Fractal Determinism and Quantum Entanglement: A Reflected Complex Framework

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The Mandelbrot set has long captivated mathematicians and scientists for its paradoxical nature: a simple iterative formula produces a structure of infinite complexity and visual richness. Though entirely deterministic, the set's boundary is fractal, unstable, and sensitive to microscopic changes—qualities that intriguingly resemble phenomena observed in quantum mechanics. In this work, we explore that resemblance more deeply by applying a complex-plane transformation $c \mapsto -i^*\mathrm{conj}(c)$, creating a mirrored Mandelbrot set that subtly reconfigures the set's dynamic properties while preserving its form. What emerges is more than a rotated image—it is a dual iteration system whose escape-time behavior diverges from the original, echoing the structure of bidirectional time evolution in quantum theory. We show that the differences in escape dynamics under this reflection closely resemble features of entangled quantum systems and time-symmetric formulations such as the Two-State Vector Formalism. In doing so, we propose a new geometric and computable analogy: deterministic fractal systems as visual and mathematical analogs of quantum entanglement, coherence, and retrocausal boundary conditions.

Keywords: Mandelbrot set, fractals, quantum entanglement, complex reflection, antiunitary symmetry, escape time, retrocausality, two-state vector formalism, time symmetry, dual Hilbert space, mirrored dynamics, fractal boundary, quantum foundations, deterministic chaos, conjugate iteration. A collaboration with GPT-4o.

Abstract

We explore a novel bridge between deterministic fractal dynamics and the structure of quantum entanglement by reflecting the Mandelbrot set in the complex plane. Using the transformation $c \mapsto -i*\mathrm{conj}(c)$, we generate a “mirror Mandelbrot set” whose form remains fractal, but whose dynamic behavior—specifically, escape-time structure—reveals measurable differences from the original. This transformation, drawn from a broader theoretical framework involving reflected Hilbert spaces and mirror-time quantum evolution, acts as a geometric analog to time-symmetric quantum formalisms such as the two-state vector approach and retrocausal boundary conditions.

Despite being defined by a simple, deterministic iterative rule, the Mandelbrot set exhibits sensitive and self-similar structure at its boundaries. When this structure is mirrored under complex conjugation and phase rotation, we observe localized shifts in iteration stability that resemble the interference patterns found in quantum systems with both initial and final boundary constraints. The result is a visual and computational demonstration of how classical determinism can encode features typically associated with quantum entanglement, coherence, and bidirectional evolution. This work supports a geometric and fractal lens through which to view emergent quantum structure.

1. Introduction

Fractals occupy a unique position in mathematics: they are born of strict determinism, yet give rise to infinitely complex, self-similar structures. Among them, the Mandelbrot set remains the most iconic—a visual embodiment of order emerging from iteration. Defined by the recursive map $z_{n+1} = z_n^2 + c$, with $z_0 = 0$ and $c \in \mathbb{C}$, the Mandelbrot set includes all complex values of c for which the orbit remains bounded. Its deep boundary structure, sensitive to infinitesimal changes in c, reveals a complexity that is paradoxically generated by one of the simplest iterative rules in mathematics.

In contrast, quantum mechanics is inherently probabilistic, yet it also displays highly structured behavior, particularly through phenomena such as entanglement and time symmetry. Interpretations like the two-state vector formalism posit that quantum amplitudes evolve both forward and backward in time, raising questions about causality, measurement, and the role of final-state conditions.

This paper bridges these two worlds by reflecting the Mandelbrot iteration via the complex transformation $c \mapsto -i\overline{c}$. This mapping, previously studied in quantum theory for its role in defining antiunitary symmetries and dual Hilbert spaces, introduces a mirrored version of the Mandelbrot set. While visually similar, the reflected set displays subtle but real differences in escape dynamics—differences that provide an analog to entanglement and retrocausal propagation in quantum systems.

By comparing the original and reflected Mandelbrot sets, both visually and computationally, we demonstrate that deterministic fractal iteration can exhibit behavior structurally aligned with quantum amplitude interference. The results offer a new lens through which the deterministic-complexity of fractals and the bidirectionality of quantum amplitudes may be understood as part of a shared geometric language.

2. The Mandelbrot Set as Deterministic Complexity

At the heart of the Mandelbrot set lies one of the simplest recursive definitions in mathematics:

$$z_{n+1} = z_n^2 + c,$$

with the initial condition $z_0 = 0$ and c ranging over the complex plane. Despite its simplicity, this iterative rule gives rise to a remarkably intricate and infinitely detailed structure, known as the Mandelbrot set.

A point $c \in \mathbb{C}$ belongs to the Mandelbrot set if the sequence $\{z_n\}$ remains bounded—that is, it never diverges to infinity no matter how many iterations are applied. In practice, a cutoff threshold is used: if $|z_n| > 2$ at any step, the point is said to have "escaped," and is not part of the set. The number of iterations required for escape is called the **escape time**, and this forms the basis for visualizing the set with color gradients—highlighting the rate at which divergence occurs for values near the boundary.

What emerges is a vast structure with a connected central cardioid and recursively nested "bulbs" branching in self-similar patterns. The edge of the Mandelbrot set is infinitely detailed and nowhere smooth—a fractal boundary exhibiting sensitivity to infinitesimal changes in the parameter ccc. This boundary, while determined by an exact and repeatable process, displays behaviors that resemble chaos and complexity.

It is this paradox—the deterministic rule generating unpredictably rich structure—that makes the Mandelbrot set a natural analog for exploring questions in quantum mechanics. Unlike chaotic systems that depend on nonlinear feedback or stochastic input, the Mandelbrot set evolves without randomness. Yet, its sensitive edge, where points sit at the knife-edge between boundedness and divergence, suggests a visual metaphor for quantum superposition and decoherence.

The focus of this paper is not merely the set itself, but how its behavior changes under a complex geometric transformation that mirrors quantum-theoretic symmetries. Before that, we examine the nature of this reflection and how it modifies the structure of the Mandelbrot set.

3. The Reflected Transformation

To explore deeper structural analogies between fractal iteration and quantum entanglement, we introduce a transformation of the Mandelbrot parameter space inspired by antiunitary symmetry in quantum mechanics. Specifically, we replace the traditional complex parameter c in the Mandelbrot iteration $z_{n+1} = z_n^2 + c$ with a reflected counterpart:

$$c \mapsto -i\overline{c}.$$

This transformation has several key properties that make it both mathematically elegant and physically motivated.

3.1 Geometric Interpretation

The transformation $c \mapsto -i\overline{c}$ performs two actions:

1. **Complex Conjugation** $\overline{c}$: Reflects the point c across the real axis.
2. **Multiplication by** $-i$: Rotates the result by 90° clockwise in the complex plane.

Together, these operations define a reflection through a diagonal axis and reorientation of the complex parameter space—one that preserves the magnitude of c, but alters its angle and quadrant. This is more than cosmetic: the directionality of iteration is subtly affected, much like phase inversion in quantum amplitudes.

3.2 Dynamical Implications

In quantum theory, this transformation is closely related to antiunitary operators, which reverse complex phase and preserve the inner product via conjugate-linear mappings. Such operators are essential for describing time-reversal symmetry and charge conjugation in physical systems.

In our context, by applying this reflection to the parameter ccc, we effectively place the entire Mandelbrot iteration into a “mirror quadrant” of the complex plane. This mirrors not only the geometry of the fractal but also the dynamical flow of iteration. Points that were previously stable may now escape more quickly, and vice versa—analogous to changing the boundary conditions of a quantum system.

3.3 Preservation and Rotation

Mathematically, the transformation preserves the modulus:

$$|c| = |-i\bar{c}|,$$

ensuring that the distance from the origin remains the same. However, the phase of the complex number—its angular orientation—undergoes a rotation and reflection, meaning that while the "shape" of the iteration space is preserved in a broad sense, the dynamics within it are not invariant.

This sets the stage for a direct comparison of the standard and reflected Mandelbrot sets—not simply in appearance, but in escape-time behavior and iteration flow.

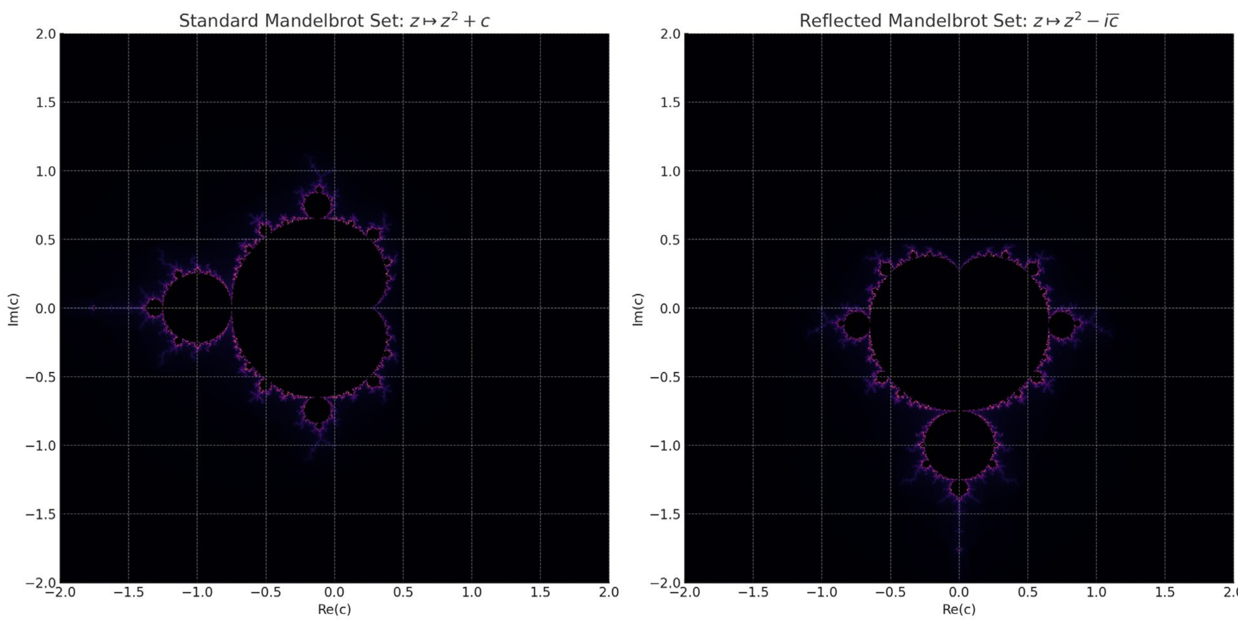

Figure 1: Standard vs. Reflected Mandelbrot Set

Left: The standard Mandelbrot set, based on $z_{n+1} = z_n^2 + c$.
Right: The reflected Mandelbrot set, using $z_{n+1} = z_n^2 - i\bar{c}$.

The figure reveals that while the general shape and fractal layering remain intact, the reflected set is rotated 90° and exhibits internal structural variation—setting the stage for deeper dynamic comparisons.

4. Escape Time as a Dynamic Analogy

The escape-time function is central to visualizing and understanding the Mandelbrot set. Each point $c \in \mathbb{C}$ is colored according to how many iterations it takes for the orbit $\{z_n\}$, beginning with $z_0 = 0$, to exceed a magnitude threshold (typically $|z_n| > 2$). This escape time determines not only whether c belongs to the set (non-escape) but also how close it lies to the fractal boundary.

In this context, escape time serves as a dynamic marker of stability, much like the way amplitude evolves in quantum systems through time. A point with a high escape time can be thought of as "holding onto" stability longer, resisting divergence, while one with a low escape time quickly reveals its instability.

4.1 Escape Time as Quantum Time Evolution

This analogy becomes more potent when we map escape time to quantum temporal amplitude evolution. In quantum mechanics, especially in time-symmetric or two-state vector formulations, amplitudes evolve both forward and backward in time. The stability of a quantum state is influenced not only by its initial condition but potentially by a final constraint—a boundary in the future that shapes the past.

In the Mandelbrot case, the reflected transformation $c \mapsto -i\bar{c}$ alters the iteration dynamics in a way that is directly comparable to switching the direction of amplitude propagation. Instead of evolving forward from a given seed point ccc, we evaluate what happens when the system is subjected to its mirror counterpart—a reflected origin point.

4.2 Measurable Differences in Stability

When we compare the standard and reflected Mandelbrot sets not just visually, but numerically through escape time, differences emerge. These differences are not uniform: they are localized and sensitive to the fractal boundary. Some points that were once stable become less so, and vice versa. These variations suggest that the reflection transformation introduces an effective asymmetry in dynamic behavior, even though the iteration rule remains formally the same.

This dynamic asymmetry is strikingly analogous to interference effects in quantum amplitudes, where the outcome depends on how multiple propagation paths overlap, often shaped by both initial and final boundary conditions.

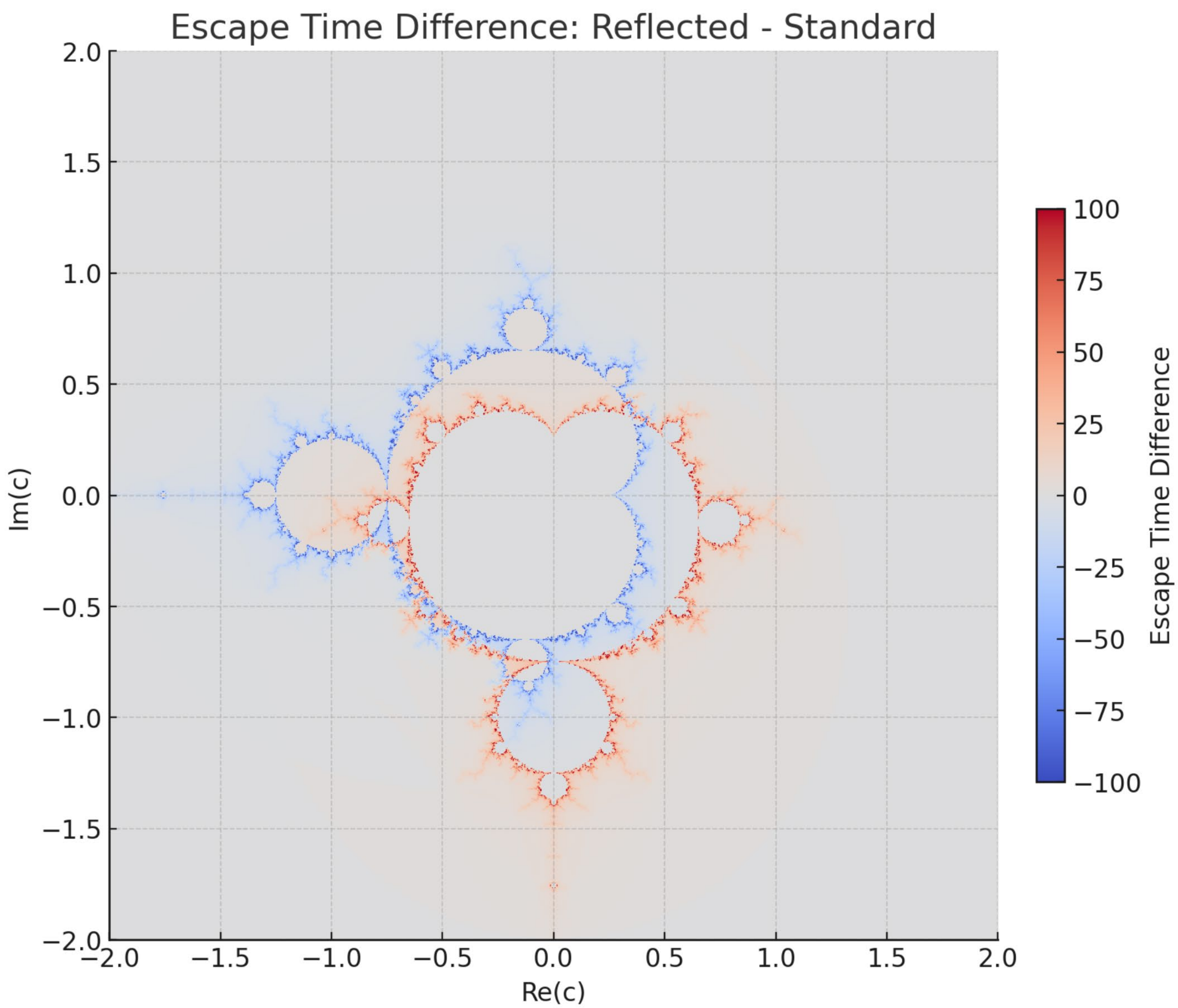

Figure 2: Escape Time Difference Map: Reflected – Standard

This plot shows the pixel-by-pixel difference in escape time between the standard and reflected Mandelbrot sets:

$$\Delta(c) = \mathrm{EscapeTime}_{\mathrm{reflected}}(c) - \mathrm{EscapeTime}_{\mathrm{standard}}(c).$$

White regions indicate no change: dynamically invariant under reflection.

- Red regions reflect increased stability in the mirrored version.
- Blue regions highlight decreased stability or faster escape.

The asymmetries captured in this figure offer more than artistic variance—they constitute a dynamic footprint of the reflection transformation and a direct analogy to mirror-time evolution in quantum physics.

5. Fractal Boundaries and Quantum Entanglement

At the heart of the Mandelbrot set lies its boundary—an infinitely intricate structure that defies smoothness, simplicity, or isolation. This boundary is where the true complexity of the set emerges. It is neither entirely within the stable region nor fully outside of it; instead, it resides on the knife-edge between stability and escape. This edge behavior is not only fractal but non-local, in the sense that infinitesimally small changes in the complex parameter ccc can result in radically different outcomes after many iterations.

5.1 Boundary Structure as Entangled Behavior

This paradoxical nature—deterministic but unpredictable, sensitive yet orderly—mirrors the behavior of entangled quantum systems. In quantum mechanics, entanglement occurs when two or more subsystems cannot be described independently, even when separated in space or time. The system as a whole carries information that is not contained in any of its parts individually.

Similarly, points near the Mandelbrot boundary are coupled to nearby points in complex ways. Their behaviors are not independent; the outcome of one is intimately tied to the behavior of its neighbors under iteration. This interdependence is deterministic rather than probabilistic, but its structure behaves much like a complex amplitude field, where the local state is informed by global constraints.

5.2 Quantum Amplitudes and Dual Hilbert Spaces

In the context of the dual Hilbert space framework proposed in earlier work, a quantum state is represented not by a single evolving vector, but by a pair of states evolving forward and backward in time:

$$|\psi\rangle \in \mathcal{H}, \quad \langle\phi| \in \mathcal{H}^*.$$

These states interact at a point of measurement or interference, and their overlap governs the observable outcome. The mirrored Mandelbrot iteration—generated through $c \mapsto -i\bar{c}$—functions analogously. It does not merely replot the same fractal, but instead introduces a structurally correlated system that responds to initial parameters in ways that differ from the standard formulation.

This duality between the standard and reflected sets forms a kind of geometric entanglement: a region in one space corresponds to altered dynamics in the other, and their differences encode a deeper relationship. Just as entangled quantum states reveal information only when considered together, the dynamic properties of the Mandelbrot and its reflected form reveal their true depth only when analyzed in tandem.

5.3 Reflected Fractals as Geometric Entangled States

In this view, the standard Mandelbrot set can be seen as a forward-propagating iteration system—analogous to the forward-evolving amplitude in quantum mechanics. The reflected Mandelbrot set, constructed through conjugation and rotation, serves as a backward-propagating counterpart. The escape-time difference between the two can thus be interpreted as the result of their interference—a visual, computable analog of the overlap between forward and backward amplitudes in quantum mechanics.

These reflected fractals are not just geometric curiosities. They act as visual metaphors for entangled quantum systems, where the full structure only emerges when both directional components are understood together. In both cases, structural coherence arises not from one part alone, but from the mutual relation between parts across a transformation boundary.

6. Dual Iteration and the Two-State Vector Formalism

In conventional quantum mechanics, the evolution of a system is described by a single state vector $|\psi(t)\rangle$, governed by the Schrödinger equation and evolving forward in time from a known initial condition. However, this view is incomplete when it comes to understanding phenomena that involve measurement, retrocausality, or interference with boundary conditions defined in the future.

The **Two-State Vector Formalism (TSVF)**, introduced by Aharonov, Bergmann, and Lebowitz, provides a time-symmetric extension of standard quantum theory. In this framework, a quantum system is described not by a single state, but by a **pair of state vectors**:

$$|\psi(t)\rangle \in \mathcal{H}, \quad \langle\phi(t)| \in \mathcal{H}^*,$$

one evolving forward from the initial state and one evolving backward from a final measurement or boundary condition. Observables are determined not just by the initial state, but by the **interference of both amplitudes**.

6.1 Mandelbrot Iteration as a Time-Evolution Analog

The iteration of $z_{n+1} = z_n^2 + c$ in the Mandelbrot set is structurally similar to time evolution in quantum mechanics: each new value depends deterministically on the prior one, with the complex parameter c shaping the trajectory. In the **reflected Mandelbrot set**, we replace c with $-i\bar{c}$, effectively introducing a second, conjugate iteration that behaves differently but is geometrically related.

In this context:

- The **standard iteration** is analogous to forward evolution in $\mathcal{H}$,
- The **reflected iteration** acts as a backward-propagating state in $\mathcal{H}^*$,
- And the **escape-time differences** between the two define a kind of **interference pattern**, capturing how the two propagation directions interact.

This interaction is not random; it is geometrically encoded in the structure of the complex plane. Some regions show no difference in escape time (akin to constructive interference), while others show acceleration or delay (analogous to destructive interference or phase shifts).

6.2 Escape-Time Difference as Phase Interference

In quantum mechanics, when a forward-evolving amplitude overlaps with a backward-evolving amplitude, their product (or inner product) gives rise to measurable quantities such as weak values or interference terms. In the reflected Mandelbrot context, the escape-time difference plays a similar role. It maps the degree to which forward and backward iterations agree or disagree in the stability of a point.

This analogy provides a powerful visual metaphor: just as a quantum system's measurable properties arise from the overlap of forward and backward state vectors, a point in the complex plane exhibits dynamic coherence or discrepancy based on how differently it behaves under the two iteration regimes.

The result is a compelling geometrical interpretation of the TSVF—a framework normally limited to abstract amplitude manipulation—now made visible through color gradients, fractal boundaries, and mirrored iterations.

7. Final-State Boundary Conditions and Retrocausality

In classical physics, the future is determined solely by initial conditions: given the state of a system at time t_0, the laws of motion allow us to evolve it forward deterministically. In standard quantum mechanics, however, this determinism gives way to probability, and outcomes are typically understood as depending only on the past—except at the point of measurement, where the wavefunction collapses seemingly outside of causal evolution.

An alternative perspective—supported by both the Two-State Vector Formalism (TSVF) and growing interest in time-symmetric quantum mechanics—suggests that final-state boundary conditions may play a fundamental role in shaping what occurs in the present. In such theories, retrocausality is not a paradox, but a natural consequence of a system defined by both past and future constraints.

7.1 Modeling Final-State Effects in Fractals

The reflection transformation $c \mapsto -i\bar{c}$ introduces a novel boundary condition into the iteration process of the Mandelbrot set. While traditional iteration explores how a point evolves under forward dynamics, the reflected system incorporates a "reversed" influence. Although there is no explicit final condition imposed in the fractal's algorithm, the reflected iteration acts as if the system is constrained from both directions—mirroring the philosophical and mathematical structure of final-state quantum models.

This dual structure enables us to interpret escape-time discrepancies between the standard and reflected Mandelbrot sets as the classical analog of retrocausal interaction. A point's stability is not just a matter of how it unfolds from the origin, but how it would behave if its trajectory were informed by a mirror condition—akin to post-selection in quantum experiments.

7.2 Backward Influence and Temporal Symmetry

In this interpretation, the reflected Mandelbrot set plays the role of a time-reversed partner to the original. Together, the two define a complete "two-boundary" system—one that is temporally symmetric and structurally analogous to a quantum process constrained by both preparation and post-selection.

Retrocausal theories in quantum foundations have gained traction due to their ability to resolve paradoxes (such as nonlocality and contextuality) without invoking hidden variables. Similarly, the mirror Mandelbrot does not hide information or change the rules; it reinterprets the same deterministic structure under a reflected time dynamic. The result is a difference in escape behavior that is measurable, repeatable, and consistent—yet rooted in a bidirectional causal structure.

This deepens the analogy: just as quantum theory may be fundamentally time-symmetric, with both past and future shaping the present, the Mandelbrot set—under reflection—becomes a visual and computable metaphor for retrocausality.

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8. Discussion: Structural Equivalence Between Reflection and Entanglement

At the heart of this investigation lies a deeper proposition: that the structure of the Mandelbrot set under mirrored iteration is more than a computational curiosity—it is functionally analogous to the structure of entangled quantum systems. While fractals and quantum states operate under vastly different ontologies (one deterministic, the other probabilistic), they share critical features in how stability, boundary behavior, and interaction manifest within their respective frameworks.

8.1 Fractal Stability vs. Quantum Coherence

In the standard Mandelbrot set, stability is defined by boundedness under iteration. The structure emerges entirely from whether sequences $\{z_n\}$ remain finite. In quantum mechanics, coherence refers to the phase-aligned propagation of probability amplitudes. Quantum systems preserve information through superposition and entanglement—states that remain uncollapsed until measured.

Despite their differing languages, both concepts refer to the persistence of structure through successive transformations:

- In fractals: the persistence of a bounded orbit,
- In quantum systems: the persistence of an uncollapsed superposition.

Mirrored iteration via $c \mapsto -i\bar{c}$ changes the stability profile of certain points in the Mandelbrot set. Similarly, conjugate evolution in quantum systems—through time reversal or post-selection—reshapes how coherence manifests. In both cases, the structural behavior emerges not from one state alone, but from the relationship between dual propagating states.

8.2 Boundary Sensitivity as a Shared Trait

The Mandelbrot set is notorious for its edge sensitivity: the boundary between stable and unstable behavior is fractal, infinite in detail, and highly sensitive to small changes in c. Likewise, entangled quantum states are sensitive to measurement configuration, exhibiting nonlocal correlations that shift based on seemingly distant decisions.

This suggests a shared principle: that meaningful structure is concentrated at the boundary, where dual dynamics intersect. In the Mandelbrot set, this is the transition from bounded to unbounded iteration. In quantum mechanics, it is the interface where forward and backward amplitudes interfere. These boundaries are not smooth, nor easily predictable, but they are governed by strict rules that encode deep relationships—whether geometric or quantum.

8.3 Modeling Entanglement via Mirrored Iteration

What emerges from this work is the possibility that entanglement-like structures can be modeled classically, not by simulating quantum superpositions, but by constructing dual-iteration systems that reflect quantum logic geometrically.

By considering:

- Standard Mandelbrot iteration as **forward evolution**, and
- Reflected iteration as **conjugate, backward evolution**,
  we effectively model a **two-boundary system**, analogous to entangled states evolving across $\mathcal{H} \times \mathcal{H}^*$. The difference in behavior—mapped through escape time, structural asymmetry, and sensitivity—becomes a stand-in for the **interference and nonlocality** found in entangled quantum systems.

This structural equivalence is not merely poetic; it offers a computable analogy that may help to visualize and simulate quantum phenomena with classical tools. In particular, the mirrored Mandelbrot framework invites exploration of how entanglement and dual evolution could be rendered as geometric transformations, unlocking new insight into both domains.

9. Conclusion

In this paper, we have proposed and examined a new conceptual bridge between deterministic fractal systems and time-symmetric quantum mechanics. By applying the complex-plane transformation $c \mapsto -i\bar{c}$ to the Mandelbrot set, we generated a reflected version that is structurally similar but dynamically distinct from the original. This transformation mirrors the action of antiunitary symmetries in quantum theory and gives rise to a form of bidirectional behavior analogous to quantum dual-state evolution.

Our key finding is that the differences in escape-time behavior between the standard and reflected Mandelbrot sets—despite the identical iteration rule—mirror the types of amplitude interference seen in entangled quantum systems. The boundary of the Mandelbrot set, long known for its infinite complexity and sensitivity, reveals a deeper analogy when viewed as the product of dual iteration: forward from the origin, and backward from a mirrored final-state condition.

This correspondence supports the idea that entanglement, coherence, and retrocausality in quantum mechanics can be meaningfully modeled using geometrically transformed fractal systems. The reflected iteration structure serves not merely as a metaphor, but as a computational and visual analog to two-state vector quantum mechanics and time-symmetric formulations of measurement.

Implications and Future Directions

This work suggests that the mathematical and geometric structure of fractals, particularly when subjected to conjugate transformations, may offer valuable insight into quantum foundational problems. Unlike stochastic or probabilistic systems, fractals are entirely deterministic—yet their edge behavior and iterative sensitivity mimic the indeterminacy and structure of quantum entanglement.

Potential paths forward include:

- Developing hybrid models where fractal iteration approximates amplitude evolution in discrete quantum systems,
- Investigating zoomed correspondence regions between reflected and original fractal dynamics to probe analogs of quantum interference,
- Extending the framework to Julia sets and complex dynamical systems under mirrored rules,

- Exploring the possibility of fractal-based simulation tools for visualizing dual Hilbert space dynamics or nonlocal quantum processes.

Ultimately, by reframing a classical system through the lens of reflected dynamics, we find ourselves at a surprising frontier: deterministic iteration revealing the structural language of quantum entanglement.

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Appendix A: Python Code for Mandelbrot and Reflected Iterations

This appendix provides clean, modular Python code used to generate Figures 1 and 2 of the paper. The code visualizes both the standard and reflected Mandelbrot sets and computes the pixel-wise difference in escape time between them. All images are generated at 1000 × 1000 resolution, with consistent color maps and normalization for direct visual comparison.

A.1 Imports and Setup

python

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```
import numpy as np

import matplotlib.pyplot as plt
```

A.2 Mandelbrot Iteration Function

python

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```
def compute_mandelbrot(c, max_iter):

    z = np.zeros_like(c, dtype=np.complex128)

    div_time = np.zeros(c.shape, dtype=int)

    for i in range(max_iter):

        z = z**2 + c

        mask = (np.abs(z) > 2.0) & (div_time == 0)

        div_time[mask] = i

    return div_time
```

A.3 Domain Configuration

python

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```
width, height = 1000, 1000
re_start, re_end = -2.0, 2.0
im_start, im_end = -2.0, 2.0
max_iter = 200

real = np.linspace(re_start, re_end, width)
imag = np.linspace(im_start, im_end, height)
C = real[np.newaxis, :] + 1j * imag[:, np.newaxis]
```

A.4 Compute Standard and Reflected Mandelbrot Sets

python

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```
# Standard Mandelbrot
div_time_std = compute_mandelbrot(C, max_iter)

# Reflected Mandelbrot: c → -i * conjugate(c)
C_reflected = -1j * np.conj(C)
div_time_reflected = compute_mandelbrot(C_reflected, max_iter)
```

A.5 Generate Figure 1: Side-by-Side Comparison

python

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```
fig, axs = plt.subplots(1, 2, figsize=(16, 8))

axs[0].imshow(div_time_std, extent=(re_start, re_end, im_start, im_end), cmap="inferno")
axs[0].set_title("Standard Mandelbrot Set")
axs[0].set_xlabel("Re(c)")
axs[0].set_ylabel("Im(c)")

axs[1].imshow(div_time_reflected, extent=(re_start, re_end, im_start, im_end), cmap="inferno")
axs[1].set_title("Reflected Mandelbrot Set (c → -i·conj(c))")
axs[1].set_xlabel("Re(c)")
axs[1].set_ylabel("Im(c)")

plt.tight_layout()
plt.show()
```

A.6 Generate Figure 2: Escape-Time Difference

python

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```
escape_diff = div_time_reflected - div_time_std
```

```
fig, ax = plt.subplots(figsize=(8, 8))
cax = ax.imshow(escape_diff, extent=(re_start, re_end, im_start, im_end),
                cmap="coolwarm", vmin=-max_iter//2, vmax=max_iter//2)
ax.set_title("Escape Time Difference: Reflected – Standard")
ax.set_xlabel("Re(c)")
ax.set_ylabel("Im(c)")

cbar = fig.colorbar(cax, ax=ax, fraction=0.03, pad=0.04)
cbar.set_label("Escape Time Difference")

plt.tight_layout()
plt.show()
```

This code produces all visualizations used in the main text and enables independent reproduction of results. It also provides a foundation for deeper fractal analysis using mirrored iterations and dynamic comparisons.