter: resource audit shows added SWAP depth; Bell-pair routing without SWAP/teleportation fails.
- **No-monodromy null.**
  Prediction: encircling a singularity does not change interference.
  Refuter: phase shift scales with winding number; direction reversal flips the sign or inverts the matrix.
- **Crossing-triviality null (CFT).**
  Prediction: s-channel amplitude equals t-channel without basis change.
  Refuter: equality only after applying the F/R transform; pre-transform discrepancy is systematic.
- **Global abelianization null (anyons).**
  Prediction: R_{b,a} R_{a,b} = id for all channels.
  Refuter: extracted topological spins and S,T data violate that identity; link invariants are nontrivial.

**Robustness playbook.**

1. **Precompute margins.** Simulate both the coherent (order-sensitive) and symmetric-null models to set expected gaps vs noise.
2. **Vary one knob.** Loop area, braid length, gate angles—look for nonlinearity characteristic of non-commutation (e.g., BCH cross terms).
3. **Cross-platform replication.** Run the same AB vs BA on different hardware/material platforms; a consistent gap argues against platform-specific artifacts.

4. **Blind swaps.** Randomize AB/BA order in acquisition; decode only after locking analysis to prevent confirmation bias.

**Bottom line.** An experiment "passes" coherence and "fails" symmetry when: (i) AB vs BA differences persist above noise, (ii) encirclement monodromy appears with the right winding/direction dependence, and (iii) any equalities hold only *after* the mandated change-of-basis (F/R, gauge conjugation, routing swaps) rather than by naïve swapping.

## 9. Limits, Caveats, and False Positives

### 9.1 Systems that effectively abelianize

**Mechanisms that hide order-sensitivity.**

- **Small-parameter regimes.** If $\|[A,B]\| \ll |A||B|$, Baker–Campbell–Hausdorff terms are below noise; AB and BA look equal. (E.g., weak fields, short loops, near-commuting Hamiltonians.)
- **Projection to 1D subspaces.** Degeneracy lifting or strong symmetry can reduce non-Abelian transport to an effective U(1).
- **Decoherence/averaging.** Environmental dephasing, disorder or ensemble averaging can wash out monodromy phases; measured observables appear commutative.
- **Stroboscopic/effective frames.** Floquet or interaction-picture descriptions can "move" noncommutation into the frame; naive lab-frame comparisons then look symmetric.

**Guardrail.** Treat "AB ≈ BA" as *instrumental*, not structural. Report bounds: $\|[A,B]\| \le \varepsilon$ with $\varepsilon$ tied to time/length scales, temperature, and readout bandwidth.

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### 9.2 Emergent symmetries and scaling limits

**Where symmetry is real—but only in the IR.**

- **Renormalization-group flow.** Non-symmetric microscopics can flow to symmetric fixed points (e.g., abelianization of braids at long scales; rotational symmetry emerging from lattices).
- **Hydrodynamic/mean-field limits.** Collective variables commute approximately; Poisson brackets linearize; effective field theories hide microscopic braid/ordering data.

- **Large-N or central-limit effects.** Matrix commutators average to zero in certain sectors; fluctuations shrink as 1/N.

**Pitfall.** Declaring the emergent symmetry as a **microscopic axiom**. That erases domains where the symmetry has not yet emerged (short scales, high energies, finite-N).

**Practice.** Always tag symmetry with its **regime of validity** (scale, N, temperature). Design probes that can leave the IR (short loops, fast pulses, finite-size scaling) to re-expose non-symmetric structure.

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### 9.3 When coherence checks are necessary but not sufficient

**Passing coherence ≠ being the right theory.** You can satisfy pentagon/hexagon, functoriality, Jacobi, microcausality—and still be wrong or incomplete.

**Failure modes beyond coherence.**

- **Wrong universality class.** Consistent anyon data (F,R) could describe a different phase than the sample realizes. Coherence only says "internally consistent," not "empirically correct."
- **Parameter misfit.** Crossing equations can hold for many spectra; only one matches measured dimensions/OPE coefficients.
- **Anomaly *matching* without dynamics.** IR anomaly constraints can be satisfied by multiple distinct theories; coherence doesn't pick the realized one.
- **Duality confusion.** Two dual descriptions both coherent but not both applicable in a given regime; misapplied dual variables can fake or hide order-sensitivity.
- **Regularization/scheme artifacts.** A calculation can be coherent in one scheme yet misrepresent physical observables (contact terms, counterterm choices).
- **Numerical impostors.** Finite precision can make commutators "zero," crossing "pass," or hexagons "close" within tolerance; overfitting can tune data to tests without predictive power.

**Sufficiency add-ons (what to layer on top).**

- **Predictive sharpness.** Out-of-sample predictions with quantified error bars (not just postdictions).
- **Parameter economy.** Prefer models that pass coherence with fewer arbitrary knobs (Occam).

- **Robust observables.** Choose quantities invariant under scheme/duality (e.g., topological spins, modular S,T eigenvalues, Wilson loop spectra).
- **Scale tests.** Verify claimed symmetry/emergence across scales; look for crossover behavior predicted by the model.
- **Ablation studies.** Remove or vary specific structural assumptions (e.g., turn off braiding, set R^2 = id) and confirm predicted qualitative failures appear.
- **Reproducibility across platforms.** Same coherence-sensitive signature in distinct materials/devices argues against artifact.

**Bottom line.**

- **Coherence checks are a *gate*.** Fail them → stop; pass them → proceed to empirical fit, regime tagging, and robustness analyses.
- Treat symmetry as *earned* and *regime-bound*.
- Report limits quantitatively so "effective abelianization" is documented as a scale-dependent phenomenon, not mistaken for a fundamental law.

## 10. Conclusion: Measure Twice (Coherence), Cut Once (Symmetry)

### 10.1 What this heuristic screens out

- **Swap-for-free mistakes.** Models that quietly identify $A \otimes B \equiv B \otimes A$in order-sensitive domains (anyons, circuits, defects), erasing monodromy, routing costs, or left/right actions.
- **Ordering amnesia.** Derivations that drop $P$- or $T$-ordering (holonomy, time-ordered products) and thus predict commuting effects where none exist.
- **Unproven "obvious by symmetry."** Arguments that rely on symmetric reasoning in braided/plain monoidal environments, or use Exchange in ordered logics.
- **Incoherent recoupling.** Anyon/CFT data that fail pentagon/hexagon/crossing; algebraic proposals that fail Jacobi or unit/associativity diagrams.
- **Anomaly blind spots.** Symmetry stories that cannot be consistently implemented (uncanceled gauge/global/modular anomalies).

- **Tooling illusions.** Compilers and simulators that relabel wires without SWAP/teleportation, or numerical pipelines that "pass" coherence due to tolerance, not truth.

Result: the heuristic eliminates **category errors** (using the wrong ambient structure) and **consistency errors** (failing the ambient's coherence), before one argues dynamics or fit.

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### 10.2 A short checklist for reviewers and builders

**Ambient & typing**

1. What is the ambient structure—plain, braided, symmetric (ribbon?)—and why is it physically justified?
2. Are wires/paths/defects **typed** so that permutations require explicit morphisms (SWAP, braids, rebase)?

**Minimal coherence**
3) Triangle + pentagon verified? (Always.)
4) Hexagon (and ribbon twist) verified if braiding/twist is used?
5) For VOA/CFT: Moore–Seiberg (crossing) checked on 4-point blocks?
6) For algebras/brackets: associativity/unit/Jacobi (or $A_\infty/L_\infty$ with explicit higher coherences) satisfied?

**Ordering & locality**
7) Is $P$- or $T$-ordering present wherever operators/potentials don't commute? Regime stated when it can be dropped?
8) Microcausality/locality honored: commutators vanish **only** where they should?

**Anomalies & invariance**
9) Gauge/global/modular anomalies computed and canceled/matched?
10) Basepoint/gauge covariance (holonomy) demonstrated; categorical traces well-defined (ribbon)?

**Resource realism (for circuits/systems)**
11) Explicit permutation/routing costs accounted for; interchange law holds; entanglement isn't teleported "for free."

**Experimental posture**
12) AB vs BA and encirclement probes specified; symmetric-null predictions written down as falsifiable baselines.

If an item fails on the **smallest nontrivial diagram/test**, stop and repair before scaling.

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### 10.3 Open problems and standardization prospects

**(A) Universal "coherence test suites."**

- **Anyon/CFT pack:** canonical JSON for $(F, R, \theta)$with auto-checks (pentagon/hexagon/crossing), tolerance policies, and human-readable counterexamples on failure.
- **Holonomy pack:** reference geometries and gauge fields with expected $U(\gamma)$matrices; standardized $P$-ordering integrators and gauge-covariance checks.
- **Circuit pack:** device-graph–aware SWAP benchmarks; interchange-law validators; entanglement-routing litmus tests.

**(B) Ordering-aware intermediate representations.**

- A minimal, category-theoretic IR where tensor order is explicit, permutations are morphisms, and coherence constraints are first-class. Target for compilers, notebooks, and proof assistants.

**(C) Reporting standards (Methods sections).**

- Mandatory "Ambient & Coherence" box: state monoidal level, list verified diagrams, specify where symmetry/Exchange is assumed, and the scale/regime that justifies it.
- Mandatory "Ordering Justification" line: where $P/T$-ordering is used or safely omitted, with a bound on $\| [A, B] \|$.

**(D) Numeric robustness & certification.**

- Interval-arithmetic or proof-assistant–backed certificates for pentagon/hexagon/crossing checks; machine-readable artifacts to accompany publications.
- Standardized tolerance profiles to prevent "false passes."

**(E) Bridging $E_n$structure to experiments.**

- Operational criteria to distinguish $E_1$vs $E_2$vs $E_\infty$behavior in lab platforms (signatures, minimal probes, finite-size scaling).

**(F) Anomaly ledgers.**

- Shared registries that track anomaly budgets (UV/IR, inflow, counterterms) per model, with machine-checkable cohomology representatives.

**(G) Education & review culture.**

- Reviewer training modules built around the **smallest necessary diagrams**; conference artifacts (posters/short talks) that present a model's coherence sheet up front.

**Closing note.**
This paper's motto—**measure twice (coherence), cut once (symmetry)**—asks builders to choose the correct ambient first, prove the minimal commuting diagrams, and only then elevate symmetry from convenience to claim. Do that, and many seductive but flawed models never make it past page one; what survives is clearer, portable across contexts, and far easier to test in the lab.

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*Notes:* Items 1–10 establish categorical and anyonic structure; 11–18 cover TQFT and CFT coherence; 19–24 give quantum group/Yang–Baxter/holonomy foundations; 25–29 treat locality and anomalies; 30–33 cover $E_n$, Higher Algebra, and HoTT; 34–36 address directed/ordered logics; 37–38 relate circuits and categorical quantum mechanics; 39–40 give classical background and quasi-Hopf coherence.