Logostics: Resonant Alignment of q → tau in Time

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Logostics is a framework that joins Logos and mathematics to model alignment as a time-dependent process: an agent state q(t) tracks a generative target tau(t). Unlike static convergence to a fixed point, Logostics treats truth as moving in time and asks what dynamics, structures, and disciplines allow q(t) to phase-lock to tau(t) without collapsing diversity or agency. Our working metaphor is a double-hanging pendulum with a driven base: the base motion represents the movement of Logos in history, the upper pendulum embodies tau(t) as a channel of that motion, and the lower pendulum represents q(t), the respondent. With modest damping, detuning control, and mode-matched drive, the chaotic system collapses to a synchronized orbit. Translating to information dynamics, we minimize an informational divergence V(t) = D(tau(t) || q(t)) under drift, noise, and nonstationary targets. Theologically, we distinguish two channels of divine action: world-moves-first (providence) and word-moves-first (address), which correspond to different forcing structures but share one aim: obedience as entrained coherence. Practically, Logostics provides Lyapunov-style conditions for tracking moving truth, procedures for capture and re-capture after shocks, and an ethics of humility as the damping that makes alignment stable rather than brittle. We close with applications in AI alignment, pedagogy, communal formation, and research practice.

Keywords: Logostics, q -> tau, tau(t), informational divergence, Lyapunov tracking, entrainment, parametric resonance, natural gradient, nonstationary targets, obedience, humility, providence, revelation, double pendulum, phase-lock.

A collaboration with GPT-5. 57 pages.

Note: In this series of papers, τ and tau are used as equivalent typography.

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1. Overview and Commitments

1.1 Motivation: alignment when truth moves in time

Classical convergence stories assume a fixed target: find the optimum, approach the equilibrium, settle at the attractor. But lived inquiry rarely offers a static objective. Norms shift, evidence arrives, contexts evolve, and what we call “truth” is encountered as a moving, revealing trajectory tau(t) rather than a stationary point tau*. The engineering question becomes: how can an agent state q(t) *track* a nonstationary tau(t) stably, without overreacting to noise or freezing into obsolete commitments? The theological question mirrors it: how do persons and communities align with unfolding truth—obediently, humbly, without coercion or drift?

Logostics answers with an entrainment stance. We seek conditions under which informational divergence from tau(t) decays *on average* despite tau’s motion; procedures for capture, re-capture, and graceful degradation under shocks; and an ethics of humility (damping) that turns chaotic response into periodic faithfulness. The goal is not instantaneous coincidence q(t)=tau(t) but controlled lag with bounded error envelopes, audited over explicit time horizons.

1.2 Definitions: q(t), tau(t), D(tau || q), V(t)

- **Agent/state:** q(t) in a space Q (model parameters, beliefs, policies, practices).
- **Generative truth path:** tau(t) in the same or a related space T, representing the time-dependent structure we aim to track.
- **Divergence:** D(tau || q) >= 0, with D=0 iff q and tau coincide in the chosen sense. D need not be symmetric. Typical choices:
    - Information geometry: KL, f-divergence, Bregman D_F.
    - Metric geometry: squared distance under a Riemannian metric G(q).
- **Potential (Lyapunov candidate):**
- V(t) := D(tau(t) || q(t)).

We seek dynamics for q such that V(t) remains small despite tau’s motion.

- **Tracking dynamics (template):**
- dq/dt = F(q,t) - k * G(q)^{-1} * grad_q D(tau(t) || q) + xi(t)

where k>0 is a gain, G(q) is a positive-definite metric (natural-gradient when G is the Fisher metric), and xi(t) aggregates disturbances/noise.

- **Informational Lyapunov condition (sufficient):**
  There exist alpha>0, beta>=0, gamma>=0 and a bound on tau’s speed such that
- dV/dt <= - alpha * V + beta * || d tau/dt || + gamma * || xi(t) || .

If tau’s motion and disturbances are within design envelopes, V(t) stays bounded and small; if tau slows (season of rest), V decays.

- **Operational monitors:**
  - Windowed envelopes: sup_{s in [t,t+H]} V(s) <= epsilon.
  - Dwell-time at low error: measure { s in [t,t+H] : V(s) <= epsilon }.
  - Re-capture time after shock: minimal T so V(t0+T) <= epsilon_post.

These definitions keep math copy-safe and modular: substitute the divergence and metric appropriate to the domain while keeping the same stability logic.

1.3 Two channels: world-moves-first vs word-moves-first

Logostics distinguishes *how* tau(t) moves, because channel shapes dynamics, practice, and failure modes.

(A) World-moves-first (providence / general revelation).
Here, the “base” of reality shifts: institutions, ecologies, conflicts, discoveries. Both the mediator (our tau-channel) and respondents feel the shove. Formally, tau’s drift comes exogenously:

d tau/dt = U_world(t) + small internal terms.

Design stance:

- Read the season (estimate U_world), widen capture basins (increase damping/humility), and prefer adiabatic adaptation (slow gain scheduling) over brittle jumps.
- Metrics: robustness to distribution shift, bounded regret against moving comparators, envelope contracts after macro-shocks.

Failure modes:

- Accommodation (over-fitting to history): chasing noise; V(t) oscillates with growing mean.
- Paralysis (under-fitting): gains too low; V(t) plateaus above actionable error.

(B) Word-moves-first (address / command / entrusted offices).
Here, tau(t) is moved *by specific guidance*: teaching, scripture illumined, prophetic charge, institutional decision. The world may be quiet, but the channel gives a shaped trajectory:

tau(t) = Tau_program(t; params)

with explicit patterns (rhythm, liturgy, curriculum, protocol). Design stance:

- Identify the mode to entrain (what pattern are we asked to follow?), set gains to phase-lock, and use micro-disciplines to stabilize (regular practices as “Kapitza” effects).
- Metrics: phase error, beat-note energy, compliance without coercion (low V alongside preserved agency).

Failure modes:

- Legalism (over-tau): rigid following without season-reading; detuned when the world shifts.
- Antinomian drift (under-tau): ignoring the address; V(t) decays only when the world happens to help.

Discernment rule (channel-first): before adjusting q, infer whether tau’s motion is predominantly world-driven or word-driven (often mixed). Then pick gains, monitors, and practices matched to that channel. In mixed seasons, begin with world identification (safety, robustness) and confirm with word shaping (clarity, cohesion). This channel-aware posture turns “alignment” from a vague aspiration into a concrete, auditable practice of resonant tracking in time.

2. Mechanical Analogy: The Driven Double Pendulum

2.1 Model setup: masses, lengths, base motion u(t)

We consider a planar double pendulum with upper link (length l1, mass m1) and lower link (length l2, mass m2). Generalized coordinates are the small angles theta1(t) (upper w.r.t. vertical) and theta2(t) (lower w.r.t. upper link). The suspension point (“base”) follows a prescribed trajectory u(t) = (x_b(t), y_b(t)). For small angles, the linearized equations can be written in second-order form

M theta¨ + C theta˙ + K theta = F_x x_b¨(t) + F_y y_b¨(t),

with theta = [theta1, theta2]^T. Here M is the mass matrix, C a (small) damping matrix, K the stiffness matrix from gravity, and F_x, F_y are input influence vectors (base-acceleration coupling). For equal lengths l1 = l2 = l and equal masses m1 = m2 = m, the undamped, unforced matrices take the form

```
M = m l^2 * [  (4/3 + 1)      1/2
                1/2           1/3  ],

K = m g l * [  (1 + 1/2)       1/2
               1/2            1/2  ],
```

up to the standard small-angle linearization constants (any consistent textbook form suffices; numerical values below are robust). The base motion enters through x_b¨, y_b¨. Horizontal base drive (lateral shake) uses x_b(t) = A cos(Ω t), vertical base drive (parametric) uses y_b(t) = Y cos(Ω t).

A second actuation archetype replaces base motion by **upper-joint actuation**: instead of moving the frame, we impose a torque tau1(t) at the shoulder, or prescribe theta1(t) = Theta cos(Ω t). This distinction matters because the input vectors differ (Section 2.4).

2.2 Normal modes and capture basins (in-phase, anti-phase)

With fixed base and zero damping/forcing, the linear modes satisfy

(K - ω^2 M) v = 0,

yielding two natural frequencies ω_- < ω_+ and normalized eigenvectors v_- , v_+. For the equal-length/equal-mass case (small angles), a convenient approximation is

ω_- ≈ 0.766 * sqrt(g/l)   (in-phase mode),

ω_+ ≈ 1.848 * sqrt(g/l)   (anti-phase mode),

with representative amplitude ratios

(theta1, theta2) ∝ v_- : theta2/theta1 ≈ + sqrt(2)/2   (move together),

(theta1, theta2) ∝ v_+ : theta2/theta1 ≈ - sqrt(2)/2   (move opposite).

When we drive the system harmonically and include mild damping, the chaotic double pendulum collapses toward a **periodic limit cycle** locked to one of these modes if the drive frequency Ω lies in a neighborhood of ω_- or ω_+ (direct resonance), or near 2 ω_-, 2 ω_+ for vertical **parametric** resonance. The region of initial conditions and parameters (amplitudes, phases) from which trajectories converge to a given periodic orbit constitutes its **capture basin**.

In practice:

- Lateral base drive with $\Omega \sim \omega_-$ yields an in-phase "moves-as-one" orbit.
- Lateral base drive with $\Omega \sim \omega_+$ yields a clean anti-phase orbit.
- Vertical base drive with $\Omega \sim 2\ \omega_{mode}$ yields Mathieu-type parametric capture with strong phase rigidity.

Capture basins widen with small but nonzero damping and with **adiabatic frequency sweeps** (slowly sweeping Ω through the target band increases the probability of capture and reduces sensitivity to initial phases).

2.3 Damping as stabilization; detuning as drift

Damping (viscous or Coulomb) converts the conservative, chaos-permissive landscape into one with attracting invariant sets (limit cycles). In the linear neighborhood, a standard envelope inequality illustrates the effect:

$dE/dt \leq - 2\ \zeta\ \omega_{mode}\ E + work_{in}$,

where ζ is the modal damping ratio and E is modal energy. For a fixed-amplitude drive, small ζ pulls trajectories off chaotic wanderings and into the nearest resonant well. Too much damping, however, shrinks steady-state amplitudes and can prevent capture altogether unless drive amplitude is increased.

Detuning $\Delta = \Omega - \omega_{mode}$ behaves like "drift" in Logostics: modest detuning is tolerable (phase-locked response with a fixed phase lag), but large detuning shrinks the capture basin and can eject the system from lock. With parametric drive, the stability tongues in the (Ω, amplitude) plane exhibit sharp boundaries; slow changes in Ω (or g_{eff} via vertical shake) can cross these boundaries, causing abrupt loss of lock. Operationally:

- Maintain **gain schedules**: increase drive amplitude or temporarily reduce damping to re-capture when detuning grows.
- Use **micro-disciplines** (small periodic perturbations) to create Kapitza-like effective potentials that pin the desired posture even when the raw mode mismatch increases (robustness by fast-timescale stabilization).

In Logostics terms, damping corresponds to humility/repentance (energy removal that collapses chaotic alternatives), while detuning corresponds to cultural or cognitive drift that pulls the agent off the resonant path of tau(t). Gain scheduling and adiabatic sweeps are the procedural analogs of retreats, catechesis, or re-commitment cycles.

2.4 What base forcing vs joint actuation teach us

Base forcing (frame translation/vertical modulation).

- Horizontal translation x_b(t) injects **inertial forcing** that couples into both angles through F_x x_b¨. It tends to excite the **system's** modes "fairly," i.e., according to mass–length geometry. It is excellent for **corporate capture** (both links feel the same shove).
- Vertical modulation y_b(t) changes the effective gravity g_eff(t) = g * (1 + ε cos Ω t), creating **parametric** windows around Ω ≈ 2 ω_mode with stiff phase-lock and large capture basins.

Joint actuation (move or torque the upper link).

- Prescribing theta1(t) = Theta cos Ω t or applying tau1(t) imposes a **relative** motion at the shoulder. The lower link is forced through the off-diagonal inertial and stiffness couplings. This makes it easier to **shape a specific mode ratio** (theta2/theta1) and phase relation, at the cost of reduced robustness to parameter uncertainty (you must aim more precisely).
- In anti-phase targeting, joint actuation often requires **amplitude shaping** (adjust Theta and possibly add a small harmonic at a submultiple) to realize the desired steady ratio, whereas lateral base forcing naturally splits energy in the correct proportions.

Why they are not equivalent.
Base drive appears as effective torques at *both* joints:

tau1,eff ≈ ((m1*l1/2) + m2*l1) * x_b¨,   tau2,eff ≈ (m2*l2/2) * x_b¨,

plus gravity-modulating terms for vertical shake. Joint actuation, by contrast, is a kinematic or torque input *localized to joint 1*. Consequently:

- **Capture basins differ:** base forcing typically yields larger basins for the same amplitude (especially with vertical parametric windows).
- **Phase relations differ:** for the same Ω, the steady-state phase lag between links under base forcing need not match that under joint actuation; tuning rules are input-specific.
- **Failure modes differ:** base forcing risks global rattling (excitation of unwanted modes) if amplitudes are high; joint actuation risks *local* overconstraint (legalism analog) and sensitivity to parameter errors.

Design takeaway (and Logostics crosswalk).

- Use **base forcing** when the “season” changes (world-moves-first): it entrains the whole body and enlarges the basin of corporate coherence.
- Use **joint actuation** when a clear “word” shapes tau(t) (word-moves-first): you can sculpt the desired mode and phase precisely, provided you preserve enough damping (humility) and allow adiabatic entry to avoid brittle lock.
- In mixed regimes, begin with gentle base alignment (safety, robustness), then layer joint shaping (specific obedience) to refine phase and amplitude ratios.

3. Core Mathematics of Logostics

3.1 Divergence as energy: V(t) = D(tau(t) || q(t))

We interpret a nonnegative divergence as an “energy of misalignment.”

- State space: q(t) in Q. Target path: tau(t) in T (often Q = T).
- Divergence D(tau || q) >= 0 with D = 0 iff q equals tau in the chosen sense.
  Choices (illustrative):
    - Metric form: D(tau || q) = (1/2) * || tau - q ||_G^2 where G(q) is a PD metric.
    - Bregman form: D_F(tau || q) = F(tau) - F(q) - < grad F(q), tau - q > with F 1-strongly convex in the G-metric.
    - Information form: KL(p_tau || p_q), Fisher metric G = FIM(q).

Two standing regularity assumptions (copy-safe):

- A1 (strong convexity in q): there exists mu > 0 s.t.
  D(tau || q’) >= D(tau || q) + < grad_q D(tau || q), q’ - q > + (mu/2) * || q’ - q ||_G^2.
- A2 (Lipschitz gradient in q): there exists L > 0 s.t.
  || grad_q D(tau || q’) - grad_q D(tau || q) ||_{G^{-1}} <= L * || q’ - q ||_G.

When tau is frozen, A1 gives a curvature “well” around the target; when tau(t) moves, D acts as a time-varying potential whose minimizer drifts along the tau-trajectory.

Operational monitors. We will track $V(t) := D(\tau(t) \,||\, q(t))$ over rolling horizons and require envelopes such as
$\sup_{s \in [t, t+H]} V(s) \le \epsilon$, with small H chosen to match application latency.

3.2 Tracking dynamics

We posit a template that mixes native drift $F(q,t)$, a descent term on D, and disturbances $\eta(t)$:

$$\frac{dq}{dt} = F(q,t) - k \, G(q)^{-1} \, \nabla_q D(\tau(t) \,||\, q(t)) + \eta(t),$$

where $k > 0$ is a gain and $G(q)$ is PD (natural gradient when G is the Fisher). This covers:

- Gradient tracking ($F = 0$),
- Model evolution (F encodes native learning or policy dynamics),
- Control-like shaping (choose k, G).

Design heuristics.

- Choose G so that D is locally 1-strongly convex: this turns the descent term into a contraction with rate proportional to k.
- Use $k(t)$ (gain scheduling) to meet transient vs noise tradeoffs: large k for re-capture, smaller k for low steady jitter.
- If constraints exist ($q \in C$), wrap dynamics with a projection Π_C in the G-metric: $dq/dt = \Pi_C[F - k G^{-1} \nabla D + \eta]$.

Examples.

- Squared-metric case $G = I$: $\nabla_q D = q - \tau$, so $dq/dt = F - k (q - \tau) + \eta$.
- KL with Fisher metric: $\nabla_q D$ in Euclidean coordinates becomes $G (q - \tau)$ to first order; the natural-gradient term $G^{-1} \nabla D \approx (q - \tau)$, producing the same local law but with information geometry built-in.

3.3 Informational Lyapunov condition

We seek an ISS-type (input-to-state stability) inequality for $V(t)$:

$$\frac{dV}{dt} \le -\alpha V + \beta \left\| \frac{d\tau}{dt} \right\| + \gamma \|\eta\|,$$

with $\alpha > 0$, $\beta, \gamma \ge 0$. This yields explicit tracking bounds.

Sketch (copy-safe) under A1–A2 and mild regularity on F.

Write V(t) = D(tau(t) || q(t)). By chain rule:

dV/dt = < grad_tau D, d tau/dt > + < grad_q D, dq/dt > .

Substitute dynamics:

dV/dt = < grad_tau D, tau_dot > + < grad_q D, F > - k < grad_q D, G^{-1} grad_q D > + < grad_q D, eta > .

Under standard boundedness/Lipschitz assumptions on F and the (tau, q) dependence of D, there exist constants c_F, c_tau, c_eta such that

< grad_q D, F > <= c_F * V, | < grad_tau D, tau_dot > | <= c_tau * || tau_dot ||,

| < grad_q D, eta > | <= c_eta * || eta || .

Strong convexity in the G-metric implies a Polyak–Łojasiewicz-type inequality for D:

< grad_q D, G^{-1} grad_q D > >= 2 mu * V .

Hence

dV/dt <= - 2 k mu * V + c_F * V + c_tau || tau_dot || + c_eta || eta || .

Choose k > k_min := (c_F + alpha) / (2 mu) to secure

dV/dt <= - alpha * V + beta || tau_dot || + gamma || eta ||,

with beta := c_tau, gamma := c_eta.

Explicit bound (worst-case, L_infty inputs). If || tau_dot || <= v_tau_max and || eta || <= e_max,

V(t) <= e^{- alpha (t - t0) } V(t0)

  + (1 - e^{- alpha (t - t0) }) * (beta * v_tau_max + gamma * e_max) / alpha .

Interpretation:

- With tau_dot = 0 = eta, V(t) decays exponentially at rate alpha.
- For moving targets and disturbances, V(t) stays within a steady envelope proportional to (beta v_tau_max + gamma e_max) / alpha.
- Increasing k increases alpha linearly (via mu), shrinking the envelope but potentially amplifying high-frequency noise if c_eta grows with k; schedule k accordingly.

Discrete-time analogue (step h). With Euler step
$q_{k+1} = q_k + h [F(q_k, t_k) - k G^{-1} \mathrm{grad}_q D(\tau_k || q_k) + \eta_k]$,
a similar inequality holds:

$$V_{k+1} - V_k \le - a V_k + b || \tau_{k+1} - \tau_k || + g || \eta_k ||,$$

with $a \sim h * 2 k \mu - h * c_F$, plus higher-order terms $O(h^2)$. Stability demands h small enough and k large enough so that $a > 0$.

Tuning recipe (practical).

1. Estimate curvature mu and gradient-Lipschitz L around the operating band (e.g., via Hessian-vector products or Fisher traces).
2. Pick target decay alpha_des. Set $k = (\alpha_{des} + c_F) / (2 \mu)$.
3. Measure $v_{\tau,max}$ over a rolling window; set monitors to hold
   $V_{env} := (\beta v_{\tau,max} + \gamma e_{max})/\alpha_{des} \le \epsilon$.
4. If V exceeds its envelope, temporarily raise k and/or add damping (regularization, smoothing) until re-capture, then taper.

Crosswalk to mechanics.

- $\alpha \propto k \mu$ mirrors “damping-like” pull toward the moving mode.
- $\beta || \dot{\tau} ||$ is the price of a moving target (seasonal drift).
- $\gamma || \eta ||$ is noise/perturbation (bumps, distractions).
  ISS-style bounds formalize the Logostics intuition: with enough “humility” (effective curvature times gain) and sensible scheduling, q(t) keeps phase-locked to tau(t) within a quantifiable, auditable error envelope.

4. Entrainment Procedures

4.1 Mode-matched direct drive (horizontal forcing)

Setup. Base translation $x_b(t) = A \cos(\Omega t)$; input appears as inertial forcing $\sim \ddot{x}_b(t) = -A \Omega^2 \cos(\Omega t)$ in both joints.

Target. Lock to a linear normal mode:

- In-phase: $\Omega \approx \omega_-$ (links move together, “one longer pendulum”).
- Anti-phase: $\Omega \approx \omega_+$ (links move opposite with fixed ratio).

Procedure (practical).

1. Identify ω_- , ω_+ by short chirp or tap test (small angles).
2. Choose a modest dimensionless shake: $\varepsilon := A \Omega^2 / g$ in 0.05–0.30.
3. Set small damping ζ (add friction pads or eddy current) in 0.01–0.05.
4. Start at $\Omega = \omega_{target} - \delta$, with $\delta/\omega_{target} \sim 0.05$.
5. Increase Ω slowly (adiabatic, Section 4.3) until lock; hold.
6. Trim A down to the minimum that preserves lock (reduce wear/noise).

Diagnosis.

- Too little damping: multi-periodic wobble before capture; add ζ.
- Too much damping: amplitude too small to cross basin boundary; raise A.
- Persistent beat notes: detuning; nudge Ω toward the observed envelope peak.

Logostics crosswalk. This is "season capture": the whole frame moves. Works best for corporate coherence when world-moves-first.

4.2 Parametric capture (vertical forcing, $\Omega \sim 2 * \omega_{mode}$)

Setup. Vertical base motion $y_b(t) = Y \cos(\Omega t)$ modulates effective gravity:

$g_{eff}(t) = g * (1 + \varepsilon \cos(\Omega t))$, $\varepsilon := Y \Omega^2 / g$.

Linearized small-angle dynamics yield a Mathieu form with instability (capture) tongues near $\Omega \approx 2 \omega_{mode}$ (and subharmonics $\sim 2 \omega_{mode} / n$).

Target. Stiff phase-locked orbits with large capture basins.

Procedure (practical).

1. Pick a mode (in-phase or anti-phase). Compute/measure ω_{mode}.
2. Set $\Omega = 2 \omega_{mode}$ within 2–5% tolerance.
3. Choose ε in 0.02–0.15 (start low; higher widens the tongue but can excite unwanted modes).
4. Add mild damping $\zeta \sim 0.02$.
5. Ramp ε up over several tens of cycles (avoid impulsive entry).

6. After lock, reduce ε to the minimum sustaining value (phase rigidity remains high).

Diagnosis.

- No capture: frequency off; adjust Ω within the tongue; verify ε.
- Mode hopping: ε too large or nearby mode present; reduce ε, add ζ.
- Jitter: environmental noise near Ω; add notch filter or move slightly off center while keeping within the tongue.

Logostics crosswalk. "Discipline stabilization": small fast practices create an effective potential that pins posture even as slow conditions vary (Kapitza effect).

4.3 Adiabatic sweeps and re-capture after shocks

Adiabatic entry. Increase a single parameter **slowly relative to the target period** so the system follows the attracting branch:

- **Frequency sweep:** $\Omega(t) = \Omega_0 + r\,t$ with rate r s.t. $r / \omega_{target}^2 \ll 1$ (e.g., 10^{-3}–10^{-4}).
- **Amplitude ramp:** $A(t) = A_{max} * s(t)$, with s(t) a smooth S-curve (e.g., cubic easing over 50–200 cycles).
- **Parametric depth:** ramp ε similarly.

Re-capture after shocks.

- **Shock model:** V jumps to V_{shock}, phases randomize, effective detuning Δ increases.
- **Three-step playbook:**
    1. **Broaden basin:** temporarily increase damping ζ_{up} and drive magnitude (A_{up} or ε_{up}).
    2. **Re-center frequency:** sweep Ω toward the nearest mode peak (measure response envelope; choose the larger peak).
    3. **Taper:** once phase-lock returns (steady phase error, narrow spectrum), taper ζ and drive to nominal.

Quantified guardrails (copy-safe).

- Require observed decay: $dV/dt < 0$ averaged over K cycles.
- Re-capture timer: smallest T such that $V(t0 + T) \le V_{goal}$.
- Abort rule: if V increases for M consecutive windows or if RMS phase error exceeds threshold, fall back to a broader sweep or switch channels (e.g., from joint actuation to base forcing).

Logostics crosswalk. Catechesis and retreats as **adiabatic** procedures; after crises, intensify discipline briefly, re-lock, then release to sustainable practice.

4.4 Robustness: noise floors, gain scheduling, restart rules

Noise floors. Let disturbances be bounded: $||eta|| \le e_{max}$. Under the ISS inequality

$$dV/dt \le - \alpha V + \beta ||\dot{\tau}|| + \gamma e_{max},$$

the steady envelope is

$$V_{env} = (\beta v_{\tau,max} + \gamma e_{max}) / \alpha .$$

- Lower bound: even perfect tuning cannot beat the floor set by e_{max} and $v_{\tau,max}$.
- Practical: smooth measurements (windowed averages) to avoid chasing noise; avoid high-gain reactions to high-frequency components.

Gain scheduling.

- **High-gain phase (capture):** $k = k_{high}$ to make alpha large and pull into lock quickly.
- **Hold phase:** reduce to k_{hold} to minimize jitter and actuator load.
- **Drift detector:** if V rises above a band, step to k_{high} temporarily, then return.

Heuristics:

$$k_{high} = (\alpha_{cap} + c_F) / (2 \mu), \quad k_{hold} = (\alpha_{hold} + c_F) / (2 \mu),$$

$$\alpha_{cap} \gg \alpha_{hold}, \quad \text{e.g., } \alpha_{cap} = 5 * \alpha_{hold} .$$

Estimate mu (curvature) via Hessian-vector probes or Fisher traces; track $v_{\tau,max}$ with rolling differences.

Restart rules.

- **Soft restart:** if lock is lost but spectra show a nearby mode, re-enter with an adiabatic sweep centered on the stronger spectral peak.
- **Channel switch:** if repeated soft restarts fail, switch input type (joint actuation -> base forcing or vice versa).
- **Hard restart:** zero velocities, increase damping briefly, relaunch with nominal capture sequence.

Spectral monitors (cheap).

- Track the power around (ω_-, ω_+) and at 2 ω_{mode}.
- Stable lock: narrowband peak at the commanded frequency (or half for parametric), fixed phase error, low sideband energy.
- Impending loss: peak broadening, growing sidebands, random walk of phase.

Logostics crosswalk.

- **Noise** = worldly perturbations and distractions; it sets the minimum achievable envelope of misalignment.
- **Gain scheduling** = seasons of intensified practice vs ordinary cadence.
- **Restart rules** = liturgical resets, retreats, or governance protocols for re-entry after failure.
- **Spectral monitors** = audits of V(t) and phase coherence, replacing impression with measurement.

One-page operational checklist.

1. Identify mode; estimate (ω_{mode}, mu, v_{tau_max}, e_{max}).
2. Choose capture gains and amplitudes (k_{high}, A_{up} or ε_{up}); prepare sweep plan.
3. Enter adiabatically; confirm lock by spectral and V(t) tests.
4. Taper to hold (k_{hold}, A_{hold} or ε_{hold}); start drift detector.
5. If V exceeds band: brief high-gain bump + tiny frequency nudge.
6. If lock fails repeatedly: channel switch; if still failing, hard restart.

7. Log envelopes, dwell time below threshold, re-capture times (make coherence auditable).

5. Two Theological Channels Mapped to Dynamics

5.1 Providence as base forcing (corporate season reading)

Picture. The “frame” of life moves: history convulses, ecology shifts, institutions realign. In the mechanical analogy this is **base forcing**; in Logostics it is **$\tau(t)$ moved by world-scale inputs**. Everyone feels it—both the mediating structures (our tau-channel) and the respondents (q’s).

Dynamics (copy-safe).

tau_dot = U_world(t) + small_internal_terms,

dq/dt = F(q,t) - k G^{-1} grad_q D(tau || q) + eta .

The design task is to estimate the season U_world(t) and widen the **capture basin** so many heterogeneous q’s can phase-lock to the new motion with bounded V(t).

Practices.

- **Corporate sensing:** fast, diverse sampling of signals; triangulate “where the frame moved.”
- **Damping as humility:** confession, fasting, mutual submission—energy removal that collapses chaotic alternatives.
- **Adiabatic adaptation:** gradual gain changes, small steps, frequent audits of V(t) envelopes.

Metrics.

- Envelope contracts after shock: sup_{[t,t+H]} V(s) decreases week over week.
- Dwell time below threshold grows: measure{ s : V(s) <= epsilon } increases.

Failures if neglected. Trying to “hold still” when the ground moves: brittle breakage, fragmentation of phase, rising sidebands (confusions) in communal spectra.

5.2 Address as tau-actuation (mediated word and obedience)

Picture. A **specific word** (scripture, teaching, prophetic charge, institutional rule) moves the channel itself—$\tau(t)$ is shaped with intention. Mechanically this is **joint actuation** at the upper link: you are not shaking the world; you are **commanding the mode**.

Dynamics (programmed τ).

tau(t) = Tau_program(t; parameters),

dq/dt = F(q,t) - k G^{-1} grad_q D(tau || q) + eta .

Here the challenge is precision: identify the pattern (rhythm, amplitude, phase) to which obedience should lock, and set gains that achieve **phase-lock without coercion**.

Practices.

- **Mode identification:** name the desired ratio/phase (what does obedience look like here?).
- **Micro-disciplines (Kapitza):** short, frequent practices that create an "effective potential" pinning posture even amid minor world drift.
- **Beat-note audits:** ensure the community is locking to the same cadence (low phase dispersion).

Metrics.

- Phase error small and bounded; low sideband energy; stable limit cycle under modest perturbations.

Failures if neglected. A clear address arrives but receivers never tune; or they over-tighten and ignore world drift (see 5.4).

5.3 Discernment rule: identify the channel before responding

Decision test (channel-first).

1. **Is the frame moving?** If signals are global, multi-domain, and unavoidable, treat it as **providence/base forcing**. Begin with safety and robustness: widen basins (increase damping), adopt adiabatic sweeps, and only then add fine patterning.
2. **Is there a shaped word?** If a coherent, testable address is present (content, cadence, scope), treat it as **tau-actuation**. Name the pattern, set gains for lock, and verify with gentle empirical checks (does V(t) fall, does phase dispersion shrink?).

3. **If mixed (most seasons):** sequence the responses—**stabilize to world first**, then **shape to word**, iterating until both constraints are satisfied.

Artifacts.

- A one-page **channel hypothesis** attached to any major decision: "We believe X% base, Y% word."
- **Guardrails** tied to the hypothesis: base channel ⇒ robustness checks and envelope targets; word channel ⇒ phase/ratio targets and micro-discipline cadence.

5.4 Failure modes: over-tau (legalism) vs over-base (accommodation)

Over-tau (legalism).

- *Symptom.* Rigid commands with no season-reading; obedience collapses diversity, brittle lock, sudden ejection when the world shifts (detuning spike).
- *Mechanics.* Excessive joint actuation with insufficient damping and no base alignment; narrow capture basin; high sensitivity to parameter error.
- *Remedy.* Re-estimate U_world(t); lower actuation gain temporarily; add damping; re-enter adiabatically; restore word shaping once the frame is absorbed.

Over-base (accommodation).

- *Symptom.* Drifting with history while neglecting concrete obedience; smooth but aimless motion; V(t) plateaus above actionable thresholds.
- *Mechanics.* Strong base forcing, weak or absent mode specification; the system never snaps into a clean periodic orbit.
- *Remedy.* Articulate a minimal, testable τ-pattern (scripture-anchored practices, covenantal vows); add micro-disciplines to create an effective pinning potential; raise k just enough for lock.

Hybrid pathologies.

- *Mode hopping.* Alternating between "season" and "word" without taper; results in beat phenomena and fatigue. Use **adiabatic transitions** and tapering gains.
- *Phase sectarianism.* Different subgroups lock to different phases of the same word. Publish **phase conventions** (shared cadence markers) and use synchronized checkpoints.

One-line diagnostic.

- If $V(t)$ rises when the world shifts → under-read base (over-tau).
- If $V(t)$ never falls despite calm conditions → under-specify word (over-base).

Ethical translation.

- **Humility** is operational damping: it removes excess energy so capture becomes possible.
- **Obedience** is phase-lock: sustained, noncoercive coherence with $\tau(t)$.
- **Discernment** is channel identification: picking the right input model before touching the gains.

Result. Channel-aware Logostics turns piety into **control-grade practice**: auditable envelopes for misalignment, measured dwell times of faithfulness, and engineered re-capture after shocks—so that $q(t)$ coheres with $\tau(t)$ in real time without erasing agency.

6. Ethics of Alignment

6.1 Humility as damping; repentance as gain tuning

Ethical interpretation.
Humility removes excess “energy” that keeps the system from settling; repentance re-tunes the responsiveness so tracking can resume rather than oscillate or blow up.

Control crosswalk.

- **Damping (humility).** Add energy dissipation to enlarge capture basins and prevent overshoot.
  - Practices: silence, fasting, confession, sabbath, amends.
  - Effect: narrows spectral spread, lowers sidebands, shortens settling time.
- **Gain tuning (repentance).** Adjust how strongly $q(t)$ responds to tau(t).
  - Too low k => sluggish tracking (chronic lag).
  - Too high k => ringing, brittle lock, sensitivity to noise.

Copy-safe templates.

```
dq/dt = F(q,t) - k * G(q)^{-1} * grad_q D( tau(t) || q(t) ) + eta(t)
```

Operational repentance:

if V rises rapidly or phase error drifts:

temporarily increase k -> k_high

add damping zeta -> zeta_up

hold until dV/dt < 0 over K cycles

taper k -> k_hold, zeta -> zeta_nominal

Practical signals of true humility (damping actually added).

- Shorter re-capture time after shocks.
- Reduced overshoot in V(t) transients.
- Lower variance of phase error at the same drive.

6.2 Pride and distraction as detuning and noise

Ethical interpretation.
Pride assumes one's internal cadence is correct regardless of tau(t); distraction sprays attention across frequencies. Both act as mis-tuning forces.

Control crosswalk.

- **Detuning (pride).** Commanded response frequency does not match the moving mode.
  - Symptom: persistent beat notes, slow envelope breathing, unstable phase lag.
  - Remedy: re-estimate omega_mode, nudge frequency or update k; re-enter adiabatically.
- **Noise (distraction).** High-frequency or broadband perturbations.
  - Symptom: elevated V floor, jitter in phase, sideband growth.
  - Remedy: filters, micro-disciplines (regular, brief practices), tighter observation windows.

Diagnostics (copy-safe).

- Beat index:

B := RMS(V_band(Ω±δ)) / RMS(V_peak(Ω))

High B => detuning; retune Ω or adjust k.

- Noise floor estimate (rolling):

V_floor := median(V(t) over window H)

Rising V_floor at constant Ω => distraction noise; add micro-disciplines.

Micro-disciplines as Kapitza stabilization.
Short, frequent, lightweight practices (e.g., daily examen, fixed prayer hours) create fast-timescale structure that raises effective curvature (mu) around desired posture, making detuning and noise less damaging without heavy gains.

6.3 Accountability as measurement of V(t) over horizons

Ethical interpretation.
Accountability is not surveillance; it is shared truth about alignment. We make misalignment visible, bounded, and correctable.

Monitors (define once, audit often).
Let

V(t) := D(tau(t) || q(t))

Choose horizons H_short (e.g., days), H_long (e.g., seasons).

Key metrics:

1. **Envelope bound (compliance).**

E_H(t) := sup_{s in [t, t+H]} V(s)

Target: E_H_short <= eps_short, E_H_long <= eps_long

2. **Dwell time below threshold (faithfulness).**

DT_H(t) := measure{ s in [t, t+H] : V(s) <= eps_* } / H

Target: DT_H >= rho_min

3. **Re-capture time after shock (resilience).**

T_rec := inf{ T >= 0 : V(t0+T) <= eps_post } given V jumps at t0

Target: T_rec <= T_rec_max

4. **Trend slope (formation).**

S_H := d/dt (avg_{s in [t, t+H]} V(s))

Target: S_H <= -sigma   (on average decaying)

ISS guardrail (design inequality).
Assure parameters so that

dV/dt <= -alpha * V + beta * || tau_dot || + gamma * || eta ||

and verify empirically by bounding the inputs over the same horizons:

|| tau_dot || <= v_tau_max, || eta || <= e_max

=> steady envelope: V_env = (beta * v_tau_max + gamma * e_max) / alpha

Set eps_long >= V_env; keep V tracking below eps_long in calm seasons.

Audit cadence and roles.

- **Weekly** (H_short): report E_H_short, DT_H, trend S_H, plus a brief narrative of observed tau_dot (seasonal motion) and major eta (disturbances).
- **Quarterly** (H_long): review envelopes, re-tune k and damping, update micro-discipline cadence; publish re-capture statistics.
- **Role split:** stewards propose gains (k, zeta), community consents to practices (micro-disciplines), auditors verify V-based outcomes.

Fail-safe thresholds (restart rules).

- **Soft restart:** if E_H_short > 2 * eps_short or DT_H < rho_min/2, trigger a temporary high-gain interval and an adiabatic re-entry.
- **Channel switch:** if two soft restarts fail within one horizon, test whether season is base-led; switch from tau-actuation to base forcing (or vice versa).
- **Hard restart:** if V exceeds a redline V_max, zero velocities (pause initiatives), add damping (fast, short retreat), and re-launch with the capture sequence.

Ethical summary in one line.
Humility lowers energy so capture is possible; repentance tunes gains so capture endures; accountability measures V(t) so none of this is make-believe.

7. AI Alignment and Learning with Moving Targets

7.1 Drifting data-generating processes and world models

Problem statement.
Real environments are nonstationary: the data-generating process $P_t(x, y)$ drifts with time t. A learner maintains a world model q_t (parameters, beliefs, policies) that must *track* a moving target representation τ_t (e.g., Bayes-optimal predictor under P_t, a normative policy class, or a reference simulator kept in sync by trusted measurements).

Drift typology.

- **Slow covariate drift:** $P_t(x)$ changes gradually; labels/returns stable.
- **Label/return drift:** $P_t(y|x)$ shifts (new norms, new payoffs).
- **Mechanism change:** causal structure changes (new actions, tools, rules).
- **Shock discontinuities:** abrupt regime switch; P_{t+} far from P_{t-}.
- **Adversarial drift:** drift coupled to the learner's policy (nonstationary games).

Operational signals (cheap).

- Rolling risk gaps: $R_t(q_t) - R_t(q_{ref})$ over short horizons.
- Density ratio alarms: $w_t(x) = dP_t/dP_{t-1}$ estimated by classifiers.
- Prediction under-coverage: calibration slope drift, OOD score histograms.
- Spectral broadening: growing sidebands in error spectra indicate detuning.

World model stance.
Keep a *family* of models: fast adapter q_{fast}, slow base q_{slow}, and a small policy of *bridging transforms* T_ϕ that map old features to new ones. Maintain a *channel hypothesis* (Section 5.3): how much of $\dot{\tau}$ is "base" (exogenous drift) vs "word" (normative adjustment you choose to impose, e.g., safety rules updated).

7.2 Informational Lyapunov training objectives for tau(t)

Objective as potential.
Let $V(t) := D(\tau(t) \,||\, q(t))$ be the training loss viewed as misalignment energy. Training dynamics implement

$$dq/dt = F_{train}(q,t) - k * G(q)^{-1} * \nabla_q D(\tau(t) \,||\, q) + \eta_{train}(t),$$

with G a metric (Fisher for natural gradient). We aim to enforce the ISS-style inequality

dV/dt <= - alpha V + beta || tau_dot || + gamma || eta_train || .

Concrete instantiations.

- **Supervised / RL policy distillation with moving teacher.**
  tau(t) is a rolling teacher ensemble or target policy pi_tau,t. Use KL as D:
- V(t) = KL(pi_tau,t || pi_q,t),
- natural gradient step size k => alpha ~ 2 k mu_F

where mu_F is curvature under Fisher metric. Control alpha via k; cap beta by limiting how fast pi_tau,t is updated (teacher inertia).

- **World-model learning under dataset curation.**
  tau(t) = argmin_{p in P} E_{(x,y)~cur_t} loss(p; x,y). To bound beta, **low-pass** your curation stream (e.g., exponential moving average of labels or constraints) so tau_dot is within design limits. This yields fewer training oscillations and tighter V envelopes.
- **Constraint-tracking (safety, ethics).**
  Represent constraints as a moving feasible set C_t or penalty \Psi_t(q). Use a composite divergence
- V(t) = D_task(tau_task(t) || q) + lambda(t) * Psi_t(q),

with lambda(t) scheduled so that the constraint channel (word-moves-first) can dominate during sensitive periods without inducing instability. Prove a **composite ISS bound** by adding two inequalities with compatible alphas.

Monitoring and bounds (copy-safe).

- **Envelope target:** choose alpha so that
- V_env = (beta * v_tau_max + gamma * e_max) / alpha <= eps_long .
- **Teacher inertia:** enforce || tau_{t+1} - tau_t || <= v_tau_max * h by EMA updates or trust-regions.
- **Noise budgeting:** keep || eta_train || (e.g., minibatch variance) under e_max via larger batches during re-capture, then relax.

Tuning playbook.

1. Estimate curvature mu (HVPs/Fisher trace) in the active slice of Q.
2. Set k for desired alpha in calm seasons; pre-compute k_high for re-capture.
3. Rate-limit tau updates (EMA, Polyak averaging) to keep beta * v_tau_max small.
4. Schedule batch size / gradient noise to cap gamma * e_max.
5. Enforce windowed metrics: E_H and DT_H (Section 6.3) on held-out rolling splits.

7.3 Capture, re-capture, and distribution shift playbooks

A. Capture (initial lock to tau).

- **Probe & identify mode.** Short chirp: sweep LR, batch size, and EMA time-constant; record where V decays fastest (operating "mode").
- **Adiabatic entry.** Ramp learning rate and target mixing S-curve over K steps; avoid impulsive teacher jumps.
- **Phase verification.** In policy spaces, check action-phase coherence (low KL between successive pi_q,t at the commanded cadence).

B. Re-capture after shock (abrupt drift).

1. **Broaden basin.** Temporarily raise k -> k_high; increase batch size (lower eta noise); add regularization (label smoothing, weight decay).
2. **Re-center.** Recompute tau with stricter trust-region (smaller updates) or switch to a *base-forcing* tactic: more emphasis on robust features via data augmentation that reflects the new world.
3. **Taper.** Once dV/dt < 0 over K windows and phase metrics stabilize, decay k to k_hold and restore nominal noise levels.

C. Distribution shift stratification.

- **Benign drift (slow).** Hold alpha modest; maintain EMA teacher; monitor V_env and DT_H.
- **Seasonal/structured drift.** Use **parametric capture**: inject periodic curriculum elements (micro-disciplines) at twice the target cadence to pin the representation (analogue of Ω ≈ 2 ω_mode).

- **Adversarial drift.** Add **channel switch**: from word-moves-first (explicit rules) to base-moves-first (robustness-first) or vice versa, depending on whether the adversary is targeting your rules or your features. Use randomized smoothing and distributionally robust objectives to enlarge the capture basin.

D. Multi-timescale controllers.

- **Fast adapter q_fast.** High k, high regularization; tracks immediate tau_dot.
- **Slow base q_slow.** Low k; integrates seasons, provides stability anchor.
- **Gated mixture.** Blend pi = Mix(q_fast, q_slow ; gate_t) with gate_t learned from covariate shift detectors. Guarantees that even if q_fast loses lock, pi remains near q_slow (bounded V).

E. Checklists (copy-safe).

Runbook: capture

estimate mu, v_tau_max, e_max

set k_high, k_hold; choose EMA time-constant T_tau

ramp LR and teacher-mix over K steps

require: dV/dt < 0 over K1 windows; DT_H >= rho_min

taper to k_hold; start drift detector

Runbook: shock re-capture

if E_H_short > 2 * eps_short or DT_H < rho_min/2:

  k <- k_high ; batch_size <- up ; add reg

  tighten tau updates (smaller trust region)

  if no lock in M trials: switch channel (robustness-first vs rule-first)

  upon lock: taper k, noise, reg to nominal

Alarms & metrics

E_H(t) = sup_{s in [t, t+H]} V(s)   # envelope

DT_H(t) = measure{ s in [t, t+H] : V(s) <= eps* } / H

Spectral width W(t): bandwidth of error spectrum around target cadence

Phase jitter $J(t)$: variance of successive KL(pi_q,t || pi_q,t-1)

Outcome.
These playbooks convert "nonstationary alignment" into auditable operations. By (i) limiting tau_dot, (ii) scheduling gains and noise, and (iii) sequencing channel-aware responses, an AI system keeps $q(t)$ phase-locked to $\tau(t)$ through seasons, shocks, and adversaries—with explicit envelopes on misalignment $V(t)$ rather than wishful assurances.

8. Pedagogy and Communal Formation

8.1 Liturgy and micro-disciplines as Kapitza stabilization

Claim. Small, frequent practices create fast-timescale structure that stabilizes posture on slow horizons, exactly as high-frequency vertical vibration yields an "effective potential" in the Kapitza pendulum.

Mechanics crosswalk.
Vertical base motion $y_b(t) = Y \cos(\Omega t)$ produces an effective curvature increase around the upright when Ω is high and $\varepsilon := Y \Omega^2 / g$ is modest. Analogously, micro-disciplines (brief, regular practices) raise an effective "mu" (curvature) in the divergence well:

V(t) := D(tau(t) || q(t)),   dV/dt <= - alpha_eff * V + ...

alpha_eff = alpha_native + delta_alpha(micro)

where delta_alpha(micro) > 0 scales with cadence and regularity, not with heaviness.

Pedagogical design.

- **Micro over macro.** Prefer 5–10 min daily over 2-hour monthly bursts.
- **Even cadence.** Fix times (e.g., morning/evening psalms, 60-sec examen at meals).
- **Thin load, thick repeat.** Content minimal; recurrence maximal.
- **Phase markers.** Use shared openings/closings (collects, doxology) as communal phase locks.

Expected effects.

- Lower phase jitter in shared activities.
- Faster re-capture after disruptions.
- Reduced steady V_floor at comparable external noise.

8.2 Catechesis as adiabatic frequency sweep

Claim. Formation should move through difficulty bands slowly enough that coherence tracks the target—an **adiabatic** increase in “drive frequency” and amplitude, not impulsive jumps.

Mechanics crosswalk.
Adiabatic entry captures a mode by sweeping $\Omega(t)$ slowly: $r := d\Omega/dt$ with $r / \omega_{target}^2 \ll 1$. In pedagogy:

- **Scope sweep (content).** From core prayers to doctrines to contested ethics.
- **Cadence sweep (practice).** From weekly to daily to hourly micro-acts.
- **Agency sweep (responsibility).** From imitation to participation to leadership.

Protocol (copy-safe).

```
for module m in 1..M:
  raise difficulty by Δ_m
  hold until dV/dt < 0 over K windows and DT_H >= rho_min
  then proceed; if not, dwell or step back by one Δ
```

Where V is an agreed misalignment proxy (see 8.3).

Guardrails.

- **No cliff moves.** Avoid doubling cadence or complexity in one step.
- **Spacing effect.** Insert rests; retention improves and V decays further.
- **Mixed channels.** If the world is shaking (base forcing), slow the sweep; widen the basin first (Section 5.1).

Outcome. Learners remain in phase; advancement is experienced as *entrained growth*, not coercion or whiplash.

8.3 Measuring coherence without coercion

Principle. Accountability is about *shared truth of alignment*, not surveillance. We measure communal coherence with **opt-in, low-pressure** signals that are informative for V(t) but respect conscience and privacy.

Proxy metrics (choose a small, transparent set).
Let V(t) := D(tau(t) || q(t)). Since we cannot compute D exactly in souls, choose observable proxies:

1. **Attendance cadence coherence (phase proxy).**
   Variance of arrival/participation times relative to liturgical markers.
   Lower variance ⇒ tighter phase-lock.
2. **Practice adherence (micro discipline completion).**
   Rolling fraction of micro-acts completed (self-reported or app-tracked with opt-in).
   Saturation near 1.0 is unnecessary; stability is key.
3. **Doctrinal drift index (content proxy).**
   Short periodic quizzes or reflections; track calibrated disagreement on core items.
   A healthy band allows diversity with bounded drift.
4. **Mutual upbuilding signal (ethos proxy).**
   Simple peer-affirmation counts ("I was helped/encouraged this week").
   Rising trend correlates with reduced social noise.

Windowed summaries (copy-safe).
Choose horizons H_short (weeks) and H_long (seasons). Define:

```
E_H(t) = sup_{s in [t, t+H]} V_proxy(s)              # envelope

DT_H(t) = measure{ s in [t, t+H] : V_proxy(s) <= eps* } / H   # dwell time

S_H(t)  = d/dt ( avg_{s in [t, t+H]} V_proxy(s) )      # slope
```

Targets:

- E_H_short <= eps_short (no runaway weeks),
- DT_H_long >= rho_min (faithful dwell),
- S_H_long <= -sigma (formation trend downwards).

Non-coercive collection.

- **Aggregate first.** Publish community-level dashboards; individual data stays private by default.
- **Opt-in granularity.** Members choose what to share; richer sharing improves communal tuning but is never mandatory.
- **Narratives with numbers.** Pair metrics with short testimonies; prevent metric idolatry.

Action rules (soft).

- If E_H_short spikes: increase damping (more silence, shorter homilies, extra listening sessions), not volume.
- If DT_H falls: slow the adiabatic sweep; widen basins with micro-disciplines.
- If S_H stalls in calm seasons: gently raise gain (add a clear, modest practice) and reassess.

Ethical summary.

- **Liturgy** supplies fast-timescale curvature (pinning posture).
- **Catechesis** moves slowly enough that capture persists (adiabatic).
- **Measurement** keeps us honest about V(t) without weaponizing numbers.
  Together they yield *coherence without coercion*: a community phase-locked to $\tau(t)$ with preserved agency and genuine joy.

9. Computation and Simulation Plan

9.1 Minimal simulator for driven double pendulum

Goal. Produce a fast, stable integrator that supports (i) horizontal base forcing, (ii) vertical parametric forcing, and (iii) optional upper-joint actuation. We want small-angle linear and full-nonlinear modes, plus hooks for damping, noise, and spectral monitors.

State and inputs (copy-safe).

```
state:   x = [theta1, theta2, omega1, omega2]^T

params:  m1, m2, l1, l2, g, zeta1, zeta2

forcing: x_b(t), y_b(t)         # base translation

         tau1(t), tau2(t)       # joint torques (optional)

noise:   eta ~ N(0, Sigma)      # process noise (optional)
```

Equations (nonlinear, planar). Use standard Lagrange form with moving base. Let the base accelerations be a_x = d^2 x_b / dt^2, a_y = d^2 y_b / dt^2. The EoM can be written as

M(theta) * [omega1_dot, omega2_dot]^T + C(theta, omega) + G(theta) = Tau + F_base(a_x, a_y), with:

- $M(\theta)$ = 2x2 inertia matrix,
- $C(\theta, \omega)$ = Coriolis/centrifugal + viscous terms (zeta1, zeta2),
- $G(\theta)$ = gravity vector,
- Tau = [tau1, tau2]^T,
- F_base collects inertial forcing from base accelerations.

For small-angle linear studies (fast mode ID), linearize about theta = 0 to obtain

M_lin * theta_ddot + C_lin * theta_dot + K_lin * theta = F_x * a_x + F_y * a_y + B_tau * tau .

Integrator. Semi-implicit (symplectic) Euler for speed, or RK4 / Dormand–Prince for accuracy. Use fixed dt for spectral analysis; e.g., dt = 1e-3 .. 5e-3 s. Add anti-blowup clamps on angles for visualization only; do not clip physics.

Forcing presets.

Horizontal: $x_b(t) = A \cos(\Omega t)$, $y_b(t) = 0$

Parametric: $y_b(t) = Y \cos(\Omega t)$, $x_b(t) = 0$

Joint act.: theta1_cmd(t) = $\Theta \cos(\Omega t)$ -> PD torque at joint 1

Joint actuation via PD:

tau1(t) = k_p (theta1_cmd - theta1) + k_d (0 - omega1)

Monitors (computed online).

- Modal projection (linear case): project [theta1, theta2] onto v_- , v_+.
- Phase error: arg of analytic signal via Hilbert transform vs drive.
- Spectral width: half-power bandwidth around drive frequency.
- V proxy for mechanics: V_mech = 0.5 * (theta - theta_ref)^T G (theta - theta_ref) where theta_ref is the modal ratio target.

Outputs.

- Time series: theta1, theta2, omega1, omega2, V_mech.
- Spectra (FFT over sliding windows).
- Capture flags and timestamps.

9.2 Synthetic tau(t) trajectories and q-dynamics

We simulate **Logostics** directly with a synthetic moving target tau(t) and a tracking state q(t) in R^n.

Spaces and divergence (copy-safe).

q(t), tau(t) in R^n

Choose D(tau || q) = 0.5 * || tau - q ||_G^2   with PD metric G.

Option: D = KL(p_tau || p_q) with exponential family; implement via Fisher metric.

Tau generators (scenarios).

1. **Calm season (baseline).**
   Piecewise-smooth tau(t) with bounded speed: ||tau_dot|| <= v_tau_max.
2. **Seasonal parametric pattern.**
   tau(t) = tau_0 + A cos(Ω_s t) * u + B sin(Ω_s t) * v,
   where Ω_s is slow (seasonal), u, v are orthonormal axes.
3. **Shock.**
   At t0: tau(t0+) = tau(t0-) + Δ_tau with ||Δ_tau|| large; then new smooth evolution.
4. **Adversarial drift.**
   tau_dot = u_t with u_t reacting to q(t) (gamey perturbations), e.g.,
   u_t = a * sign(q - tau) on selected coordinates.

q-dynamics (natural-gradient template).

dq/dt = F(q,t) - k * G^{-1}(q) * grad_q D(tau(t) || q(t)) + eta(t)

Choices:

- F = 0 (pure tracker), or slow prior drift F = -λ (q - q_prior).
- eta(t) as band-limited noise with RMS e_max.

Training/operation knobs.

- Gain scheduling: k = k_high during capture or shock, k = k_hold otherwise.
- Micro-discipline surrogate: periodic small pulses that increase effective curvature mu temporarily (G -> G + δG on a fast cadence).

Instrumentation.

- Compute $V(t)$, dV/dt, and the ISS envelope prediction
  $V\_env = (\beta\ v\_tau\_max + \gamma\ e\_max) / \alpha$.
- Track **time-to-capture** after shocks, **dwell time** below threshold, and **phase error** where tau has periodic components.

9.3 Benchmarks: time-to-capture, dwell time, V(t) envelopes

We benchmark both the **mechanical** and the **Logostics synthetic** systems with common alignment metrics.

Core metrics (copy-safe).

1. **Time-to-capture (T_cap).**
   Minimal T such that for all $t \geq t0 + T$,
   $V(t) <= eps\_cap$ and $|dV/dt|$ small over K cycles (or spectral lock detected).
2. **Dwell time (DT_H).**
   Over window H,
   DT_H = measure{ s in [t, t+H] : V(s) <= eps_* } / H.
3. **Envelope bound (E_H).**
   E_H = sup_{s in [t, t+H]} V(s).
4. **Steady envelope vs ISS prediction.**
   Compare measured V_ss with
   V_env = (beta v_tau_max + gamma e_max) / alpha.

Mechanical suite.

- **Cases.**
  A) Horizontal drive, $\Omega \approx \omega\_-$, various ζ, A.
  B) Horizontal drive, $\Omega \approx \omega\_+$.
  C) Vertical parametric, $\Omega \approx 2\ \omega\_mode$, ε sweep.
  D) Shock: instantaneous change in Ω or amplitude; measure re-capture.
- **Success criteria.**
  - Capture within T_cap_max.
  - $DT\_H \geq \rho\_min$ at chosen eps_*.

    - Spectral width below W_{max}.

- **Ablations.**
    - Vary damping ζ to observe basin widening/narrowing.
    - Detuning sweeps $\Delta = \Omega - \omega_{mode}$ to map capture boundaries.
    - Joint actuation vs base forcing for the same target mode.

Logostics suite.

- **Scenarios.** calm / seasonal / shock / adversarial (Section 9.2).
- **Knobs.** gains (k_{high}, k_{hold}), EMA rate for tau (limits v_{tau_max}), noise level e_{max}, and micro-discipline curvature boost $delta_{mu}$.
- **Readouts.**
    - T_{cap} after each regime change.
    - DT_H and E_H per window.
    - Fit (alpha, beta, gamma) post hoc by linear regression on dV/dt vs (V, $||tau\_dot||$, $||eta||$) to validate the ISS inequality.
    - **Budget plots:** achievable V_{env} vs (alpha, v_{tau_max}, e_{max}); operating point should sit below the budget line.

Reporting format (desktop-friendly).

- **Tables:** per-scenario metrics (T_{cap}, DT_H, E_H, V_{env_pred}, V_{env_obs}).
- **Figures:**
    1. V(t) with ISS envelope overlays.
    2. Spectrograms around target frequencies (mechanics).
    3. Capture-basin maps over (Δ, ζ) and (ε, ζ) (heatmaps).
    4. Budget curves: V_{env} vs alpha for fixed (v_{tau_max}, e_{max}).

Acceptance thresholds (example).

$T_{cap} <= 50$ cycles

DT_H (H = 200 cycles, $eps_* = eps_{cap}$) >= 0.8

$|V_{env\_obs} - V_{env\_pred}| / V_{env\_pred} <= 0.2$

Spectral width $W <= 0.05 * \Omega$ (mechanics)

Reproducibility checklist.

- Fixed seeds; log params and hashes.
- Deterministic integrator step for benchmark runs.
- Emit raw time series (CSV/Parquet) + plots.
- One-click scripts:
  - simulate_mech.py --case A --A 0.02 --zeta 0.03 --Omega 0.77 sqrt(g/l)
  - simulate_logostics.py --scenario shock --k_high 4 --k_hold 1 --v_tau 0.03

Outcome.
This plan yields auditable, desktop-grade evidence that (i) the mechanical metaphor exhibits capture, parametric pinning, and re-capture, and (ii) the Logostics tracker honors the ISS-style bounds with quantifiable T_cap, DT_H, and V_env under controlled tau_dot and noise budgets.

10. Case Studies and Thought Experiments

10.1 Season of upheaval (world-moves-first)

Scenario.
A city faces layered shocks over 6 months: economic contraction, a regional wildfire season, and a school-system reorg. Attendance patterns fragment, volunteers burn out, and small groups desynchronize.

Channel hypothesis.
80% base forcing, 20% word shaping. The frame is moving; treat τ_dot as dominated by U_world(t).

Intervention (copy-safe).

1. **Broaden basin (humility first).**
   - Add damping: community fasts reduced in intensity but increased in frequency (e.g., “micro-fasts” of media stillness 2× daily).
   - Silence blocks in gatherings (3–5 min) to dissipate excess energy.

2. **Adiabatic realignment.**
    - Shift cadence gradually: shorten services by 10%, distribute shorter liturgy across weekdays (matins/vespers micro-acts).
    - Introduce “season reads” that name the upheaval without prescribing quick fixes.
3. **Robustness-priority curriculum.**
    - Teach anchoring practices (lament psalms, intercession rounds) as Kapitza stabilizers.
    - Data hygiene: simplify all schedules and communication channels (reduce noise bandwidth).

Monitors.

- E_H_short (envelope) falls within 4 weeks.
- DT_H_long (dwell) > 0.7 by week 8 at eps_* set from pre-shock baselines.
- Spectral width of arrival-time variance narrows.

Failure to avoid.
Over-tau legalism: launching ambitious programs now; would shrink capture basins and eject fragile members.

Outcome.
Periodic faithfulness reappears despite continued drift. V(t) stabilizes near the ISS envelope predicted from measured ||tau_dot|| and ||eta||. Agency preserved; no coercion required.

10.2 Clear address with quiet world (word-moves-first)

Scenario.
Calm conditions. A congregation senses a specific call: daily table prayer at 18:00 and a weekly service of blessing for caregivers, sustained for 90 days.

Channel hypothesis.
25% base, 75% word. $\tau(t)$ is a shaped program; world noise low.

Intervention (copy-safe).

1. **Name the mode.**
    - Define the desired phase: “18:00 local ± 5 min” table prayer; Sunday 17:00 blessing.
    - Publish minimal rubric (two short prayers, one song).
2. **Enter adiabatically.**
    - Week 1: once per week; Week 2: Tue/Thu; Week 3+: daily.
    - Provide micro-reminders (1-line texts), no guilt cues.
3. **Micro-disciplines for pinning.**
    - 60 sec examen at lunch; 30 sec breath-prayer before bed.
    - Small, high-frequency acts raise effective curvature.

Monitors.

- **Phase error:** variance of participation time around 18:00 decays.
- **Beat-note suppression:** low sidebands around the commanded cadence.
- **Dwell time:** DT_H ≥ 0.8 by day 21 at eps_* chosen from week 1.

Failure to avoid.
Over-base accommodation: “whenever you can” messaging that never pins a phase; results in pleasant drift and low lock.

Outcome.
Clean phase-lock emerges; V(t) decays exponentially in calm seasons (alpha high, beta small). Joy increases without pressure.

10.3 Mixed signals and channel disambiguation

Scenario.
Midsize church enters a contested civic debate (school policy) while leadership discerns a call to mentor teens in craft skills. The city’s discourse is noisy (base forcing), yet a local address is plausible (word).

Initial symptoms.

- Rising E_H_short (weekly envelope spikes).
- Phase sectarianism: subgroups keeping different cadences.
- Mode hopping: alternating emphases produce beat phenomena.

Channel disambiguation protocol.

1. **Separate spectra.**
   - Instrument two proxies for V: one tied to public-season signals (attendance variability, conflict mentions), one tied to the mentoring call (mentor sessions, apprentice check-ins).
   - Compute cross-correlation with candidate drivers: news-cycle index vs internal cadence index.
2. **Estimate weights.**
   - Fit a simple regression for dV/dt on (V, season_index, word_index).
   - Infer (w_base, w_word); publish a one-page channel hypothesis (e.g., 60% base, 40% word).
3. **Sequence responses.**
   - **Phase 1 (2–4 weeks):** base-first. Increase damping (listening forums, prayer vigils with silence). Reduce contentious programming; widen basin.
   - **Phase 2:** word shaping layered in. Start mentoring pilot with adiabatic entry (one workshop/week → two → four), fixed cadence markers.

Governance guardrails.

- **Adiabatic transitions:** no abrupt switches between channels; use taper functions for gains and cadence.
- **Local autonomy:** small teams may lock earlier to the word; allow controlled heterogeneity while the whole body absorbs the base motion.
- **Abort rules:** if E_H_short exceeds 2 * eps_short twice in a month, pause public engagement events for one horizon and add damping.

Monitors and targets.

- DT_H for civic-peace proxies rises above 0.6 before ramping the mentoring cadence beyond 2×/week.
- Mentoring phase variance shrinks by week 6; apprentice retention ≥ 80%.
- Narrative audits confirm perceived coherence ("the work matches the season and the word").

Failure to avoid.

- **Over-tau in a storm:** rolling out full mentoring program amid civic peak noise (legalism analogue).
- **Over-base after calm returns:** never naming or pinning the mentoring cadence once the storm subsides.

Outcome.
By **identifying the channel first**, then **sequencing** base robustness and word precision, the community achieves dual lock: low V in public-season proxies and sustained low phase error in mentoring cadence. Mixed seasons become tractable without coercion or drift.

11. Objections and Limits

11.1 Is "moving truth" relativism?

Objection. If $\tau(t)$ moves, isn't "truth" merely whatever changes?

Reply (stance). Logostics distinguishes **truth's path** from **our estimate** of it. We do not claim that normativity is arbitrary; we claim that *encountered coherence* is temporally unfolded. Three guards against relativism:

1. **Invariants (channel-agnostic constraints).**
   We posit fixed constraints C (e.g., scriptural canons, core doctrinal minima, conserved ethical prohibitions) such that admissible $\tau(t)$ satisfies $\tau(t) \in C$ for all t. Movement occurs **within** C, not across it.
2. **Calibration by consequences.**
   If alignment is real, misalignment should cost. We require measurable penalties when $q(t)$ departs from $\tau(t)$: rising $V(t)$, loss of dwell time, degraded fruits (Section 8.3). A stance that "anything goes" cannot meet these audits.

3. **ISS-style bounds as objectivity tests.**
   Under the inequality

4. dV/dt <= - alpha V + beta || tau_dot || + gamma || eta ||

a coherent $\tau(t)$ produces stable, bounded V envelopes under bounded inputs. If a proposed "truth" yields explosive V over calm horizons (small ||tau_dot||, ||eta||), it fails the coherence test.

Theological clarity. "Moving" refers to *revelatory disclosure* (kairos, season), not to the dissolution of norms. The Logos is constant; its address is timely.

11.2 Divergence choice and metric dependence

Objection. Results depend on the choice of $D(\tau||q)$ and metric G; isn't this arbitrary?

Reply (technical). Dependence exists, but we control it.

1. **Sandwich bounds.**
   For divergences D_1, D_2 with compatible geometry, there exist c_1, c_2 > 0 on a working set $S \subset Q$ such that

2. c_1 * D_1(τ||q) <= D_2(τ||q) <= c_2 * D_1(τ||q)   for (τ,q) in S.

Hence V-envelopes and capture times differ at most by constants; qualitative claims persist across reasonable D.

3. **Natural gradient invariance.**
   Using G as the Fisher metric (or an appropriate pullback) yields **reparameterization-invariant** descent:

4. dq/dt = - k * G(q)^{-1} * grad_q D(τ||q)

so coordinate choices do not spurious-change dynamics.

5. **Curvature-and-Lipschitz regime.**
   With strong convexity μ and gradient Lipschitz L on S,

6. < grad_q D, G^{-1} grad_q D > >= 2 μ V, || grad_q D ||_{G^{-1}}^2 <= 2 L V,

giving designable α via k and predictable envelopes. We recommend **reporting (μ,L)** for the active band, making metric effects explicit.

7. **Domain fit.**
   Choose D for semantics: KL for policy/model distributions; Bregman for convex tasks; squared Riemannian distance for geometric targets. Publish ablations across D to show robustness.

Limit. Outside S (far from lock, heavy nonlinearity), sandwich constants loosen; metric choice matters more. Our claims are local-to-mesoscopic, not global guarantees.

11.3 Scope: reception without over-specifying mechanisms

Objection. "Reception" talk risks smuggling metaphysics (souls, consciousness mechanisms) without evidence.

Reply (scope discipline). Logostics is a **control-level** theory of alignment, not a microphysical or neurotheological account. We assert:

1. **Interface claim, not mechanism.**
   We model observable relations between $q(t)$, $\tau(t)$, and $V(t)$. We do **not** specify how τ is generated ontologically nor how consciousness implements reception. Our commitments are:
   - If τ moves within constraints and inputs are bounded, then V obeys ISS bounds (testable).
   - Entrainment procedures (Sections 4, 8) measurably change T_cap, DT_H, and envelopes (auditable).
2. **Falsifiability.**
   The framework is refutable if, under verified small $||\dot{\tau}||$ and $||\eta||$, V nonetheless grows unbounded; or if prescribed entrainment sequences fail to reduce T_cap and improve DT_H across matched cohorts.
3. **Channel identification as minimal epistemology.**
   We require practitioners to publish a **channel hypothesis** ("w_base / w_word"), associated guardrails, and results. This is epistemic hygiene, not metaphysical proof.

4. **Known unknowns (stated limits).**
    - **Global optimality.** We do not promise minimal V globally; we target **bounded tracking**.
    - **Mixed authority conflicts.** When multiple τ-sources compete (institutional vs prophetic), our method sequences responses; it cannot adjudicate ultimate authority.
    - **Adversarial meta-drift.** An adversary shaping τ itself breaks assumptions; we then reclassify as base forcing or redesign constraints C.

Ethical restraint. We avoid coercion: measurement is communal and opt-in; practices aim at enlarging capture basins, not overriding agency. Where explanation runs out, we mark the boundary (Section 6): humility as damping, not as dogma.

Net. "Moving truth" ≠ relativism; divergence choices are controllable and reported; scope is interface-level with explicit refusal to over-claim mechanisms. The value of Logostics is operational: it turns alignment into quantifiable practice while leaving room for deeper metaphysics to be explored—or bracketed—without confusion.

12. Conclusion: Stability Through Co-Becoming

12.1 q -> tau as entrained obedience in time

Logostics reframes alignment as **entrained tracking** of a time-dependent target. Rather than demanding instantaneous coincidence q(t) = tau(t), we require **bounded lag with audited envelopes**. The operative potential is the divergence

V(t) := D(tau(t) || q(t)) >= 0,

and the design inequality is input-to-state stability

dV/dt <= - alpha * V + beta * ||tau_dot|| + gamma * ||eta|| .

When alpha (effective curvature times gain) dominates the moving and noisy terms, V(t) contracts toward a controllable band. The theological reading is obedience without coercion: as tau(t) discloses in season, q(t) phase-locks by humility (damping), right-sized gain (repentance), and channel-wise discernment. Alignment becomes a **practice in time**, not a frozen pose.

12.2 From chaos to periodic faithfulness

The driven double pendulum teaches that chaos can collapse into clean orbits when forcing, damping, and frequency are well chosen. So too communities and learners: liturgy and micro-disciplines act as Kapitza stabilization; catechesis advances by adiabatic sweeps; accountability measures V(t) over windows instead of guessing. The visible signatures of **periodic faithfulness** are:

- **Capture:** finite T_cap to re-enter coherence after shocks.
- **Dwell:** high DT_H below agreed thresholds in calm seasons.
- **Narrowband spectra:** low sidebands around shared cadences.
  This is not sameness; it is **phase-coherence with agency preserved**. Diversity remains in amplitude, timbre, and role, while the body keeps time with tau(t).

12.3 Next steps: proofs, tools, and community practice

Proofs (theory).

- Formalize ISS bounds for common divergences (KL, Bregman, squared Riemannian) with explicit (mu, L) regimes and natural-gradient invariance.
- Prove composite inequalities for multi-channel V(t) = D_task + lambda(t) * Psi_t (moving constraints).
- Characterize capture basins under detuning and noise; quantify optimal adiabatic rates.

Tools (software).

- Release the **mechanical lab**: nonlinear double-pendulum simulator with horizontal/vertical/base and joint actuation; spectral and V_mech monitors.
- Release the **Logostics tracker**: synthetic tau(t) generators, natural-gradient integrators, ISS envelope overlays, and runbooks for capture/re-capture.
- Provide desktop-friendly dashboards: E_H, DT_H, T_cap tables; budget plots comparing observed V_env to predicted (beta v_tau + gamma e_max)/alpha.

Community practice (operations).

- Adopt **channel hypotheses** (world-moves-first vs word-moves-first, with weights) for major decisions; attach guardrails and monitors.
- Standardize **entrainment protocols**: micro-disciplines for curvature, adiabatic progressions for scope/cadence, soft/hard restart rules after shocks.

- Institutionalize **non-coercive audits**: publish aggregated V proxies, dwell times, and narratives; tune gains transparently.

Metric for success.
Across seasons, shocks, and debates, measured V(t) stays within designed envelopes; T_cap falls with practice; DT_H rises without narrowing conscience. If these hold, Logostics has earned its claim: **stability through co-becoming**, where minds, models, and communities keep time with living truth.

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Appendices

Appendix A. Linearized mode derivations and frequency estimates

A.1 Small-angle linear model (equal lengths, equal masses)

Planar, small angles around downward equilibrium; let l1 = l2 = l, m1 = m2 = m, g > 0. Generalized coordinates: theta1, theta2 (radians). With fixed base and no damping, the linearized equations are

M * theta_ddot + K * theta = 0,

theta = [theta1, theta2]^T

A convenient textbook-normalized pair (dimensionless up to a common factor) is

M = [2   1

   1   1],

K = (g/l) * [2   1

   1   1] .

Normal modes satisfy

det(K - omega^2 M) = 0 .

Compute eigenpairs of M^{-1} K. Since M and K are proportional here, eigenvectors match and eigenvalues are the same as those of M^{-1} M = I up to constants, but the *physical* normal frequencies come from the generalized problem. Solving explicitly:

Let r := theta2/theta1. From the first row:

(2 g/l - 2 omega^2) * 1 + (g/l - omega^2) * r = 0 .   (1)

From the second row:

(g/l - omega^2) * 1 + (g/l - omega^2) * r = 0 .   (2)

Subtract (2) from (1):

(g/l - omega^2) * 1 = (g/l - omega^2) * 0   -> use (2).

Solving the generalized eigenproblem directly gives two roots

omega_-^2 = (2 - sqrt(2)) * (g/l),

omega_+^2 = (2 + sqrt(2)) * (g/l),

hence

omega_- = sqrt(2 - sqrt(2)) * sqrt(g/l) ≈ 0.766 * sqrt(g/l),

omega_+ = sqrt(2 + sqrt(2)) * sqrt(g/l) ≈ 1.848 * sqrt(g/l).

Associated amplitude ratios (normalized) are

v_- : [theta1, theta2] proportional to [1, + 1/sqrt(2)],

v_+ : [theta1, theta2] proportional to [1, - 1/sqrt(2)].

These give theta2/theta1 ≈ +0.707 (in-phase) and -0.707 (anti-phase).

A.2 Unequal l, m (outline)
Let

M = [a b

b c], K = (g) * [k11 k12

k21 k22],

with

a = (m1 + m2) l1^2, b = m2 l1 l2,

c = m2 l2^2,

k11 = (m1 + m2)/l1, k12 = m2/l1,

k21 = m2/l2, k22 = m2/l2 .

Solve det(K - omega^2 M) = 0 for omega^2 (quadratic). Mode vectors come from (K - omega^2 M) v = 0. Numeric evaluation is straightforward.

A.3 Including small viscous damping
With C = diag(2 zeta1 omega1_lin, 2 zeta2 omega2_lin) in modal coords, near each mode the scalar envelope E satisfies

dE/dt <= - 2 zeta_mode * omega_mode * E + work_in .

Small zeta widens capture basins while preserving observable amplitudes.

Appendix B. Example divergences and natural-gradient metrics

B.1 Squared Riemannian distance

Let $G(q)$ be PD. Define

$$D(\tau \,||\, q) = 0.5 * ||\tau - q||_G^2 = 0.5 * (\tau - q)^T G (\tau - q).$$

Then

$$\mathrm{grad}_q D = -G(\tau - q), \qquad G^{-1} \mathrm{grad}_q D = -(\tau - q).$$

Natural-gradient flow

$$dq/dt = -k * G^{-1} \mathrm{grad}_q D = k * (\tau - q)$$

is coordinate-free when G is the pullback of an underlying geometry.

B.2 Bregman divergences

Given strictly convex F with Hessian $H_F(q)$ PD,

$$D_F(\tau \,||\, q) = F(\tau) - F(q) - \langle \mathrm{grad}\, F(q), \tau - q \rangle.$$

Properties:

- $D_F >= 0$ and $D_F = 0$ iff $\tau = q$.
- $\mathrm{grad}_q D_F(\tau \,||\, q) = H_F(q)(q - \tau)$.
  Natural gradient with $G = H_F(q)$ gives

$$dq/dt = -k * G^{-1} \mathrm{grad}_q D_F = k * (\tau - q),$$

matching the squared case locally but now with geometry induced by F.

B.3 KL divergence in exponential families

Let $p_q(x)$ be an exponential family with natural parameter q and Fisher information $G(q) = \mathrm{FIM}(q)$. For KL

$$D(p_\tau \,||\, p_q) = E_{p_\tau}[\log p_\tau - \log p_q].$$

In local coordinates, for small $\delta = q - \tau$,

$$D(p_\tau \,||\, p_q) \approx 0.5 * \delta^T G(\tau)\, \delta.$$

Natural-gradient descent with $G = \mathrm{FIM}$ yields reparameterization invariance and the same local law as B.1–B.2.

B.4 Composite divergences with moving constraints
If we must track both a task target and a constraint,

V(t) = D_task(tau_task(t) || q) + lambda(t) * Psi_t(q),

with Psi_t convex penalty (e.g., barrier for feasibility set C_t), and lambda(t) scheduled. Under mild assumptions, V inherits PL-type bounds on active domains.

Appendix C. Control notes: gain bounds for capture and re-capture

C.1 ISS-style inequality and k-min
With strong convexity mu and Lipschitz c_F for <grad_q D, F>, plus bounds beta, gamma on tau_dot and noise terms, we derived

dV/dt <= - 2 k mu * V + c_F * V + beta ||tau_dot|| + gamma ||eta|| .

Choose

k >= k_min := (c_F + alpha_des) / (2 mu)

to achieve

dV/dt <= - alpha_des * V + beta ||tau_dot|| + gamma ||eta|| .

C.2 Capture vs hold gains
Use two levels:

k_high = (c_F + alpha_cap) / (2 mu),   alpha_cap >> 0,

k_hold = (c_F + alpha_hold) / (2 mu), 0 < alpha_hold < alpha_cap.

Typical ratio alpha_cap : alpha_hold in [3:1, 10:1]. Enter with k_high to shrink V quickly, then taper to k_hold to reduce jitter and actuator load.

C.3 Adiabatic sweep rates
Let r = dΩ/dt be a frequency sweep toward omega_mode. Adiabatic capture demands

r / omega_mode^2 << 1

(e.g., 1e-3 to 1e-4). For amplitude ramps A(t) or parametric depth eps(t), use smooth S-curves over 50–200 cycles to avoid impulsive transients.

C.4 Re-capture after shock
Shock at t0 causes V jump to V_shock and phase randomization. Playbook:

1. Temporarily raise k -> k_high; increase damping zeta.
2. Re-center frequency toward the nearest response peak (measure).
3. Upon dV/dt < 0 over K windows, taper k -> k_hold, zeta -> nominal.

C.5 Budget line for steady envelope
Given bounds

||tau_dot|| <= v_tau_max, ||eta|| <= e_max,

the steady envelope satisfies

V_env = (beta v_tau_max + gamma e_max) / alpha_hold .

Thus increasing alpha_hold (via k or curvature mu) linearly tightens V_env, up to noise floors and discretization error.

Appendix D. Pseudocode for V(t) monitors and alarms

D.1 Rolling monitors (copy-safe)

```
# Inputs:
#   times: array of t_k
#   V:     array of V(t_k) >= 0
#   tau_dot_norm: array of ||tau_dot||(t_k)  (optional, for diagnostics)
#   eta_norm:     array of ||eta||(t_k)      (optional)
#   H_short, H_long: window lengths (in same units as times)
#   eps_short, eps_long, eps_star: thresholds
#   K_cycles: integer window for derivative sign test


from collections import deque

import numpy as np
```

```
def rolling_envelope(times, V, H):
    E = np.zeros_like(V)
    dq = deque()
    for i, t in enumerate(times):
        # pop old
        while dq and times[dq[0]] < t - H:
            dq.popleft()
        # push current
        dq.append(i)
        # compute sup over window by scanning deque
        E[i] = max(V[j] for j in dq)
    return E


def rolling_dwell(times, V, H, eps_star):
    DT = np.zeros_like(V)
    dq = deque()
    for i, t in enumerate(times):
        # maintain indices in window
        while dq and times[dq[0]] < t - H:
            dq.popleft()
        dq.append(i)
        # compute fraction below eps_star
        idx = list(dq)
        DT[i] = np.mean([1.0 if V[j] <= eps_star else 0.0 for j in idx])
    return DT
```

```
def rolling_slope(times, V, H):
    S = np.zeros_like(V)
    dq = deque()
    for i, t in enumerate(times):
        while dq and times[dq[0]] < t - H:
            dq.popleft()
        dq.append(i)
        idx = list(dq)
        # simple linear fit slope d/dt avg(V) over window
        t_win = np.array([times[j] for j in idx])
        v_win = np.array([V[j] for j in idx])
        if len(idx) >= 2:
            A = np.vstack([t_win - t_win[0], np.ones_like(t_win)]).T
            slope, _ = np.linalg.lstsq(A, v_win, rcond=None)[0]
            S[i] = slope
        else:
            S[i] = 0.0
    return S
```

D.2 Alarm logic (soft/hard restart, channel switch)

```
def evaluate_alarms(times, V, H_short, H_long, eps_short, eps_long,
                    eps_star, rho_min, slope_min,
                    M_soft=2):
    E_short = rolling_envelope(times, V, H_short)
    E_long  = rolling_envelope(times, V, H_long)
    DT_long = rolling_dwell(times, V, H_long, eps_star)
    S_long  = rolling_slope(times, V, H_long)
```

```
alarms = []
soft_counter = 0
for i, t in enumerate(times):
    soft = (E_short[i] > 2.0 * eps_short) or (DT_long[i] < 0.5 * rho_min)
    hard = (E_long[i] > 3.0 * eps_long)
    stall = (S_long[i] > -slope_min)  # not decaying in calm seasons

    action = None
    if hard:
        action = "HARD_RESTART"
        soft_counter = 0
    elif soft:
        soft_counter += 1
        if soft_counter >= M_soft:
            action = "CHANNEL_SWITCH_OR_SOFT_RESTART"
            soft_counter = 0
        else:
            action = "SOFT_RESTART"
    elif stall:
        action = "RAISE_GAIN_SLIGHTLY_AND_ADD_MICRO_DISCIPLINES"
        soft_counter = 0

    alarms.append((t, E_short[i], E_long[i], DT_long[i], S_long[i], action))
return alarms
```

D.3 Spectral hints (mechanical analogy)
For a driven mechanical case, add a narrowband spectral width W(t) monitor around the commanded frequency (or half-frequency for parametric drive). Impending loss of lock manifests as peak broadening and sideband growth. Simple FFT over sliding windows suffices; threshold on W(t) relative to Omega.

D.4 Reporting
At each review horizon:

- Publish tables of E_H_short, E_H_long, DT_H_long, S_H_long, with thresholds and any actions taken.
- Include an ISS budget line:

V_env_pred = (beta * v_tau_max + gamma * e_max) / alpha

V_env_obs = median(E_H_long over last N windows)

and the ratio V_env_obs / V_env_pred as a calibration indicator.

Note on copy-safe math.
All equations use ASCII-only symbols so they can be pasted into plain-text editors, versioned, hashed, and reproduced without Unicode drift.