From Horizons to Voids: A Two-Barrier Finite Algebra of Cosmic Structure

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Abstract

Gravitational collapse and cosmic-void formation are conventionally treated as opposite outcomes of density evolution. Overdense regions attract surrounding matter and may form bound structures or trapped regions; underdense regions evacuate matter toward surrounding walls and filaments and may merge into larger voids. This paper develops a finite algebraic model that places these outcomes within one conservative reaction–transport system. The local state contains matter, antimatter, radiation, and a causal-geometric label. Reaction conserves energy by converting paired matter and antimatter into radiation. Transport redistributes energy across a finite graph. Two independent geometric diagnostics are then evaluated: compactness, which detects concentration, and density contrast, which detects rarefaction. A supercritical compactness value activates a trapped-region update, whereas a sufficiently negative density contrast activates a void-region update. The resulting two-barrier model distinguishes disappearance from redistribution, darkness from underdensity, and causal trapping from sparse but observable space. It also produces finite analogues of collapse thresholds, void-in-void growth, void-in-cloud suppression, wall formation, basin merging, and critical boundaries separating dispersive, void-forming, and collapsing trajectories. The construction is not proposed as an alternative to general relativity or cosmological perturbation theory. It is an exact finite laboratory for auditing how conservation, transport, concentration, rarefaction, and causal accessibility jointly generate cosmic structure.

Keywords: cosmic void; gravitational collapse; Boötes Void; finite dynamical system; density contrast; compactness; cosmic web; reaction–transport algebra; critical phenomena.

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1. Scope

The preceding finite model described the compositional update

$$F = \mathcal{G} \circ \mathcal{K} \circ \mathcal{T}_g \circ \mathcal{A}, \qquad (1)$$

where

$$\begin{aligned} \mathcal{A} &= \text{matter–antimatter reaction}, \\ \mathcal{T}_g &= \text{geometry-dependent transport}, \\ \mathcal{K} &= \text{compactness evaluation}, \end{aligned}$$

and

$$\mathcal{G} = \text{trapping update}.$$

That architecture represented one principal geometric outcome:

$$\text{concentration} \longrightarrow \text{causal trapping}. \qquad (2)$$

Cosmic structure also contains the opposite large-scale outcome:

$$\text{rarefaction} \longrightarrow \text{void formation}. \qquad (3)$$

A cosmic void is not a region in which energy has been deleted. It is a region whose density is lower than the cosmic mean because matter has been transported preferentially toward its boundary and into the surrounding web.

The extended finite architecture is therefore

$$\boxed{F = (\mathcal{H}, \mathcal{V}) \circ \mathcal{K} \circ \mathcal{T} \circ \mathcal{A},} \qquad (4)$$

where

$$\mathcal{H} = \text{overdensity and trapping classifier} \qquad (5)$$

and

$$\mathcal{V} = \text{underdensity and void classifier}. \qquad (6)$$

The model must preserve the distinctions

$$\begin{array}{cc} \text{void} & \neq \text{vacuum}, \\ \text{void} & \neq \text{annihilated region}, \\ \text{void} & \neq \text{event horizon}, \\ \text{underdensity} & \neq \text{absence of geometry}, \\ \text{outward transport} & \neq \text{energy destruction}. \end{array} \qquad (7)$$

The intended compression is

$$\boxed{\text{reaction} + \text{transport} + \text{density contrast} + \text{compactness} + \text{two geometric outcomes.}} \qquad (8)$$

2. Finite Cosmic State

Let

$$\Gamma = (X, \mathcal{E}) \qquad (9)$$

be a finite graph representing spatial adjacency.

At each cell $x \in X$ and time $t \in \mathbb{N}$, define the local state

$$S_t(x) = (m_t(x), \bar{m}_t(x), r_t(x), h_t(x), v_t(x)). \qquad (10)$$

The nonnegative quantities

$$m_t(x), \bar{m}_t(x), r_t(x) \in \mathbb{N} \qquad (11)$$

denote matter, antimatter, and radiation energy units.

The geometric variables are

$$h_t(x), v_t(x) \in \{0,1\}, \qquad (12)$$

with

$$h_t(x) = 1$$

indicating that $x$belongs to a trapped region, and

$$v_t(x) = 1$$

indicating that $x$belongs to a void region.

The material-radiative content is

$$q_t(x) = (m_t(x), \bar{m}_t(x), r_t(x)). \qquad (13)$$

The geometric classification is

$$g_t(x) = (h_t(x), v_t(x)). \qquad (14)$$

Hence

$$S_t(x) = (q_t(x), g_t(x)). \qquad (15)$$

Definition 1. Local energy

Define

$$e_t(x) = m_t(x) + \bar{m}_t(x) + r_t(x). \qquad (16)$$

The total energy is

$$E_t = \sum_{x \in X} e_t\,(x). \qquad (17)$$

Definition 2. Mean energy density

For a uniform finite graph, define

$$\bar{e}_t = \frac{E_t}{|\,X\,|}. \qquad (18)$$

For weighted cells with volumes $V_x > 0$, define

$$\bar{\rho}_t = \frac{E_t}{\sum_{x \in X} V_x}. \qquad (19)$$

The local density is

$$\rho_t(x) = \frac{e_t(x)}{V_x}. \qquad (20)$$

For equal-volume cells,

$$\rho_t(x) \propto e_t(x). \qquad (21)$$

Definition 3. Mutually exclusive geometric labels

The simplest model requires

$$h_t(x)v_t(x) = 0. \qquad (22)$$

Thus no cell is simultaneously classified as trapped and void:

$$(h_t(x), v_t(x)) \in \{(0,0), (1,0), (0,1)\}. \qquad (23)$$

These three possibilities correspond to

$$\begin{aligned} &(0,0) && \text{:ordinary or intermediate cell,} \\ &(1,0) && \text{:trapped cell,} \\ &(0,1) && \text{:void cell.} \end{aligned} \qquad (24)$$

The global state space is therefore

$$\mathcal{S} = \prod_{x \in X} (\mathbb{N}^3 \times \{(0,0), (1,0), (0,1)\}). \qquad (25)$$

For bounded computation, let

$$E_N = \{0,1, \ldots, N\}. \qquad (26)$$

Then

$$\mathcal{S}_N = \prod_{x \in X} (E_N^3 \times \{(0,0), (1,0), (0,1)\}). \qquad (27)$$

Its cardinality is

$$| \mathcal{S}_N | = (3(N+1)^3)^{|X|}. \qquad (28)$$

3. Conservative Reaction and Transport

The local annihilation amount is

$$a_t(x) = \min\{m_t(x), \bar{m}_t(x)\}. \qquad (29)$$

The reaction map is

$$\mathcal{A}: (m, \bar{m}, r) \longmapsto (m - a, \bar{m} - a, r + 2a), \qquad (30)$$

where

$$a = \min\{m, \bar{m}\}. \qquad (31)$$

Proposition 1. Reaction conservation

For

$$E(m, \bar{m}, r) = m + \bar{m} + r, \qquad (32)$$

the reaction map satisfies

$$E(\mathcal{A}(m, \bar{m}, r)) = E(m, \bar{m}, r). \qquad (33)$$

Proof

$$\begin{aligned} E' &= (m - a) + (\bar{m} - a) + (r + 2a) \\ &= m + \bar{m} + r. \end{aligned} \qquad (34)$$

Therefore

$$E' = E.$$

■

Reaction may change the material composition while preserving the quantity that gravitates in the finite model.

Thus

$$\text{annihilation} \neq \text{evacuation}. \qquad (35)$$

Complete local annihilation gives

$$(m, m, r) \longmapsto (0,0, r + 2m), \qquad (36)$$

not

$$(m, m, r) \longmapsto (0,0, r). \qquad (37)$$

A region rich in annihilation products may remain energetically dense because the original material energy has become radiation.

Definition 4. Transport kernels

For each species

$$s \in \{m, \bar{m}, r\}, \qquad (38)$$

let

$$P_t^s(x, y) \geq 0 \qquad (39)$$

denote the fraction of species $s$transported from $x$to y.

Require

$$\sum_{y \in X} P_t^s\,(x, y) = 1. \qquad (40)$$

The transported field is

$$(\mathcal{T}_t{}^s q)(y) = \sum_{x \in X} P_t^s\,(x, y) q^s(x). \qquad (41)$$

The complete transport operator is

$$\mathcal{T}_t = \mathcal{T}_t^m \times \mathcal{T}_t^{\bar{m}} \times \mathcal{T}_t^r. \qquad (42)$$

Proposition 2. Species conservation under transport

For every transported species,

$$\sum_{y \in X} (\mathcal{T}_t^s q)(y) = \sum_{x \in X} q^s(x). \qquad (43)$$

Proof

$$\begin{aligned} \sum_y (\mathcal{T}_t^s q)(y) &= \sum_y \sum_x P_t^s(x,y) q^s(x) \\ &= \sum_x q^s(x) \sum_y P_t^s(x,y) \qquad (44) \\ &= \sum_x q^s(x). \end{aligned}$$

■

Theorem 1. Pre-geometric energy conservation

Let

$$q_{t+1}^- = \mathcal{T}_t \mathcal{A}(q_t). \qquad (45)$$

Then

$$E_{t+1}^- = E_t. \qquad (46)$$

Proof

The reaction map conserves local energy by Proposition 1. The transport operator conserves each species by Proposition 2. Their composition conserves the total sum. ■

Equation (46) is central to both branches of the model.

For collapse,

energy becomes concentrated. (47)

For void formation,

$$\text{energy leaves the interior and accumulates elsewhere.} \qquad (48)$$

Neither outcome requires destruction of energy.

Definition 5. Regional flux

For $B \subseteq X$, define the outward species flux

$$\Phi_{t,B}^{s,\text{out}} = \sum_{\substack{x \in B \\ y \notin B}} P_t^s\,(x,y) q_t^s(x), \qquad (49)$$

and the inward species flux

$$\Phi_{t,B}^{s,\text{in}} = \sum_{\substack{x \notin B \\ y \in B}} P_t^s\,(x,y) q_t^s(x). \qquad (50)$$

The net outward energy flux is

$$\Phi_{t,B} = \sum_{s} \left(\Phi_{t,B}^{s,\text{out}} - \Phi_{t,B}^{s,\text{in}}\right). \qquad (51)$$

Proposition 3. Regional energy balance

The regional energy satisfies

$$E_{t+1}^{-}(B) - E_t(B) = -\Phi_{t,B}, \qquad (52)$$

where

$$E_t(B) = \sum_{x \in B} e_t\,(x). \qquad (53)$$

Proof

Every transported unit either remains inside B, enters B, or leaves B. Internal transfers cancel in the regional sum. The only net contributions are inward and outward boundary fluxes. ■

Therefore a developing void satisfies the finite balance

$$\Phi_{t,B} > 0 \Longrightarrow E_{t+1}^{-}(B) < E_t(B). \qquad (54)$$

The energy deficit in $B$is exactly balanced by energy deposited in its exterior:

$$\Delta E(B) + \Delta E(X \setminus B) = 0. \qquad (55)$$

Hence

$$\boxed{\text{void formation is conservative evacuation.}} \qquad (56)$$

4. Density Contrast and the Void Barrier

Let

$$B_\rho(x) \subseteq X \qquad (57)$$

denote the graph ball of radius ρcentered at x.

Its volume is

$$V(B_\rho(x)) = \sum_{y \in B_\rho(x)} V_y\,. \qquad (58)$$

The enclosed energy is

$$E_t(x,\rho) = \sum_{y \in B_\rho(x)} e_t\,(y). \qquad (59)$$

The corresponding regional density is

$$\rho_t(x,\rho) = \frac{E_t(x,\rho)}{V(B_\rho(x))}. \qquad (60)$$

Definition 6. Smoothed density contrast

Define

$$\delta_t(x,\rho) = \frac{\rho_t(x,\rho) - \bar{\rho}_t}{\bar{\rho}_t}. \qquad (61)$$

Equivalently,

$$\delta_t(x,\rho) = \frac{\rho_t(x,\rho)}{\bar{\rho}_t} - 1. \qquad (62)$$

Thus

$$\delta_t(x,\rho) > 0 \qquad (63)$$

indicates an overdense region,

$$\delta_t(x,\rho) = 0 \qquad (64)$$

indicates mean density, and

$$\delta_t(x,\rho) < 0 \qquad (65)$$

indicates an underdense region.

Because all energy densities are nonnegative,

$$\delta_t(x,\rho) \geq -1. \qquad (66)$$

The limiting value

$$\delta = -1 \qquad (67)$$

corresponds to a region containing no represented energy.

A cosmic void generally satisfies

$$-1 < \delta < 0, \qquad (68)$$

not

$$\delta = -1. \qquad (69)$$

Definition 7. Void threshold

Fix a negative threshold

$$\delta_{\mathrm{v}} < 0. \qquad (70)$$

A region is void-supercritical when

$$\delta_t(x, \rho) \leq \delta_{\mathrm{v}}. \qquad (71)$$

Define the minimum smoothed contrast at $x$by

$$\delta_t^{\min}(x) = \min_{\rho \in \mathcal{R}} \delta_t(x, \rho), \qquad (72)$$

where $\mathcal{R}$ is the admitted finite set of smoothing radii.

The local void candidate is

$$\hat{v}_{t+1}(x) = \mathbf{1}\left[\delta_{t+1}^{\min}(x) \leq \delta_{\mathrm{v}}\right]. \qquad (73)$$

Equation (73) is a discrete underdensity barrier.

Remark 1. Scale dependence

A cell may be underdense at one radius and overdense at another:

$$\delta_t(x, \rho_1) < 0, \delta_t(x, \rho_2) > 0. \qquad (74)$$

Therefore void classification cannot depend only on the unsmoothed local value $e_t(x)$.

The scale $\rho$is part of the classification.

This permits a small underdensity to exist inside a larger overdense environment:

$$\delta_t(x, \rho_{\mathrm{small}}) < 0, \delta_t(x, \rho_{\mathrm{large}}) > 0. \qquad (75)$$

Such a configuration is the finite precursor of the void-in-cloud process.

Definition 8. Persistent underdensity

A void candidate is persistent over Ltime steps when

$$\delta_{t-j}^{\min}(x) \leq \delta_{\mathrm{v}} \text{for every } j = 0, \dots, L-1. \qquad (76)$$

Define

$$\hat{v}_{t+1}^{(L)}(x) = \prod_{j=0}^{L-1} \mathbf{1}\left[\delta_{t-j}^{\min}(x) \leq \delta_{\mathrm{v}}\right]. \qquad (77)$$

The persistence requirement distinguishes a dynamically maintained underdensity from a one-step fluctuation.

Definition 9. Evacuating underdensity

Let $B = B_\rho(x)$. A void candidate is dynamically evacuating when

$$\delta_t(x, \rho) \leq \delta_{\mathrm{v}} \qquad (78)$$

and

$$\Phi_{t,B} > 0. \qquad (79)$$

The combined void update is

$$\hat{v}_{t+1}(x) = \mathbf{1}\left[\delta_{t+1}^{\min}(x) \leq \delta_{\mathrm{v}}\right] \mathbf{1}\left[\Phi_{t,B_{\rho_*}(x)} > 0\right], \qquad (80)$$

where ρ_* is a radius attaining the minimum in (72).

Thus a finite void is characterized by both

low density (81)

and

net outward transport. (82)

Proposition 4. Continued evacuation lowers density contrast

Assume that the total mean density $\bar{\rho}_t$is constant and that the effective regional volume is fixed. If

$$\Phi_{t,B} > 0, \qquad (83)$$

then

$$\delta_{t+1}(B) < \delta_t(B). \qquad (84)$$

Proof

By Proposition 3,

$$E_{t+1}(B) = E_t(B) - \Phi_{t,B}. \qquad (85)$$

Since $\Phi_{t,B} > 0$,

$$E_{t+1}(B) < E_t(B). \qquad (86)$$

With fixed regional volume,

$$\rho_{t+1}(B) < \rho_t(B). \qquad (87)$$

Since $\bar{\rho}_t$is constant, equation (61) gives

$$\delta_{t+1}(B) < \delta_t(B). \qquad (88)$$

■

Corollary 1. Outward flux amplifies an underdensity

If

$$\delta_t(B) < 0 \qquad (89)$$

and

$$\Phi_{t,B} > 0, \qquad (90)$$

then the regional underdensity deepens:

$$|\ \delta_{t+1}(B)\ | > |\ \delta_t(B)\ |. \qquad (91)$$

This is the finite positive-feedback mechanism underlying void evacuation.

5. Compactness and the Collapse Barrier

The void diagnostic depends on density relative to the mean. The collapse diagnostic depends on absolute energy concentration relative to geometric scale.

Define the compactness functional

$$\mathcal{C}_t(x,\rho) = \frac{2GE_t(x,\rho)}{c^4 R(x,\rho)}, \qquad (92)$$

where $R(x,\rho)$ is the effective radius assigned to $B_\rho(x)$.

In geometrized units,

$$G = c = 1, \qquad (93)$$

equation (92) becomes

$$\mathcal{C}_t(x,\rho) = \frac{2E_t(x,\rho)}{R(x,\rho)}. \qquad (94)$$

Definition 10. Collapse threshold

Fix

$$\mathcal{C}_\mathrm{h} > 0. \qquad (95)$$

A region is collapse-supercritical when

$$\mathcal{C}_t(x,\rho) \geq \mathcal{C}_\mathrm{h}. \qquad (96)$$

Define

$$\mathcal{C}_t^{\max}(x) = \max_{\rho\in\mathcal{R}} \mathcal{C}_t(x,\rho). \qquad (97)$$

The trapped-region candidate is

$$\hat{h}_{t+1}(x) = \mathbf{1}[\mathcal{C}_{t+1}^{\max}(x) \geq \mathcal{C}_\mathrm{h}]. \qquad (98)$$

The two diagnostics are distinct:

$$\delta_t(x,\rho) = \frac{\rho_t(x,\rho)}{\bar{\rho}_t} - 1, \qquad (99)$$

while

$$\mathcal{C}_t(x,\rho) = \frac{2GE_t(x,\rho)}{c^4 R(x,\rho)}. \qquad (100)$$

A region may be overdense without being compact enough to trap:

$$\delta > 0, \mathcal{C} < \mathcal{C}_{\mathrm{h}}. \qquad (101)$$

Likewise, an underdense region is not identified merely by low compactness:

$$\mathcal{C} \ll 1 \not\Longrightarrow \delta < 0. \qquad (102)$$

A uniformly dilute universe may have low compactness everywhere while containing no distinguished void.

Proposition 5. Density contrast alone does not determine compactness

There exist regions B_1, B_2 with equal density contrast,

$$\delta(B_1) = \delta(B_2), \qquad (103)$$

but unequal compactness,

$$\mathcal{C}(B_1) \neq \mathcal{C}(B_2). \qquad (104)$$

Proof

Let the two regions have equal density ρbut different radii $R_1 \neq R_2$. In three effective dimensions,

$$E(B_i) \propto \rho R_i^3. \qquad (105)$$

Therefore

$$\mathcal{C}(B_i) \propto \frac{\rho R_i^3}{R_i} = \rho R_i^2. \tag{106}$$

Hence equal density contrast does not imply equal compactness. ■

The finite theory therefore requires two independent barriers:

$$\boxed{\delta \leq \delta_{\mathrm{v}}} \tag{107}$$

for void formation and

$$\boxed{\mathcal{C} \geq \mathcal{C}_{\mathrm{h}}} \tag{108}$$

for trapping.

6. The Two-Barrier Classification

For every cell x, define the diagnostic pair

$$D_t(x) = \left(\delta_t^{\min}(x), \mathcal{C}_t^{\max}(x)\right). \tag{109}$$

The parameter plane is partitioned by the barriers

$$\delta = \delta_{\mathrm{v}} \tag{110}$$

and

$$\mathcal{C} = \mathcal{C}_{\mathrm{h}}. \tag{111}$$

Definition 11. Intermediate regime

A cell is intermediate when

$$\delta_t^{\min}(x) > \delta_{\mathrm{v}} \tag{112}$$

and

$$\mathcal{C}_t^{\max}(x) < \mathcal{C}_{\mathrm{h}}. \tag{113}$$

This class includes ordinary background cells, walls, filaments, and overdense regions that have not crossed the trapping barrier.

Definition 12. Void regime

A cell is void-classified when

$$\delta_t^{\min}(x) \le \delta_{\mathrm{v}} \qquad (114)$$

and

$$\mathcal{C}_t^{\max}(x) < \mathcal{C}_{\mathrm{h}}. \qquad (115)$$

Definition 13. Trapped regime

A cell is trapped when

$$\mathcal{C}_t^{\max}(x) \ge \mathcal{C}_{\mathrm{h}}. \qquad (116)$$

The collapse barrier is assigned priority because causal trapping changes the permitted transport graph.

The geometric update is therefore

$$h_{t+1}(x) = h_t(x) \vee \hat{h}_{t+1}(x), \qquad (117)$$

followed by

$$v_{t+1}(x) = (1 - h_{t+1}(x))\hat{v}_{t+1}(x). \qquad (118)$$

Equation (118) guarantees

$$h_{t+1}(x)v_{t+1}(x) = 0. \qquad (119)$$

Definition 14. Two-barrier classifier

Define

$$\mathcal{B}: (\delta, \mathcal{C}) \mapsto \begin{cases} H, & \mathcal{C} \ge \mathcal{C}_{\mathrm{h}}, \\ V, & \mathcal{C} < \mathcal{C}_{\mathrm{h}} \text{ and } \delta \le \delta_{\mathrm{v}}, \\ I, & \mathcal{C} < \mathcal{C}_{\mathrm{h}} \text{ and } \delta > \delta_{\mathrm{v}}. \end{cases} \qquad (120)$$

Here

$$H = \text{trapped}, V = \text{void}, \qquad I = \text{intermediate}. \tag{121}$$

The complete finite evolution is

$$F = \mathcal{B} \circ \mathcal{K} \circ \mathcal{T}_g \circ \mathcal{A}. \tag{122}$$

More explicitly,

$$q_{t+1}^{-} = \mathcal{T}_{g_t}\mathcal{A}(q_t), \tag{123}$$
$$D_{t+1} = \mathcal{K}(q_{t+1}^{-}), \tag{124}$$

and

$$g_{t+1} = \mathcal{B}(D_{t+1}, g_t). \tag{125}$$

Theorem 2. Energy conservation under two-barrier classification

Assume that reaction and transport satisfy Propositions 1 and 2 and that the classifier $\mathcal{B}$changes only geometric labels. Then

$$E_{t+1} = E_t. \tag{126}$$

Proof

Reaction conserves total energy. Transport conserves total energy. Computation of δ and C alters no energy variable. The classifier changes only $h$and v. Therefore the complete update conserves E_t. ■

Corollary 2. Void and horizon labels are energetically neutral

The classifications

$$I \to V \tag{127}$$

and

$$I \to H \tag{128}$$

do not themselves create or remove energy.

They record different spatial organizations of the same globally conserved content.

The central two-barrier law is therefore

$$\boxed{\begin{array}{ll} \delta \le \delta_{\mathrm{v}} & \Longrightarrow \text{conservative rarefaction,} \\ \mathcal{C} \ge \mathcal{C}_{\mathrm{h}} & \Longrightarrow \text{causal trapping.} \end{array}} \tag{129}$$

7. Void Components and Void-in-Void Growth

Define the void set

$$V_t = \{x \in X : v_t(x) = 1\}. \tag{130}$$

Let

$$\mathcal{V}_t = \{V_t^{(1)}, \dots, V_t^{(n_t)}\} \tag{131}$$

be the connected components of the induced subgraph

$$\Gamma[V_t]. \tag{132}$$

Each component $V_t^{(\alpha)}$ is a finite void domain.

Definition 15. Void volume

The volume of a void component is

$$\mathrm{Vol}\left(V_t^{(\alpha)}\right) = \sum_{x \in V_t^{(\alpha)}} V_x . \tag{133}$$

Its energy is

$$E_t\left(V_t^{(\alpha)}\right) = \sum_{x \in V_t^{(\alpha)}} e_t\,(x). \tag{134}$$

Its mean density contrast is

$$\delta_t\left(V_t^{(\alpha)}\right) = \frac{E_t(V_t^{(\alpha)})}{\bar{\rho}_t \mathrm{Vol}(V_t^{(\alpha)})} - 1. \tag{135}$$

Definition 16. Void adjacency

Two distinct void components $V_t^{(\alpha)}$ and $V_t^{(\beta)}$ are adjacent when there exist

$$x \in V_t^{(\alpha)}, y \in V_t^{(\beta)} \qquad (136)$$

such that

$$(x, y) \in \mathcal{E}. \qquad (137)$$

They remain distinct at time $t$only because one or more intermediate cells are not void-classified.

Definition 17. Void merger

Suppose an intermediate region

$$C_t \subseteq X \setminus V_t \qquad (138)$$

separates void components $V_t^{(\alpha)}$ and $V_t^{(\beta)}$.

A void merger occurs at time $t + 1$ when

$$C_t \subseteq V_{t+1}. \qquad (139)$$

Then

$$V_t^{(\alpha)} \cup C_t \cup V_t^{(\beta)} \qquad (140)$$

belongs to a single connected component of V_{t+1}.

This is the finite analogue of void-in-void growth.

Proposition 6. Merger reduces the number of void components

If exactly $k \geq 2$void components become connected through newly void-classified cells, then

$$n_{t+1} \leq n_t - k + 1. \qquad (141)$$

Proof

The k previously distinct connected components are replaced by at most one connected component. All other components are unchanged or may participate in additional mergers. Therefore the component count decreases by at least $k-1$. ■

Definition 18. Void merger graph

Construct a graph

$$\mathfrak{M} \tag{142}$$

whose vertices are void components across time.

Place a directed edge

$$V_t^{(\alpha)} \longrightarrow V_{t+1}^{(\beta)} \tag{143}$$

when

$$V_t^{(\alpha)} \cap V_{t+1}^{(\beta)} \neq \emptyset. \tag{144}$$

A node with multiple incoming edges records a merger.

A node with multiple outgoing edges records fragmentation or reclassification.

If void classification is persistent and void regions do not fragment, then $\mathfrak{M}$is a directed merger forest.

Definition 19. Persistent void update

A monotone void model may use

$$v_{t+1}(x) = v_t(x) \vee \hat{v}_{t+1}(x), \tag{145}$$

provided

$$h_{t+1}(x) = 0. \tag{146}$$

Then

$$V_t \subseteq V_{t+1}. \tag{147}$$

This assumption is useful for exact finite classification but is not compulsory. Real underdense regions may be compressed or erased by their environment.

Proposition 7. Finite stabilization of persistent void labels

If X is finite and void classification obeys (145), then there exists t_V such that

$$V_t = V_{t_V} \text{for all } t \geq t_V. \qquad (148)$$

Proof

Every strict inclusion

$$V_t \subsetneq V_{t+1} \qquad (149)$$

adds at least one cell. At most $| X |$ additions are possible. ■

Persistent void classification therefore produces a finite sequence

$$V_0 \subseteq V_1 \subseteq \cdots \subseteq V_{t_V}. \qquad (150)$$

The nonpersistent model is more suitable for studying environmental suppression.

8. Void-in-Cloud Suppression

A small region may satisfy the void barrier at one scale while lying inside a larger overdense environment.

Let

$$\rho_{\mathrm{s}} < \rho_{\mathrm{L}} \qquad (151)$$

denote small and large smoothing radii.

A void-in-cloud candidate satisfies

$$\delta_t(x, \rho_{\mathrm{s}}) \leq \delta_{\mathrm{v}}, \qquad (152)$$

while

$$\delta_t(x, \rho_{\mathrm{L}}) > 0. \qquad (153)$$

The local region is underdense, but its environment is overdense.

Definition 20. Environmental compression functional

Let

$$B_{\mathrm{s}} = B_{\rho_{\mathrm{s}}}(x) \qquad (154)$$

and

$$B_{\mathrm{L}} = B_{\rho_{\mathrm{L}}}(x). \qquad (155)$$

Define the environmental inflow into the small region by

$$\Psi_t(x) = \sum_s \left[\Phi_{t,B_{\mathrm{s}}}^{s,\mathrm{in}} - \Phi_{t,B_{\mathrm{s}}}^{s,\mathrm{out}}\right]. \qquad (156)$$

Thus

$$\Psi_t(x) > 0 \qquad (157)$$

means that net transport is directed into the candidate void.

Define the normalized compression index

$$\Xi_t(x) = \frac{\Psi_t(x)}{E_t(B_{\mathrm{s}}) + \varepsilon}, \qquad (158)$$

where

$$\varepsilon > 0 \qquad (159)$$

prevents division by zero.

Definition 21. Void survival condition

A void candidate survives when

$$\delta_t(x, \rho_{\mathrm{s}}) \leq \delta_{\mathrm{v}} \qquad (160)$$

and

$$\Xi_t(x) < \Xi_c, \qquad (161)$$

where $\Xi_c > 0$ is a compression threshold.

The revised void update is

$$\hat{v}_{t+1}(x) = \mathbf{1}\left[\delta_{t+1}^{\min}(x) \leq \delta_v\right]\mathbf{1}[\Xi_t(x) < \Xi_c]. \qquad (162)$$

A sufficiently strong environmental inflow suppresses the void label.

Definition 22. Void crushing

A void-classified cell is crushed when

$$v_t(x) = 1, \qquad (163)$$

but

$$v_{t+1}(x) = 0 \qquad (164)$$

because

$$\Xi_t(x) \geq \Xi_c \qquad (165)$$

or because its density contrast rises above the void barrier:

$$\delta_{t+1}^{\min}(x) > \delta_v. \qquad (166)$$

Proposition 8. Sustained inward flux weakens an underdensity

Assume fixed regional volume and constant mean density. If

$$\Psi_t(x) > 0, \qquad (167)$$

then

$$\delta_{t+1}(x, \rho_s) > \delta_t(x, \rho_s). \qquad (168)$$

Proof

By definition,

$$\Psi_t = \Phi^{\mathrm{in}} - \Phi^{\mathrm{out}}. \qquad (169)$$

If $\Psi_t > 0$, regional energy increases:

$$E_{t+1}(B_{\mathrm{s}}) > E_t(B_{\mathrm{s}}). \qquad (170)$$

For fixed volume,

$$\rho_{t+1}(B_{\mathrm{s}}) > \rho_t(B_{\mathrm{s}}). \qquad (171)$$

With constant $\bar{\rho}$, equation (61) gives (168). ■

Thus the same conservative transport law can produce either

$$\Phi^{\mathrm{out}} > \Phi^{\mathrm{in}} \Longrightarrow \text{void deepening,} \qquad (172)$$

or

$$\Phi^{\mathrm{in}} > \Phi^{\mathrm{out}} \Longrightarrow \text{void suppression.} \qquad (173)$$

The distinction is dynamical rather than ontological.

9. Walls, Filaments, and Boundary Deposition

Void formation redistributes energy toward the boundary of the evacuating region.

For $B \subseteq X$, define its exterior boundary by

$$\partial^+ B = \{y \in X \setminus B : \exists x \in B \text{ with } (x, y) \in \mathcal{E}\}. \qquad (174)$$

Define its interior boundary by

$$\partial^- B = \{x \in B : \exists y \notin B \text{ with } (x, y) \in \mathcal{E}\}. \qquad (175)$$

Definition 23. Boundary deposition

The energy deposited from Binto its exterior boundary is

$$D_t(B) = \sum_s \sum_{\substack{x \in B \\ y \in \partial^+ B}} P_t^s(x, y) q_t^s(x). \qquad (176)$$

If transport out of Bterminates entirely in $\partial^+ B$, then

$$D_t(B) = \sum_s \Phi_{t,B}^{s,\mathrm{out}}. \qquad (177)$$

Proposition 9. Void evacuation feeds its boundary

Assume no energy passes directly from Bbeyond $\partial^+ B$in one time step. Then

$$E_{t+1}(\partial^+ B) - E_t(\partial^+ B) \geq D_t(B) - L_t, \qquad (178)$$

where $L_t$is the energy transported from $\partial^+ B$to more distant cells.

Proof

The exterior boundary gains $D_t(B)$from the void interior. It may simultaneously lose $L_t$through outward transport. Other incoming contributions are nonnegative. Therefore its net increase is at least $D_t(B) - L_t$. ■

When

$$D_t(B) > L_t, \qquad (179)$$

the boundary becomes denser:

$$E_{t+1}(\partial^+ B) > E_t(\partial^+ B). \qquad (180)$$

This produces a finite wall around the void.

Definition 24. Wall cell

Fix an overdensity threshold

$$\delta_{\mathrm{w}} > 0. \qquad (181)$$

A cell $x \notin V_t$ is a wall cell when

$$x \in \partial^+ V_t \qquad (182)$$

and

$$\delta_t(x, \rho_{\mathrm{w}}) \geq \delta_{\mathrm{w}}. \qquad (183)$$

The wall set is

$$W_t = \{x \in \partial^+ V_t : \delta_t(x, \rho_{\mathrm{w}}) \geq \delta_{\mathrm{w}}\}. \qquad (184)$$

Definition 25. Multi-void incidence number

For $x \in X \setminus V_t$, define

$$\nu_t(x) = \#\left\{V_t^{(\alpha)} : x \in \partial^+ V_t^{(\alpha)}\right\}. \qquad (185)$$

Thus $\nu_t(x)$ counts the number of distinct void components incident on x.

A simple morphological classification is

$$\begin{cases} \nu_t(x) = 1, & \text{single-void wall,} \\ \nu_t(x) = 2, & \text{wall intersection or filament candidate,} \\ \nu_t(x) \geq 3, & \text{node candidate.} \end{cases} \qquad (186)$$

Definition 26. Filament set

Let $\delta_{\mathrm{f}} > 0$. Define

$$F_t = \{x \in X : \nu_t(x) \geq 2 \text{ and } \delta_t(x, \rho_{\mathrm{f}}) \geq \delta_{\mathrm{f}}\}. \qquad (187)$$

Definition 27. Node set

Let $\delta_{\mathrm{n}} \geq \delta_{\mathrm{f}}$. Define

$$N_t = \{x \in X : \nu_t(x) \geq 3 \text{ and } \delta_t(x, \rho_{\mathrm{n}}) \geq \delta_{\mathrm{n}}\}. \qquad (188)$$

The finite cosmic web is then

$$\mathcal{W}_t = V_t \sqcup W_t \sqcup F_t \sqcup N_t \sqcup I_t, \qquad (189)$$

where I_t contains cells not assigned to the other morphological classes.

The hierarchy is

$$\boxed{\text{void interiors} \to \text{walls} \to \text{filaments} \to \text{nodes.}} \qquad (190)$$

Proposition 10. Boundary intersections amplify deposition

Suppose $x$lies on the exterior boundaries of k evacuating voids:

$$\nu_t(x) = k. \qquad (191)$$

If each incident void contributes at least $d > 0$ energy units to x, then the total void-derived deposition satisfies

$$D_t(x) \geq kd. \qquad (192)$$

Proof

The contributions from the $k$distinct incident components are nonnegative and each is at least d. Their sum is therefore at least kd. ■

Thus cells shared by multiple void boundaries are natural sites of enhanced accumulation.

The finite graph produces the structural sequence

$$\boxed{\text{interior evacuation} \Longrightarrow \text{boundary deposition} \Longrightarrow \text{wall intersections} \Longrightarrow \text{filaments and nodes.}} \qquad (193)$$

No separate wall or filament substance is required. The cosmic-web morphology arises from conservative transport between underdense and overdense regions.

12.1 Relation to Continuum Void Theory

The finite density contrast

$$\delta_t(x,\rho) = \frac{\rho_t(x,\rho)}{\bar{\rho}_t} - 1 \qquad (269)$$

is the discrete analogue of the cosmological density contrast

$$\delta(\mathbf{x},t) = \frac{\rho(\mathbf{x},t) - \bar{\rho}(t)}{\bar{\rho}(t)}. \qquad (270)$$

In an expanding continuum background, nonrelativistic matter satisfies the continuity equation

$$\frac{\partial\delta}{\partial t}+\frac{1}{a}\nabla\cdot[(1+\delta)\mathbf{v}]=0, \qquad (271)$$

where $a(t)$is the scale factor and $\mathbf{v}$is the peculiar velocity.

For small perturbations,

$$|\delta| \ll 1, \qquad (272)$$

equation (271) becomes

$$\frac{\partial\delta}{\partial t}+\frac{1}{a}\nabla\cdot\mathbf{v}=0. \qquad (273)$$

Hence

$$\nabla\cdot\mathbf{v}>0 \qquad (274)$$

drives

$$\frac{\partial\delta}{\partial t}<0. \qquad (275)$$

A divergent velocity field therefore deepens an underdensity.

The finite counterpart is

$$\Phi_{t,B}>0 \Longrightarrow \delta_{t+1}(B)<\delta_t(B). \qquad (276)$$

Thus

$$\boxed{\nabla\cdot\mathbf{v}>0 \leftrightarrow \Phi_{t,B}>0.} \qquad (277)$$

The continuum gravitational potential Φsatisfies, in the Newtonian cosmological regime,

$$\nabla^2\Phi=4\pi G a^2\bar{\rho}\,\delta. \qquad (278)$$

Underdense regions have

$$\delta < 0, \qquad (279)$$

and therefore generate gravitational acceleration away from their least-dense interiors relative to the surrounding matter distribution.

The finite transport kernel $P_t(x, y)$replaces the continuum velocity and force fields by exact cell-to-cell transfers.

The structural correspondence is

$$\begin{array}{ccc} \text{continuum cosmology} & & \text{finite model} \\ \rho(\mathbf{x}, t) & \longleftrightarrow & e_t(x)/V_x, \\ \bar{\rho}(t) & \longleftrightarrow & E_t/\sum_x V_x\,, \\ \delta(\mathbf{x}, t) & \longleftrightarrow & \delta_t(x, \rho), \\ \nabla \cdot \mathbf{v} & \longleftrightarrow & \Phi_{t,B}, \\ \text{void shell} & \longleftrightarrow & W_t, \\ \text{filament} & \longleftrightarrow & F_t, \\ \text{cluster node} & \longleftrightarrow & N_t, \\ \text{collapsed region} & \longleftrightarrow & H_t. \end{array} \qquad (280)$$

This correspondence is structural rather than metric-exact.

12.2 Expansion-Aware Extension

The present graph has fixed cell volumes unless otherwise specified.

Cosmological void growth occurs in an expanding background. To represent expansion, let the effective cell volume evolve as

$$V_x(t) = a_t^d V_x(0), \qquad (281)$$

where $d$is the effective spatial dimension.

The local density becomes

$$\rho_t(x) = \frac{e_t(x)}{a_t^d V_x(0)}. \qquad (282)$$

The global mean density is

$$\bar{\rho}_t = \frac{E_t}{a_t^d \sum_x V_x(0)}. \qquad (283)$$

If all cells expand uniformly, then the density contrast remains

$$\delta_t(x) = \frac{\rho_t(x)}{\bar{\rho}_t} - 1 = \frac{e_t(x) \sum_y V_y(0)}{E_t V_x(0)} - 1. \qquad (284)$$

Uniform expansion therefore cancels from the contrast.

The background expansion affects absolute density and compactness, while relative structure remains governed by redistribution.

Let the effective radius scale as

$$R_t(x,\rho) = a_t R_0(x,\rho). \qquad (285)$$

Then compactness becomes

$$\mathcal{C}_t(x,\rho) = \frac{2GE_t(x,\rho)}{c^4 a_t R_0(x,\rho)}. \qquad (286)$$

For fixed enclosed energy,

$$a_t \uparrow \Longrightarrow \mathcal{C}_t \downarrow. \qquad (287)$$

Background expansion therefore opposes collapse unless local concentration overcomes the increase in physical scale.

The two competing tendencies are

$$\boxed{\begin{array}{ll} \text{expansion} & : R \uparrow, \mathcal{C} \downarrow, \\ \text{concentration} & : E(R) \uparrow, \mathcal{C} \uparrow. \end{array}} \qquad (288)$$

For voids, expansion increases physical size while outward peculiar transport deepens the underdensity.

A finite void radius may be written

$$R_{\mathrm{V}}(t) = a_t R_{\mathrm{V}}^{\mathrm{com}}(t), \qquad (289)$$

where $R_{\mathrm{V}}^{\mathrm{com}}$ is the comoving radius.

Void growth beyond uniform expansion corresponds to

$$R_{\mathrm{V}}^{\mathrm{com}}(t+1) > R_{\mathrm{V}}^{\mathrm{com}}(t). \qquad (290)$$

Definition 42. Comoving void growth

A void component grows comovingly when

$$\mathrm{Vol}_{\mathrm{com}}\left(V_{t+1}^{(\alpha)}\right) > \mathrm{Vol}_{\mathrm{com}}\left(V_{t}^{(\alpha)}\right). \qquad (291)$$

Physical growth then satisfies

$$\mathrm{Vol}_{\mathrm{phys}}\left(V_{t}^{(\alpha)}\right) = a_t^d \mathrm{Vol}_{\mathrm{com}}\left(V_{t}^{(\alpha)}\right). \qquad (292)$$

The finite model can therefore distinguish

background expansion (293)

from

dynamical void enlargement. (294)

12.3 Two-Barrier Excursion Structure

Let the smoothed contrast associated with an initial state be written

$$\delta(R). \qquad (295)$$

As the smoothing scale changes, the trajectory

$$R \longmapsto \delta(R) \qquad (296)$$

may cross either an upper collapse barrier or a lower void barrier.

Let

$$\delta_{\mathrm{c}} > 0 \qquad (297)$$

be an overdensity threshold and

$$\delta_{\rm v} < 0 \qquad (298)$$

be an underdensity threshold.

The two-barrier condition is

$$\delta(R) \geq \delta_{\rm c} \qquad (299)$$

for overdense collapse and

$$\delta(R) \leq \delta_{\rm v} \qquad (300)$$

for void formation.

The present finite compactness barrier is not identical to $\delta_{\rm c}$, but the two may be related by a scale-dependent map

$$\mathcal{C}(R) = \mathfrak{C}(\delta(R), R, \bar{\rho}). \qquad (301)$$

In three dimensions,

$$E(R) = \frac{4\pi}{3} R^3 \bar{\rho}(1 + \delta(R)), \qquad (302)$$

so

$$\mathcal{C}(R) = \frac{8\pi G}{3c^4} R^2 \bar{\rho}(1 + \delta(R)). \qquad (303)$$

Thus the compactness barrier becomes

$$R^2 \bar{\rho}(1 + \delta(R)) \geq \frac{3c^4 \mathcal{C}_{\rm h}}{8\pi G}. \qquad (304)$$

Equation (304) shows explicitly that collapse depends on both contrast and scale.

A large region with moderate overdensity may have the same compactness as a smaller region with stronger overdensity.

The two-barrier phase space is therefore more accurately written

$$\boxed{(\delta_{\min}, \mathcal{C}_{\max}),} \qquad (305)$$

rather than as density contrast alone.

Definition 43. First-barrier crossing

For an initial state S_0, define

$$\tau_{\mathrm{V}} = \inf\{t: \delta_t^{\min} \le \delta_{\mathrm{v}}\}, \qquad (306)$$

and

$$\tau_{\mathrm{H}} = \inf\{t: \mathcal{C}_t^{\max} \ge \mathcal{C}_{\mathrm{h}}\}. \qquad (307)$$

The first geometric outcome is determined by

$$\tau_* = \min\{\tau_{\mathrm{V}}, \tau_{\mathrm{H}}\}. \qquad (308)$$

If

$$\tau_{\mathrm{V}} < \tau_{\mathrm{H}}, \qquad (309)$$

void formation occurs first.

If

$$\tau_{\mathrm{H}} < \tau_{\mathrm{V}}, \qquad (310)$$

trapping occurs first.

The equality

$$\tau_{\mathrm{V}} = \tau_{\mathrm{H}} \qquad (311)$$

defines a finite codimension-one competition boundary when such states exist.

12.4 Exact Computation Program

For bounded state space $\mathcal{S}_N$, every initial configuration may be evolved exactly.

The exhaustive program consists of:

$$\boxed{\begin{array}{ll} 1. & \text{enumerate initial states,} \\ 2. & \text{apply conservative reaction,} \\ 3. & \text{apply exact transport,} \\ 4. & \text{compute all admitted } \delta(x,\rho), \\ 5. & \text{compute all admitted } \mathcal{C}(x,\rho), \\ 6. & \text{classify void, intermediate, and trapped cells,} \\ 7. & \text{record walls, filaments, and nodes,} \\ 8. & \text{continue until geometric and dynamical recurrence.} \end{array}} \qquad (312)$$

For each initial state, record the certificate

$$\mathfrak{C}(S_0) = (E_0, Q_0, \tau_{\mathrm{V}}, \tau_{\mathrm{H}}, f_{\mathrm{V}}^{\max}, f_{\mathrm{H}}^{\max}, \Sigma_{\max}, \mathcal{F}_{\mathrm{V}}, \mathcal{O}), \qquad (313)$$

where

$$Q_0 = \sum_x (m_0(x) - \bar{m}_0(x)) \qquad (314)$$

is the conserved matter imbalance and $\mathcal{O}$is the eventual orbit data.

A complete atlas may partition the state space as

$$\mathcal{S}_N = \mathcal{S}_N^{\mathrm{V}} \sqcup \mathcal{S}_N^{\mathrm{I}} \sqcup \mathcal{S}_N^{\mathrm{H}}. \qquad (315)$$

The corresponding counts are

$$N_{\mathrm{V}} = |\, \mathcal{S}_N^{\mathrm{V}} \,|, \qquad (316)$$
$$N_{\mathrm{I}} = |\, \mathcal{S}_N^{\mathrm{I}} \,|, \qquad (317)$$

and

$$N_{\mathrm{H}} = |\, \mathcal{S}_N^{\mathrm{H}} \,|. \qquad (318)$$

They satisfy

$$N_{\mathrm{V}} + N_{\mathrm{I}} + N_{\mathrm{H}} = |\ \mathcal{S}_N\ |. \qquad (319)$$

Definition 44. Outcome entropy

Let

$$p_C = \frac{N_C}{|\ \mathcal{S}_N\ |}, C \in \{\mathrm{V}, \mathrm{I}, \mathrm{H}\}. \qquad (320)$$

Define

$$\mathcal{H}_{\mathrm{out}} = -\sum_C p_C \log p_C. \qquad (321)$$

A low value indicates dominance by one outcome class.

A high value indicates substantial competition among void formation, intermediate evolution, and collapse.

Definition 45. Boundary complexity

Let

$$\partial_{\mathrm{VH}} \qquad (322)$$

be the set of neighboring initial states with different void or horizon outcomes.

Define

$$B_{\mathrm{VH}} = |\ \partial_{\mathrm{VH}}\ |. \qquad (323)$$

A normalized boundary complexity is

$$\beta_{\mathrm{VH}} = \frac{B_{\mathrm{VH}}}{|\ \mathcal{S}_N\ |}. \qquad (324)$$

Refining N, the graph, or the allowed transport rules permits the study of whether

$$\beta_{\mathrm{VH}} \qquad (325)$$

converges, vanishes, or exhibits scale-dependent growth.

12.5 Limitations

The finite model does not derive the cosmological initial conditions responsible for a particular observed void.

It does not establish that the Boötes Void arose from antimatter, radiation conversion, or horizon-related processes.

It does not reproduce:

$$\text{general relativistic cosmological expansion,} \tag{326}$$
$$\text{dark matter microphysics,} \tag{327}$$
$$\text{dark energy,} \tag{328}$$
$$\text{baryonic feedback,} \tag{329}$$
$$\text{galaxy bias,} \tag{330}$$

or

$$\text{redshift-space distortions.} \tag{331}$$

The rules

$$\delta \leq \delta_{\mathrm{v}} \tag{332}$$

and

$$\mathcal{C} \geq \mathcal{C}_{\mathrm{h}} \tag{333}$$

are admitted classifiers.

They are not derived from an exact discretization of Einstein's equations.

The wall, filament, and node classifications depend on chosen thresholds and graph morphology.

The finite construction should therefore be interpreted as

$$\boxed{\text{an exact algebraic model of structural logic, not a calibrated cosmological simulation.}} \tag{334}$$

12.6 Principal Result

The principal result is the coexistence of two conservative geometric outcomes within one finite update law.

The complete evolution is

$$F = \mathcal{B} \circ \mathcal{K} \circ \mathcal{T}_g \circ \mathcal{A}. \qquad (335)$$

Reaction preserves total energy:

$$E(\mathcal{A}(q)) = E(q). \qquad (336)$$

Transport preserves total energy:

$$E(\mathcal{T}_g q) = E(q). \qquad (337)$$

Void formation occurs through net outward flux:

$$\Phi_{t,B} > 0 \Longrightarrow \delta_{t+1}(B) < \delta_t(B). \qquad (338)$$

Collapse becomes possible through increasing compactness:

$$E_t(B) \uparrow \Longrightarrow \mathcal{C}_t(B) \uparrow \qquad (339)$$

for fixed effective radius.

The same redistribution may generate both:

$$\boxed{\Delta E(B) < 0 \text{and} \Delta E(C) > 0.} \qquad (340)$$

Hence

$$\boxed{\text{rarefaction and concentration are complementary consequences of conservation.}} \qquad (341)$$

A void occupies space with relatively little energy:

$$f_{\mathrm{V}} \gg \epsilon_{\mathrm{V}}. \qquad (342)$$

A dense node occupies little space with relatively large energy:

$$\epsilon_{\mathrm{N}} \gg f_{\mathrm{N}}. \qquad (343)$$

The cosmic web is therefore represented as a segregation of conserved content across morphology.

12.7 Conclusion

The finite theory begins with a closed reaction–transport system.

Matter and antimatter may convert into radiation:

$$M + \bar{M} \longrightarrow R, \qquad (344)$$

but the total energy remains invariant.

Transport redistributes that energy across a finite spatial graph:

$$q_{\bar{t+1}} = \mathcal{T}_g \mathcal{A}(q_t). \qquad (345)$$

Two geometric diagnostics are then evaluated.

The underdensity diagnostic is

$$\delta_t(x,\rho) = \frac{\rho_t(x,\rho)}{\bar{\rho}_t} - 1. \qquad (346)$$

The compactness diagnostic is

$$\mathcal{C}_t(x,\rho) = \frac{2GE_t(x,\rho)}{c^4 R(x,\rho)}. \qquad (347)$$

The two barriers are

$$\delta \leq \delta_{\mathrm{v}} \qquad (348)$$

and

$$\mathcal{C} \geq \mathcal{C}_{\mathrm{h}}. \qquad (349)$$

They produce distinct outcomes:

$$\boxed{\begin{array}{ll} \delta \le \delta_{\mathrm{v}} & \Longrightarrow \text{void classification,} \\ \mathcal{C} \ge \mathcal{C}_{\mathrm{h}} & \Longrightarrow \text{trapped classification.} \end{array}} \tag{350}$$

Void interiors lose energy through outward transport.

Their boundaries gain energy and may become walls.

Intersections of walls may become filaments.

Intersections of filaments may become dense nodes.

Some nodes may eventually cross the compactness barrier.

Thus the full finite sequence is

$$\boxed{\text{underdensity} \to \text{evacuation} \to \text{walls} \to \text{filaments} \to \text{nodes} \to \text{possible trapping.}} \tag{351}$$

The Boötes Void belongs to the underdensity branch of this architecture.

It is modeled not as empty space, annihilated matter, or a hidden region, but as

$$\boxed{\text{a large observable domain whose energy density remains below the cosmic mean.}} \tag{352}$$

The central conservation statement is

$$\boxed{\text{what leaves the void builds the web.}} \tag{353}$$

The two-barrier model therefore unifies cosmic rarefaction and gravitational concentration without identifying them:

$$\boxed{\text{voids and horizons are opposite geometric outcomes of one conserved redistribution.}} \tag{354}$$

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