Mathematical Metaphysics

STOEM, the Physical Universe,
and the God-Man Relationship

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The mathematical examples use explicitly stated conventions. References and source notes appear at the end of the book.

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Preface

This book joins two stages of an inquiry. The first began in the Spiritual Theory of Everything Model, or STOEM, developed in papers dating from May 2023. Its central picture was a universe of infinitely many interacting fractals, each involving infinitely many variables and feeding into the others through successive iterations. Orthogonal to this physical order stood the God-Man relationship, whose nature remained undefined. The second stage examined the conditions under which mathematics describes anything at all: being, distinction, identity, interpretation, and the preservation of meaningful questions.

The joining changes both projects. STOEM gives the general inquiry a definite cosmological and spiritual proposal. Mathematical metaphysics asks what that proposal assumes, how its terms acquire meaning, and which consequences its mathematical realizations establish. The result is an account of a physical universe within a wider metaphysics of existence and relationship.

The title's God-Man relationship uses Man in the inclusive sense of humanity. It concerns the person in relation to God, with human embodiment fully included. Orthogonal expresses a distinction of explanatory kind. The word does not introduce a measured spatial direction or a second physical channel whose transmission rate has already been discovered. Keeping the relationship undefined in its mechanism leaves room for an account of its personal and theological meaning.

The book makes a positive proposal. Physical reality is understood through interacting processes; mathematical models articulate selected structures of those processes; physical laws describe their

regularities and constraints. The ground of the existing order and the spiritual significance of the person belong to the wider metaphysical inquiry. The theological development identifies that ground with God and approaches the personal relationship from a Christian standpoint. The mathematical arguments state what follows from their assumptions, so a reader can assess them even while questioning that standpoint.

Mathematics has a demanding role throughout. Explicit constructions replace schematic infinity notation. We show how infinitely many inputs can yield a well-defined update, why interaction alone does not entail chaos, how infinitely many fractal-state components can enter one all-to-all update, and when a coarse description inherits an autonomous dynamics. Results on admissibility, equivalence, and preservation become tools for understanding STOEM itself.

Physics occupies the center of the investigation. Matter, fields, energy, symmetry, gravitation, thermodynamics, quantum measurement, locality, and cosmology each constrain what a physical realization of STOEM would have to explain. The book supplies worked examples and an account of recovery from one descriptive level to another. The mathematical content of those theories remains essential to the inquiry.

The source papers record exploratory conversations and evolving formulations. This book returns to their organizing questions while revising expressions that carried more philosophical weight than their definitions justified. It distinguishes the original proposals from the reconstructions introduced here. In particular, a product of symbols for body and spirit is not treated as an established equation for a person, and an iteration number is not automatically identified with the time read by a physical clock.

No extraordinary historical rupture is required by the argument. Questions that first arose around quantum experiments have become questions about the ordinary conditions of existence, knowledge, and responsibility. The book can therefore stand independently as a metaphysical investigation.

The reader will encounter proofs, physical calculations, philosophical arguments, and theological commitments. Each has its own work to do. The aim is to make their relationships intelligible without allowing any one of them to conceal the assumptions of the others. References and source notes are collected at the end.

CHAPTER 1

From STOEM to Mathematical Metaphysics

The Initial Architecture

STOEM began with a proposal about the whole of physical existence. The universe was conceived as an infinite family of fractal processes, each with infinitely many variables. Their results would feed into one another at every iteration. What appeared within the universe as an isolated entity would belong to a larger order of interaction. The proposal also placed the spiritual relationship between God and humanity orthogonal to this physical order.

The earliest papers examined in preparing this book include Exploring the Spiritual Theory of Everything Model, dated May 23, 2023, and Spiritual Theory of Everything: A Mathematical Approach to Infinity, God, and the Nature of Reality, dated May 24. Their dialogue form matters. They show questions being sharpened, notation being attempted, and alternatives entering before a single settled presentation existed. They are records of conceptual development.

Four commitments can be separated within that development. The physical universe is an interacting whole. Its changes can be approached through iteration. Mathematical description has a central role in articulating its organization. Human existence includes a spiritual relationship that physical description does not exhaust. The commitments belong together in STOEM, but they have different meanings and require different kinds of explanation.

An interacting physical universe is a proposal about the organization of physical being. An iterative representation is a way of expressing

change. A mathematical ontology would make the stronger claim that physical being itself is mathematical. The God-Man relationship adds a claim about the scope of reality and the character of personal existence. Clarifying these differences makes it possible to develop the proposal rather than merely repeat it.

A Development Already Present in the Sources

The May 24 dialogue includes a consequential question: could an unknown equation replace the fractal while retaining infinite parallelism, infinitely many variables, iteration, and mutual feedback? That question broadens the original research program. It allows the architecture of interaction to be considered separately from the particular geometry used to represent it.

This book consequently distinguishes a broad STOEM commitment from a stronger fractal realization. The broad commitment concerns an interacting created order, meaningful succession, and a God-Man relationship that is not reduced to its physical description. The stronger proposal specifies infinitely many fractal processes as the physical constitution of that order. The distinction identifies the additional claim involved when a mathematical example is promoted to an account of the universe.

Orthogonal Divinity, dated May 29, develops the spiritual distinction. The Nature of Reality and Fractal Iteration in STOEM, dated May 30, develops iteration and time. The June 2 Seed gathers an evolving formulation into a compact statement. Mathematics and Consciousness in the STOEM Universe, dated June 10, explicitly considers mathematical realism and anti-realism. By June 22, Bridging the Metaphysical and the Mathematical places the project in conversation with earlier thinkers.

This sequence shows why mathematical metaphysics is an appropriate larger setting. Questions about what exists, what mathematics describes, how change occurs, and what a person is were present from the beginning. The present synthesis organizes them and makes their dependencies explicit.

Three Levels of Commitment

The first level is metaphysical. It concerns being, relationship, actuality, and ground. At this level the book proposes that the physical order does not exhaust the meaning of existence and that its created character is compatible with sustained scientific inquiry. The existence of equations is not sufficient to explain the existence of what they describe.

The second level is mathematical. It specifies spaces, transformations, questions, and relations. Here definitions must be adequate for proofs. An infinite sum must converge where it is used. A transformation must have a stated domain. A claimed fractal structure needs a construction or a property by which it can be recognized.

The third level is physical. It connects selected mathematical structures with clocks, matter, measurement, and physical histories. A successful connection must explain which quantities can be recovered and under what conditions. Naming a variable gravity does not supply an account of gravitation.

These levels form an order of explanation, while inquiry moves in several directions. A physical discovery can expose a poor metaphysical assumption. A mathematical obstruction can show that a proposed combination of commitments fails. A metaphysical distinction can reveal that two apparently competing statements

answer different questions. Giving metaphysics priority retains these reciprocal relationships.

From Representation to Realization

A recurrent question runs through the book: what turns an internally specified structure into a description of this world? A collection of equations can have many solutions. The actual world does not become identified with one of them simply because a variable has been named universe.

STOEM makes this question vivid because of its breadth. An account intended to encompass every physical process must also include the physical processes through which the account is written, read, and tested. The interpreter belongs to the subject. Yet belonging to the same world does not make every interpretation correct. There remains a distinction between the physical occurrence of a belief and the truth of what is believed.

The preservation framework developed later gives this distinction a practical form. A representation retains some answers and loses others. We can ask whether its transformations preserve the questions relevant to the subject. A physical model may preserve a conserved quantity while losing a detailed trajectory. A psychological score may preserve a statistical correlation while losing the person's history. The same mathematical discipline can reveal both limitations without identifying their subjects.

An Explicit Theological Orientation

The theological development of STOEM is substantive. God is treated as the ground of created existence, and the person is understood as capable of relationship with God. This orientation is not inferred from the convergence of a series or the existence of a fractal. Its warrant belongs to metaphysical and theological argument, including the Christian understanding of creation and personal relationship.

The resulting claim is stronger than saying that science has unanswered questions. An unanswered physical question may receive a physical answer. STOEM's theological question concerns the existence and dependence of the physical order, including an order whose internal laws are extensively understood. Scientific success therefore retains the question to which divine grounding is offered as an answer.

The book also retains the distinction between admitting a spiritual relationship and claiming to know its mechanism. Human relationship with God is not assigned a numerical coordinate merely to make the scheme appear complete. The account will examine recognition, responsibility, prayer, and embodiment while leaving that mechanism undefined.

How to Read the Synthesis

The next chapters develop being, priority, identity, and admissibility. Two mathematical chapters then reconstruct infinite interaction and fractal structure, culminating in one map that combines the proposed mathematical architecture. Its application to physical law remains a further task, addressed through the recovery framework. The general preservation results follow, leading into iteration and physical time. A substantial sequence on physics investigates the domain any physical realization must address.

The later chapters return to the embodied knower, the God-Man relationship, agency, and creation. Objections are considered within the same argument. Agreement with one level does not automatically require agreement with every other: a proof may stand even when its proposed application is disputed, and a theological commitment may remain meaningful while a physical realization is revised.

The proposed unity lies in making those connections answerable. STOEM supplies an architecture of existence to be developed. Mathematical metaphysics supplies a discipline for examining how that architecture is expressed, what it establishes, and how it remains connected to the world in which the inquiry takes place.

CHAPTER 2

Being Before Description

The World in Which a Model Appears

A map is something in the world. It has a material or electronic realization, a history of construction, and conventions through which its marks become informative. The landscape it represents is also something in the world. Understanding the map requires us to distinguish these two existences while explaining how they are connected.

A mathematical model presents the same philosophical situation in a more abstract form. Its elements may represent particles, temperatures, possible actions, or relations among observations. The relations inside the model can be specified with exactness. Their application still depends on an account of what they represent. A variable called position is not a physical location merely because it has been given that name.

The first commitment of this book is therefore modest but substantive: there is a reality in which representations occur. The success and failure of representation belong to that reality. An inaccurate map does not create the mountain it omits, and a correct equation is an achievement whose scope has to be understood.

This position is realist in a minimal sense. It permits a difference between what is asserted and what is the case. It does not begin by deciding the ultimate inventory of the world. Physical objects, events, relations, minds, possibilities, and values may require different accounts. The inquiry begins with the fact that there is something to be accounted for.

Three Kinds of Independence

We should distinguish independence from a particular observer, independence from all cognitive activity, and independence from every possible description. These are progressively stronger claims. A geological process can be independent of one person's awareness without establishing a general thesis about the relationship between being and thought. Likewise, rejecting the claim that a particular representation is exhaustive does not prove that reality is intrinsically indescribable.

The present framework does not require all three kinds of independence. It requires enough independence for error and correction to have a subject. When a measurement is repeated, the result must be answerable to more than the experimenter's immediate wish. When a description is revised, there must be an intelligible difference between correcting the description and changing the object it concerns.

This distinction also clarifies the role of interpretation. An interpretation connects formal expressions with a subject. It is not necessarily a subjective choice that could be made arbitrarily. Units, experimental procedures, historical reference, and causal interaction can constrain that connection. A thermometer's scale is conventional in one respect and answerable to thermal processes in another.

There can therefore be convention without unrestricted relativism. Choosing whether to express a distance in meters or feet is a choice of representation. The conversion relation preserves the distance being represented. If the conversion is performed incorrectly, the error is not merely another valid point of view.

The Explanatory Priority Proposed Here

The book adopts the following order of explanation: being provides the subject of distinction and relation; distinction and relation make certain forms of intelligibility available; interpreted mathematics articulates selected aspects of that intelligibility. This is a proposal about dependence. It is not a chronology in which the universe first existed without relationships and later acquired arithmetic.

Nor is it an instruction to scientists to stop calculating until metaphysics is finished. Mathematical investigation can reveal patterns that prompt new ontological questions. Observation can overturn an inherited picture of what kinds of things exist. Metaphysical reflection can identify a presupposition that had been mistaken for a result. Inquiry proceeds through reciprocal correction even when its account of explanation gives one consideration priority over another.

A familiar example is the difference between a temperature and its numerical reading. A number becomes a temperature reading only within a system of calibration, measurement, and physical interpretation. It is possible to investigate numerical relationships before understanding every feature of the underlying physics. The priority of the physical and semantic setting in explaining the reading does not prevent discovery from proceeding in the opposite order.

We should consequently resist treating explanatory priority as a contest for institutional authority. Mathematics has authority over the consequences of its assumptions. Empirical investigation constrains claims about physical realization. Metaphysics examines the kinds of being and dependence that the account invokes. None of these roles makes the others dispensable.

The Question Left by Success

Suppose a model predicts every outcome of a specified experiment. That achievement establishes an important relationship between the model and the experiment. It does not by itself establish that every element of the model exists as a separate physical entity, that no other model could make the predictions, or that the same interpretation applies outside the experimental domain.

The question left by success is not whether the success should be denied. It is what explains the success and what conclusions it warrants. A model can identify a real structure while idealizing other features. Different models can preserve different structures. A theory can remain useful while its philosophical interpretation changes.

This gives the book a definite task. It must explain how representations can preserve something real without presupposing that preservation amounts to exhaustive identity with reality. It must also explain how a representation can be criticized without making every criticism an excuse to abandon mathematical discipline.

The starting point is thus a world in which descriptions have consequences, standards, and limits. Being is the subject of the inquiry. Physics investigates physical being within that general subject; mathematics articulates structures used at both levels. Their relationship is the central problem of mathematical metaphysics.

The Existing Universe and Its Equations

STOEM asks us to imagine the universe through an infinitely interacting system. That system can be considered as a mathematical object or as a proposed description of physical being. The distinction becomes decisive when we ask what exists. A mathematical specification establishes its objects under mathematical assumptions; a physical interpretation identifies the processes through which the specification concerns the world.

The present synthesis gives the existing order priority in that explanation. A successful formal account can disclose its structure without being credited, merely by its consistency, with producing the world's actuality. The theological development will give a further account of this priority through created dependence. The immediate point is that the world in which an equation is understood supplies the subject whose existence the equation purports to describe.

CHAPTER 3

What Metaphysical Priority Means

Existence, Explanation, and Knowledge

The expression mathematical metaphysics can suggest that metaphysical questions are to be settled by calculation. It can also suggest that mathematical objects are the ultimate constituents of existence. Neither suggestion specifies the position of this book. The proposal here concerns the interpretation and grounding of mathematical representation, and it requires distinctions among several kinds of priority.

Existential dependence concerns what must exist for something else to exist. Explanatory dependence concerns what makes an account intelligible. Epistemic dependence concerns what a knower must have access to in order to establish a claim. These dependencies can point in different directions. A footprint may be the evidence through which an animal is identified, even though the animal's passage explains the footprint.

A numerical measurement offers a similar distinction. We may learn about a physical process through the number displayed by an instrument. The physical process, together with the instrument's operation, explains why that number appears. A calibration convention explains why the displayed number has the specified significance. The order of learning and the order of explanation need not coincide.

The metaphysical priority proposed here is primarily explanatory and semantic. It says that the application of mathematics to a subject depends on what the subject is and on the meanings assigned to the representation. It does not establish that all mathematical truths

depend for their truth on physical objects. The ontology of mathematics remains a further question.

An Argument for Explanatory Priority

The affirmative argument can be stated through four premises. It concerns an account offered as an explanation of an actual subject. It does not require that mathematical truth wait for human observation or that mathematical objects be unreal.

Premise 1. An adequate explanation must concern the subject it claims to explain. A model of a possible oscillator and an explanation of this measured oscillator can share an equation while differing in whether an actual subject has been identified. Reference and application therefore belong to explanatory adequacy.

Premise 2. Formal satisfaction alone establishes what holds in structures satisfying the stated conditions. It does not, without an additional identification, establish that a particular structure is the actual subject. A proposed identity between reality and mathematical structure is itself such an identification. Its truth cannot be inferred merely from the consistency of the structure it identifies.

Premise 3. Claims about what exists, what constitutes a subject, and why an identification is adequate are ontological claims. Their assessment belongs to metaphysics, although evidence relevant to that assessment can come from physics, biology, or ordinary experience. Calling an identification mathematical does not remove its ontological content.

Premise 4. When an explanation's adequacy depends on a warranted claim about its subject, the inquiry assessing that claim has explanatory priority over the explanation's assertion of adequacy. Priority here concerns justification and scope, rather than the order in which a researcher discovers the equation.

Conclusion. Mathematics used as an account of actuality falls within metaphysical assessment of its subject, reference, and completeness. Mathematics can constrain that assessment, but formal consequence alone does not give the account an unrestricted right to decide what exists. In this specified sense, mathematics stands under metaphysics.

The most contestable premise is the second. A mathematical realist may propose that actuality just is existence as a mathematical structure. The present argument does not refute that proposal by defining it out of consideration. It asks for its warrant: why adopt that identity, which structures does it concern, and what explains the experienced world within it? Answering those questions would supply a metaphysics of mathematical reality. It would exemplify the need for an ontological argument even if its conclusion differed from STOEM's.

Nor does the fourth premise give philosophers authority to overrule a calculation. If the application adopts a mathematical relation, its consequences remain binding. The priority concerns the grounds for applying the relation and for claiming that it exhausts the subject. A metaphysical assertion that fails its own mathematical conditions loses that application.

What STOEM Adds to the Priority Argument

Interpretive discipline alone leaves many ontologies available. STOEM makes additional affirmative commitments. It treats the physical world as an actual interacting order, regards the knowing person as a participant in that order, and understands the order and person as dependent on a divine ground. Fractal realization supplies a definite candidate organization for the physical domain. These commitments give the framework content beyond the observation that symbols need meanings.

The argument for this organization begins from explanatory integration. A common interacting order provides a setting for the same embodied person to measure, remember, act, and enter relationships. The framework retains physical regularities without treating an abstraction of those regularities as the whole person. It treats truth and responsibility as questions about beings within a shared world. The theological argument in Chapter 29 then asks whether that world's actuality, intelligibility, and personal significance are better understood through a personal ground.

This is a cumulative proposal. The priority argument explains why these ontological questions cannot be settled by calculation alone; it does not force their answers. STOEM's further commitments must earn their place through coherence, explanatory reach, and comparison with alternatives. The distinction gives the reader an affirmative thesis and identifiable reasons to challenge it.

Grounds Are Not Automatically Earlier Causes

To say that a fact holds in virtue of something else is not necessarily to describe an earlier event that produced it. A figure's being triangular explains why it has three sides under the relevant definitions. An organism's current organization may explain a capacity it exercises now. A rule can determine what counts as a legal move without preceding each move as a separate physical cause.

These examples involve different forms of explanation and should not be forced into one relation merely by calling all of them grounding. The book uses ground as a general philosophical question: what makes the specified existence, identity, or truth obtain? A particular application must then state the relation it proposes.

This restraint is necessary because the word ground can conceal uncertainty. Saying that reality grounds mathematics does not yet

explain what reality is or how mathematical necessity arises. The thesis becomes more informative when it is restricted to a definite claim: the physical significance of this equation depends on the kind of process its variables represent, or the adequacy of this equivalence relation depends on the identity criterion it is intended to preserve.

Such claims can be examined. We can change the interpretation while holding the formula fixed. We can ask which consequences survive. We can identify whether a distinction was supplied by the mathematics or by the application. The priority becomes a working practice rather than a ceremonial declaration.

Mathematical Necessity Remains Binding

Consider three requirements: $a = b$, $b = c$, and $a \neq c$. Under ordinary equality and classical reasoning, the requirements cannot be satisfied together. The conclusion does not depend on whether the letters denote numbers, measurements, or translated descriptions. If an interpretation genuinely adopts those requirements, it inherits the incompatibility.

Metaphysical examination can ask whether $a = b$ expresses numerical identity or merely agreement under one observation. It can question whether the same comparison is being used in every premise. It can discover that a premise was incorrectly attributed to the subject. What it cannot do is preserve all the meanings and rules while simply declaring the contradiction acceptable in an otherwise classical account.

A different logic is possible as a subject of investigation. It brings a different account of inference and often a different account of negation. The move requires an explicit interpretation of those differences. Naming a logic does not make every inconsistency informative or every inference sound.

The same discipline applies to more sophisticated mathematics. An impossibility theorem has assumptions and a scope. Understanding those assumptions may reveal alternatives, but an alternative earns its standing by specifying which assumption changes and what follows. The vocabulary of metaphysics should make this process clearer.

A Reciprocal Method

The relation between mathematics and metaphysics is therefore reciprocal in inquiry and ordered in explanation. A proposed ontology supplies objects and relations to be represented. Mathematics exposes consequences. Observation and reflection test the application. The ontology can then be revised.

This method does not guarantee convergence on a final picture. It does provide a standard for progress. A revision should explain more, remove an ambiguity, resolve a contradiction, or clarify a limitation. Adding terminology without improving any of those things does not advance the argument.

An especially useful test is to ask what the account would have to give up if a particular requirement failed. If every possible outcome can be accommodated by silently changing the meaning of the terms, the interpretation has become difficult to assess. If the commitments are explicit, their consequences can be compared with alternatives.

The hierarchy defended here is thus compatible with intellectual vulnerability. Metaphysics examines the meaning and scope of mathematical assumptions, while mathematical results can expose a metaphysical mistake. That relationship is central to the book's claim to rigor.

Priority Within STOEM

The hierarchy does not require that God first construct a stock of mathematical objects and then use them as physical materials. That would add a speculative sequence to an account of dependence. The theological proposal is that the order is created and intelligible; mathematical articulation expresses relationships within that order. How mathematical truth itself relates to the divine intellect is a further philosophical question.

The early STOEM papers sometimes describe reality as a system of equations. Here that phrase receives a disciplined reading: the physical order may possess the structures expressed by an iterative system. Identifying the instantiated world with an abstract mathematical structure remains a stronger thesis. The book need not settle every version of mathematical Platonism to insist that an account of actuality and interpretation is required.

CHAPTER 4

A Place Among Earlier Approaches

Several Projects Under One Name

Mathematical metaphysics is not the name of a single settled doctrine. It can refer to the use of mathematical methods in ontology, to a claim that reality is mathematical, or to an investigation of how mathematical abstraction relates to existence. A useful comparison must identify which of these tasks a writer undertakes.

The issue is older than the phrase. Questions about number, form, substance, order, and necessity have long been part of metaphysics. The present book enters that history through a particular problem: how the meaning of a representation constrains what counts as a successful preservation of its subject. It makes no claim to have originated the distinction between reality and its description.

The following comparisons identify selected points of contact from antiquity to modern approaches. They locate questions and disagreements rather than recruiting historical figures as imagined endorsers of STOEM. A full account of any one of these philosophies would require a book of its own.

Plato and Aristotle: Order and the Scope of Inquiry

The opening cosmological argument of Plato's Timaeus connects a mathematically ordered cosmos with an account of intelligent ordering. That conjunction makes it relevant to STOEM's attempt to think about physical structure and divine significance together. The comparison does not establish agreement about infinitely interacting fractals or about the Christian God-Man relationship. It identifies an enduring question: how are mathematical intelligibility and the source of cosmic order related?

Aristotle's Metaphysics distinguishes the inquiry into being as being from investigations of particular domains. Book IV, chapter 1, states the general inquiry; Book VI, chapter 1, considers the relationships among physics, mathematics, and first philosophy. This provides a historical point of contact for treating physics as a study of physical being within a broader metaphysical inquiry. Aristotle's own account of substance and the divine should nevertheless be examined on its own terms. The present hierarchy is not a claim that modern physics can simply be inserted unchanged into his system.

Spinoza and Leibniz: Unity and Multiplicity

Part I of Spinoza's Ethics gives a systematic account of God, substance, and the dependence of finite things. It offers a significant comparison for STOEM's ambition to understand the universe as a unified order. The present book retains a distinction between Creator and creation, however. Its theological ground is not identified with the aggregate of physical states. Agreement that finite things belong to a comprehensive order therefore leaves a substantial disagreement about that order's relation to God.

Leibniz's Monadology, especially sections 1-7 and 56-57, treats a plurality of simple substances and their relation to the universe. Its

picture of a many-sided order invites comparison with STOEM's multiplicity. The difference is equally instructive: Leibniz does not describe monads as exchanging physical causal inputs in the manner of a coupled dynamical network. His account of their coordination cannot be equated with the mutual feedback proposed here. Historical comparison becomes useful precisely when it identifies such divergent explanatory strategies.

Whitehead and the Criticism of Abstraction

In Science and the Modern World, Whitehead examines the tendency to treat scientific abstractions as though they were the concrete realities from which the abstractions were drawn. His discussion assigns philosophy a role in examining the abstractions through which inquiry proceeds. He also connects the problem with the treatment of matter, mind, and spatial location.

This is a substantial predecessor to the present project. Our claim that a mathematically useful account needs examination of its scope does not by itself move beyond Whitehead. The further task undertaken here is to state selected preservation requirements explicitly and to calculate how those requirements behave under transformations of a representation.

The comparison also sets a standard of scope. A criticism of abstraction should explain why the abstraction succeeds where it does. Otherwise the criticism risks treating explanatory power as an embarrassment instead of something to be understood.

Badiou and Mathematics as Ontology

Badiou's Being and Event develops an account of being through multiplicity and set theory. His later discussion of mathematics as a philosophical condition makes the relation between mathematical development and philosophical concepts explicit. Mathematics is not confined to supplying illustrations of independently formed opinions.

The present book shares the demand that mathematics have a consequential role. It differs in leaving the identification of formal structure with an account of being subject to a further interpretation. Our sets of states and relations are chosen representations. Their ontology is not established merely by their appearance in the notation.

This difference should not be exaggerated into a claim that Badiou simply identifies every physical object with a set. The point of comparison is the role mathematical discourse occupies in an account of being, and the responsibility assigned to philosophical interpretation.

Tegmark and a Mathematical Reality

Tegmark's mathematical universe hypothesis proposes that external physical reality is an abstract mathematical structure. His argument explores the consequences of describing a world independently of human-specific concepts and extends to questions about computation, observers, and other mathematical structures.

Our framework treats the identity between physical reality and mathematical structure as a position to assess. A faithful representation of selected relationships does not alone establish that the represented world is exhausted by those relationships. The difference is especially visible when two interpretations attach different physical significance to formally similar structures.

The resulting challenge is constructive. We must explain what the additional interpretation contributes and why it cannot always be absorbed into a richer mathematical structure. The book offers a method for locating that contribution in particular applications; it does not pretend to have proved a universal impossibility of complete mathematical representation.

Zalamea and the Joining of Perspectives

Zalamea's Synthetic Philosophy of Contemporary Mathematics explicitly explores the joining of complementary philosophical perspectives through ideas associated with sheaves. His treatment emphasizes relationships among local and global descriptions, mathematical transformations, and the historical movement of concepts.

This is a close precedent for any project centered on contexts and reconciliation. The use of a collection of viewpoints or a gluing construction is therefore not, by itself, a novel philosophical contribution. Our more restricted question concerns the requirements a proposed joining must preserve, and what happens when different requirements support different comparisons.

The distinction matters because successful joining is always joining of something in a specified respect. A vocabulary can unify the numerical outputs of several practices while omitting what makes the practices different. The book turns that issue into finite examples in which the omitted distinctions can be identified exactly.

Langan and a Comprehensive Metaphysical Language

Langan's An Introduction to Mathematical Metaphysics uses the phrase directly and proposes an account connecting cognition, perceptual reality, and a self-referential language of reality. His article aims at a comprehensive identity between the mental and physical aspects within that account.

The present project also asks how the inquirer belongs to the world being described. It proceeds by declaring limited commitments and proving conditional results about selected representations. Its mathematical models do not certify a comprehensive ontology merely through their internal consistency.

The overlap makes a comparison necessary. Shared words such as relation, identity, and self-reference can conceal different definitions. The appropriate question is what each definition permits us to infer and how the proposed inference relates to the world.

The Contribution Sought Here

The book's candidate contribution is a disciplined account of preservation. Given a subject, a family of descriptions, and stated requirements of meaning and identity, what transformations preserve the account? What information is lost? When can two descriptions be reconciled, and when does an apparent reconciliation simply discard the disputed feature?

These questions have relatives in logic, measurement, philosophy of science, and mathematics. Theoretical equivalence already has sophisticated formal treatments, including the distinctions studied by Barrett and Halvorson. The aim here is to bring a limited part of that territory into a general metaphysical method with accessible examples.

A claim of progress must consequently be specific. It might consist in a clearer distinction, a productive application, or a new result about a defined class of representations. It cannot be established by the size of a synthesis or by adopting a more comprehensive title. The remainder of the book supplies the argument on which its own contribution should be judged.

STOEM contributes a particular architecture to this landscape: infinite interaction, an iterative account of change, and a spiritual relationship orthogonal to physical description. The reconstruction offered here adds a disciplined comparison between that architecture and the structures used to represent it. Its originality, where present, must be assessed at that level of argument and construction, rather than inferred from the breadth of its subject.

CHAPTER 5

Distinction, Identity, and Equivalence

A Difference That Matters

A distinction becomes informative when a difference matters to an inquiry. The letters A and B differ as written signs. Two specimens differ in origin. Two measurements differ in value. These are not interchangeable kinds of difference. Their significance depends on the question being asked.

This does not make every distinction arbitrary. A chosen criterion can be inappropriate, unstable, or contradicted by the subject. An instrument designed to distinguish temperatures may respond mainly to humidity. A classification can group together cases whose differences are essential to the explanation it claims to provide. The act of choosing a criterion creates an obligation to justify its relevance.

The framework therefore begins with explicit discrimination. Before counting objects, it asks what counts as one object. Before comparing states, it asks which differences are retained. Before identifying a repeated event, it asks which continuity makes repetition an appropriate description.

Counting itself illustrates the issue. A library can contain one work in three editions and eight physical copies. Each answer is exact under a different criterion. The numbers are not contradictory until the criteria are confused. Mathematical exactness does not select the criterion on its own.

Four Uses of Sameness

Numerical identity says that two references concern one entity. Qualitative similarity says that distinct entities resemble one another in some respect. Functional equivalence says that they can play the same role under stated conditions. Observational indistinguishability says that a selected procedure gives the same result for them.

A replacement component may be functionally equivalent to the original while remaining a different physical component. Two people may be indistinguishable under a coarse demographic description while remaining different persons. A digitized image may preserve a document's visible text while failing to preserve the original sheet of paper.

These differences matter because sameness supports inference. If two descriptions refer to one person, information obtained under either description may concern that person's life. If they merely assign the same score, the inference is much weaker. Confusing the two can turn an accurate classification into a mistaken identification.

A metaphysics that gives relations a central role must therefore specify the relations it uses. The assertion that everything is connected does not tell us whether a connection preserves reference, causation, resemblance, or membership in a category.

When an Equivalence Relation Is Appropriate

An equivalence relation on a set S is a relation that is reflexive, symmetric, and transitive. Reflexivity requires each element to be related to itself. Symmetry permits reversal of a related pair. Transitivity requires that if x is related to y and y to z, then x is related to z.

A relation defined by having the same value under a function q satisfies these conditions. Write x ~q y when q(x) = q(y). Equality of outputs is reflexive and symmetric, and equality through an intermediate output is transitive. The function partitions S into classes of elements that receive the same output.

Resemblance need not behave this way. If two lengths count as similar whenever they differ by less than a centimeter, one length can resemble a second and the second a third while the first and third do not resemble one another. Treating this resemblance as equivalence would add a property the original criterion does not possess.

The example shows why formalization is already an interpretive act. Selecting an equivalence relation commits the account to a particular pattern of inference. That commitment can be useful and justified. It should not enter invisibly through the ordinary word same.

Quotients and the Cost of Compression

The quotient S/~ is the set of equivalence classes under a relation ~. It can be a powerful representation because it removes differences irrelevant to a chosen purpose. A table organized by output does not need a separate row for every input that yields that output.

The benefit has a cost. Once two inputs have been identified in the quotient, a question that distinguishes them cannot be answered from the quotient alone. The lost distinction can be restored only by adding information, returning to a finer representation, or changing the task.

A simple example uses four cards labeled a, b, c, and d. Suppose a and b are red, while c and d are blue. A color classification has two classes. It preserves the answer to the question of color, but it cannot tell a from b. If a and c also carry a circle while b and d carry a square, color alone does not preserve shape.

Neither classification is universally superior. Their adequacy depends on what must remain answerable. A description preserving color and shape distinguishes all four cards in this example. The finer account is more informative about those questions, but it may be unnecessary for a task concerned only with color.

Identity Through Change

Persistence introduces another dimension. Something can change while remaining identifiable. A building can be repaired, an organism can develop, and a theory can be reformulated. Each case requires a criterion for what continues through the change.

The present framework does not supply one criterion for every domain. It supplies a demand for explicitness. A physical continuity criterion differs from a criterion of functional continuity or autobiographical continuity. A dispute about persistence may concern evidence, but it may also concern which criterion is appropriate.

This makes identity a central point of contact between mathematics and metaphysics. Mathematics can represent equivalence, continuity, and transformation. Metaphysical interpretation explains why the selected representation answers the intended question about what exists and persists. The next chapter places these identity claims within a common world of relationships and perspectives.

A Component Across Iterations

In STOEM, writing the same component index i at successive iterations already assumes a criterion of continuity. The index may identify an enduring process, a fixed position in a model, or a role occupied by different physical constituents. Those interpretations differ. The repeated label does not decide between them.

A physical realization must therefore explain what continues when a coordinate changes. The issue becomes especially important for organisms: material exchange can coexist with maintained organization. The mathematical framework can represent that continuity, while the metaphysical account explains why it identifies the same being. This prepares the later distinction between a person's physical description and the person's relation to God.

CHAPTER 6

Relations, Contexts, and a Common World

Relations Without Premature Reduction

To understand a thing is often to understand how it stands in relation to other things. An object's location relates it to a spatial account. Its ancestry relates it to a history. Its function relates it to activities and environments. Its identification relates an encounter to a continuing subject.

This establishes an important role for relations without proving that beings have no intrinsic features. The claim that intelligibility requires relations is weaker than the claim that reality consists exclusively of relations. This book adopts the first claim. The second remains a possible ontology requiring additional argument.

The distinction protects the account from a circular shortcut. If everything about an object is explained by relations to other objects, and everything about those objects is explained in the same way, we still need to understand what makes the whole network an instantiated reality. A relational description can be rich and exact while leaving the question of existence open.

Relations also differ in kind. Causal dependence, spatial adjacency, semantic reference, and moral obligation should not be merged merely because each can be represented by ordered pairs. Their common mathematical form does not establish an identity of meaning.

What a Context Contains

A context is a specified setting in which a description has conditions of application. It may include an instrument, a vocabulary, a scale, a preparation procedure, or a practical objective. The context determines which distinctions are available and what a recorded result is taken to mean.

Consider a photograph of a room. One context concerns its visible geometry. Another concerns ownership of the objects pictured. A third concerns the photographer's purpose. The same image can participate in all three, but the evidence and relevant relationships differ. The pixels alone do not answer every question posed in every context.

The point is not that the image has no determinate content. It is that content becomes informative through a question and an interpretive connection. An image can answer a geometric question accurately while offering no basis for an assertion about ownership. Extending its authority to the second question requires additional evidence.

A scientific context can be much more tightly specified. A preparation procedure, a measurement setting, and a recording rule together define an experimental question. Even there, the distinction between the numerical outcome and its physical significance remains important.

Translation Is Selective Preservation

A translation between contexts changes a representation while retaining something of its content. A temperature conversion preserves a physical quantity through different units. A summary preserves selected claims while omitting wording. A change of coordinates preserves specified geometrical relationships.

Successful translation is therefore relative to a requirement. It may preserve order without preserving distance, or preserve the existence of a relationship without preserving its direction. A translation that is adequate for one purpose can fail for another.

The common phrase lossless translation is meaningful only after the relevant information has been identified. A representation can be lossless with respect to a family of questions and lossy with respect to a larger family. This is one reason the requirements of preservation occupy the center of the formal account developed below.

Different descriptions can also be complementary without translating into one another directly. A diagram and a verbal account may each contribute information that becomes intelligible only through a larger practice. The absence of a simple conversion formula does not make them descriptions of unrelated realities.

Unity Is Not One Universal Vocabulary

A common world need not be represented by one vocabulary in which every distinction is made simultaneously. Different accounts can be connected through partial translations, common records, or stable relationships among their predictions. Their unity can consist in a structured network of comparisons.

The requirement of unity is nevertheless substantive. If every assertion can be insulated from every other assertion by assigning it a private context, the idea of a common world loses explanatory force. There must be some account of where comparisons are possible, what they preserve, and how disagreements are investigated.

A useful standard is corrigibility. An account belongs to a common practice of inquiry when some errors can be identified through its relationships with other accounts. An isolated vocabulary that cannot

encounter correction may remain internally coherent, but its connection with shared reality requires further explanation.

This standard is not a definition of all existence. Unobserved events can exist without entering a current human practice. It is a standard for the epistemic claim that our descriptions participate in a common inquiry.

Local Success and Global Questions

A collection of descriptions can each work locally while creating difficulty when combined. The cause may be incompatible assumptions, different meanings assigned to shared terms, or a mistaken demand that all local descriptions admit one joint assignment.

The difference between these possibilities matters. A failure of combination does not automatically establish a failure in the world. It may establish that the proposed combination is wrong. Equally, local success does not automatically settle every question about how the descriptions fit together.

The mathematical problem is to state the proposed conditions of combination and determine whether they can be satisfied. The philosophical problem is to explain why those conditions are appropriate. Keeping both problems visible permits exact reasoning without treating the first chosen formalism as the final definition of reality.

The Whole and Its Internal Access

STOEM proposes a common physical order containing the processes through which observers encounter it. The global physical state used in a model and the local descriptions available to observers are different objects. A complete underlying configuration does not automatically assign simultaneous context-independent values to every conceivable measurement. That further assignment requires a measurement model.

The distinction is essential to the synthesis. Infinite interaction concerns how the proposed order is constituted. Contextual access concerns what participants can describe or measure. The mathematics must relate those levels explicitly, so a philosophical commitment to one world does not silently import a particular theory of observation.

CHAPTER 7

Admissibility and the Meaning of May

Four Questions of Possibility

A configuration may satisfy a set of equations without being physically realizable. A physical possibility may never occur. An actual event may expose a limitation in the model through which it had been judged impossible. The word possible conceals these differences unless its conditions are stated.

Formal admissibility asks whether a description satisfies a specified collection of mathematical requirements. Metaphysical possibility asks whether the proposed beings and relationships could exist under an account of being. Physical realizability asks whether a configuration is compatible with relevant laws and conditions. Actuality asks what is instantiated.

These questions should not be arranged into an unconditional hierarchy of sets. A faulty model can exclude an actual case. A mistaken metaphysical commitment can misdescribe a physical possibility. The differences concern the grounds on which a claim is made, and each ground remains open to examination.

Claim	What Must Be Supplied
Formally admissible	A representation and requirements it satisfies.
Metaphysically possible	An account of the relevant kinds of being and dependence.
Physically realizable	Applicable physical laws, conditions, and an account of realization.
Actual	Grounds for asserting that the configuration is instantiated.

Requirements Have Different Sources

A model can contain several kinds of requirement. Some express mathematical well-formedness. Others represent observed regularities, identity criteria, or practical objectives. These sources differ, and their differences should remain visible after they have been encoded in a common formal language.

A bridge model might require a positive span, a specified material response, a load limit, and a budget. The positivity condition may belong to the definition of the modeled geometry. The material response concerns physical behavior. The load limit combines physical analysis with a performance requirement. The budget comes from an institutional decision.

Encoding all four as inequalities does not give them one kind of authority. A violation can call for a different response depending on which requirement fails. The geometry might be ill-defined, the material model inaccurate, the design inadequate, or the budget revisable.

The same issue arises in metaphysics. A requirement of consistent reference differs from an assumption that every observable possesses a context-independent value. A condition that preserves personal identity differs from a criterion that merely preserves a score assigned to a person. Their mathematical expressions should not conceal those differences.

The Selection of P

The formal model below uses P for a collection of requirements. The symbol is useful because it makes their presence explicit. It is also dangerous if the requirements are introduced without justification. An account can be made to admit or exclude almost anything by changing its conditions after the fact.

The discipline proposed here has three parts. State a requirement before interpreting the result it produces. Identify the question it serves and the grounds for adopting it. Then examine the sensitivity of the result to changing or removing it.

This procedure does not provide an algorithm for choosing every metaphysical commitment. It creates a record of argumentative responsibility. If two accounts disagree, one can ask whether they disagree about evidence, interpretation, or a prior condition of admissibility.

Requirements should also be distinguished from desired conclusions. Suppose an investigator wants a single account of several contexts. Adding a requirement that the contexts must have a common assignment does not show that the desired account exists. It merely states the question that the remaining constraints must answer.

Minimal Commitment and Adequate Commitment

Fewer assumptions can make an account easier to satisfy, but this does not automatically make it better. Removing all requirements produces an account that excludes little and explains little. A representation gains content by making discriminations that matter to its subject.

The aim is adequate commitment. Each requirement should contribute to the intended explanation, and its contribution should be

visible. An unnecessary condition narrows the account without improving its connection to the subject. An omitted necessary condition can make a formally successful result irrelevant.

For example, a conversion that preserves the ordering of temperatures can support comparisons of hotter and colder. It may fail to preserve a question about a temperature difference if the conversion rule has been specified inadequately. The requirement needed depends on the task. A purely order-preserving account should not claim to preserve every metric fact.

This is one reason the book connects admissibility with preservation. The first asks which descriptions are allowed. The second asks what remains meaningful when a description changes. A defensible model needs both.

Contingency Without Arbitrariness

Some requirements may be contingent: a different world or a different physical regime might obey different conditions. Contingency does not make the actual requirement arbitrary. Its authority can come from the world it describes even if it is not necessary in every possible world.

Conversely, a condition can be mathematically necessary within a representation while its application remains contingent. Once equality is defined in the ordinary way, transitivity follows. Whether two actual records should be treated as records of the same event is a further question.

The framework distinguishes necessities arising from definitions, from the chosen logic, and from an account of essence or law. It also asks which conditions have merely been assumed because they are familiar.

A Ledger of STOEM Commitments

A STOEM realization can now identify its requirements by source. The range of a coordinate and the convergence of a coupling belong to mathematical specification. A claim that a recovered quantity represents energy belongs to physical interpretation. Infinite physical fractal organization is an ontological proposal requiring an account of realization. Divine grounding and the God-Man relationship belong to the metaphysical and theological orientation.

These distinctions allow a revision to have a definite scope. A failure of one coupling formula need not refute every metaphysical commitment. Equally, accepting the theological orientation does not make every proposed physical equation admissible. The argument can develop by correcting a specified connection while retaining the commitments that the correction does not address.

CHAPTER 8

Infinite Interaction: A Mathematical Realization

Replacing a Schematic Infinity

An infinite collection is not obtained by adding a last entry labeled infinity to a finite list. The positive integers have no final member. An expression involving the variable immediately before the last infinite variable therefore does not define an ordinary sequence. The early STOEM notation expressed an intention that needs a more precise representation.

Let I be the set of positive integers. For every pair (i, j) in I × I, introduce a real coordinate x(i, j) in [0, 1]. The index i identifies a component, and j identifies one of its variables. A complete state x is the whole array of coordinates. Denote the collection of these arrays by X. Each component has countably many variables, and there are countably many components.

This choice makes one meaning of infinite explicit. It does not claim that countability is forced by metaphysics. Nor does the countability of the coordinate labels make X a countable set: even one real coordinate admits uncountably many values. The number of labels and the number of possible states answer different questions.

A general update would be a function T from X to X, with x at iteration n + 1 obtained from T applied to x at iteration n. That expression becomes informative when T is actually defined. We now give two definitions, chosen to illustrate possibilities within an architecture of infinite interaction. They are mathematical realizations introduced in this book, rather than equations already established for the physical universe.

A Convergent Coupling of Every Coordinate

Assign the positive weight $w(i, j) = 2$ raised to the power $-(i + j)$. The weights sum to one because each of the two geometric series beginning with 1/2 sums to one. Every coordinate receives a positive weight, although distant indices receive progressively smaller weights.

Define $r(u) = 4u(1 - u)$ for u in $[0, 1]$. This function also takes values in $[0, 1]$. For a complete state, define the collective quantity:

$$M(x) = \Sigma_i \Sigma_j w(i, j) r(x(i, j)).$$

Both sums run over the positive integers. The series converges absolutely and uniformly over X because each summand lies between zero and its weight. Consequently $0 \leq M(x) \leq 1$. The collection has infinitely many contributors and a finite, well-defined aggregate.

Choose a fixed parameter α with $0 < \alpha \leq 1$. Define the next value of each coordinate by:

$$T(x)(i, j) = (1 - \alpha) r(x(i, j)) + \alpha M(x).$$

Every output depends on the same aggregate of every input, together with a local contribution when $\alpha < 1$. Changing a remote coordinate can alter M and thereby alter all outputs. The strength and form of that dependence are specified rather than inferred from the word interconnected.

The coordinate labels here have no assigned physical distances. A rapidly decreasing weight in index space is not yet a law of attenuation in physical space. A physical interpretation would have to supply the geometry, units, and measurement relations through which that distinction becomes meaningful.

Proposition A: Existence and a Stability Bound

The update T maps X into itself. Given any initial x_0 in X, it therefore defines a unique forward sequence x_0, x_1, x_2, and so on by repeated application. For the distance $d(x, y) = \sup_{ij} |x(i, j) - y(i, j)|$, it satisfies:

$$d(T(x), T(y)) \leq 4\, d(x, y).$$

Proof. Each output is a convex combination of two numbers in [0, 1], so it remains in that interval. The aggregate has already been shown to converge, making every output unambiguous. This defines T everywhere on X and makes finite forward recursion unique.

For u and v in [0, 1], factor the difference of the quadratic expressions. It gives $|r(u) - r(v)| = 4\,|u - v|\,|1 - u - v| \leq 4\,|u - v|$. Applying this bound to the weighted sum gives $|M(x) - M(y)| \leq 4\, d(x, y)$. The local and aggregate terms then give the displayed inequality. Repeated application yields $d(x_n, y_n) \leq 4^n\, d(x_0, y_0)$.

The bound is a statement of controlled dependence over a specified number of iterations. It is an upper bound, so it does not establish that distances actually grow at that rate. In particular, the proof is not a demonstration that every trajectory is chaotic. Existence of a map, sensitivity of a particular trajectory, and chaotic behavior of a system require different arguments.

This construction also separates defining a universe-sized state transformation from running it on a finite computer. The mathematical rule is well-defined. Exact execution of arbitrary infinitely specified inputs is another question. There is no implication that an observer inside the represented universe possesses infinite computational resources.

What a Finite Approximation Omits

Retain the coordinates with $1 \le i, j \le m$, and use the same weights without renormalizing. The retained weights sum to $(1 - 2^{-m})^2$. The omitted weight is therefore:

$$\delta_m = 1 - (1 - 2^{-m})^2.$$

Because $0 \le r \le 1$, dropping the omitted terms changes M by at most δ_m when the retained values are exact. This is a rigorous error statement about a particular truncation. Renormalizing the retained weights would define a different approximation and require its own estimate.

Suppose the finite update starts with the exact retained coordinates of x_0. Let e_n be the largest error among those retained coordinates after n updates. The omitted aggregate contributes at most $\alpha\, \delta_m$, while the retained errors contribute at most $4e_n$. Thus:

$$e_{n+1} \le 4e_n + \alpha\, \delta_m,\ e_0 = 0.$$

Summing the geometric progression gives $e_n \le \alpha\, \delta_m(4^n - 1)/3$. The actual error is also at most one because all coordinates stay in [0, 1]. A growing upper bound can cease to be informative without the approximation becoming undefined.

For $\alpha = 1/4$, $m = 20$, and $n = 5$, the bound is less than 0.000163. This certifies an accuracy over five iterations for every initial state under the stipulated comparison. It does not certify the long-term behavior from a short numerical run. The number of retained variables, the horizon, and the required tolerance belong together in an approximation claim.

Proposition B: Infinite Interaction Can Converge to Rest

A second example changes the dynamics while retaining the same state space and positive coupling weights. Define $L(x) = \Sigma_i \Sigma_j w(i, j)x(i, j)$, and set:

$$U(x)(i, j) = x(i, j)/2 + L(x)/4 + 1/8.$$

The range of each output lies within [1/8, 7/8]. For the same distance d, the triangle inequality gives $d(U(x), U(y)) \leq (3/4)d(x, y)$. The constant state c with every coordinate equal to 1/2 is a fixed point: substituting $L(c) = 1/2$ gives $U(c)(i, j) = 1/2$.

It follows by induction that $d(U^n(x), c) \leq (3/4)^n d(x, c)$, which tends to zero. This proves convergence to c from every initial state in X. If another fixed point existed, its distance from c would be at most three quarters of itself, forcing that distance to be zero. The fixed point is unique.

This proof directly supplies the conclusion; no appeal to an undefined tendency toward order is needed. Infinitely many inputs influence every output, yet the dynamics converges to a single constant state. Infinite interaction alone therefore does not entail chaos, novelty, consciousness, or an inexhaustible increase of complexity.

The Metaphysical Work Still Required

The examples establish that infinite interaction can be expressed coherently and that its consequences depend on the chosen update. They improve the mathematical articulation of STOEM while leaving the question of physical realization open. Neither the logistic construction nor the contracting construction has been derived from an ontology of created being.

They also leave fractal structure to be addressed separately. A nonlinear iteration can generate complicated trajectories without supplying a fractal geometry for the state space or the physical universe. A complete STOEM realization must say where the fractal resides: in a set, an attractor, a distribution, a history, or a physical pattern.

That requirement leads to the next chapter. The purpose is to make the word fractal carry a definite structural claim. Once its meaning is specified, we can ask how that structure participates in interaction and what would be required to recognize it within physical reality.

CHAPTER 9

Fractals, Feedback, and the Limits of Scale

What Is Repeated?

Fractals entered STOEM as a way of imagining inexhaustible structure within an interacting universe. The intuition joins two ideas: a pattern can retain organization across scales, and repeated operations can generate details not visible at a coarse level. These ideas have mathematical realizations, but they should be distinguished before either becomes a claim about the physical world.

Iteration concerns the repeated application of an operation. Self-similarity concerns a relation between a structure and rescaled portions of it. A dynamics can be iterative without its states being fractals. A fractal set can be specified without being a physical process that repeatedly reconstructs itself. The mathematical setting must identify what is changing and what is being compared across scales.

The next construction uses the familiar middle-thirds Cantor set because every step can be inspected. It illustrates what a precise fractal claim looks like. It is not offered as a geometric picture of the observable universe.

A Fractal With an Explicit Construction

Begin with the closed interval [0, 1]. Remove its open middle third. From each of the two remaining intervals remove its open middle third, and repeat. After m stages there are 2^m intervals, each of length 3^{-m}. Their total length is $(2/3)^m$. The limiting set C consists of the points surviving every stage.

Define two contractions $s_0(u) = u/3$ and $s_1(u) = (u + 2)/3$. The surviving set satisfies $C = s_0(C) \cup s_1(C)$. Its two parts are rescaled copies of the whole. Hutchinson's work on iterated function systems provides a general setting for such constructions. Here the elementary interval construction is enough to identify the set and its self-similarity.

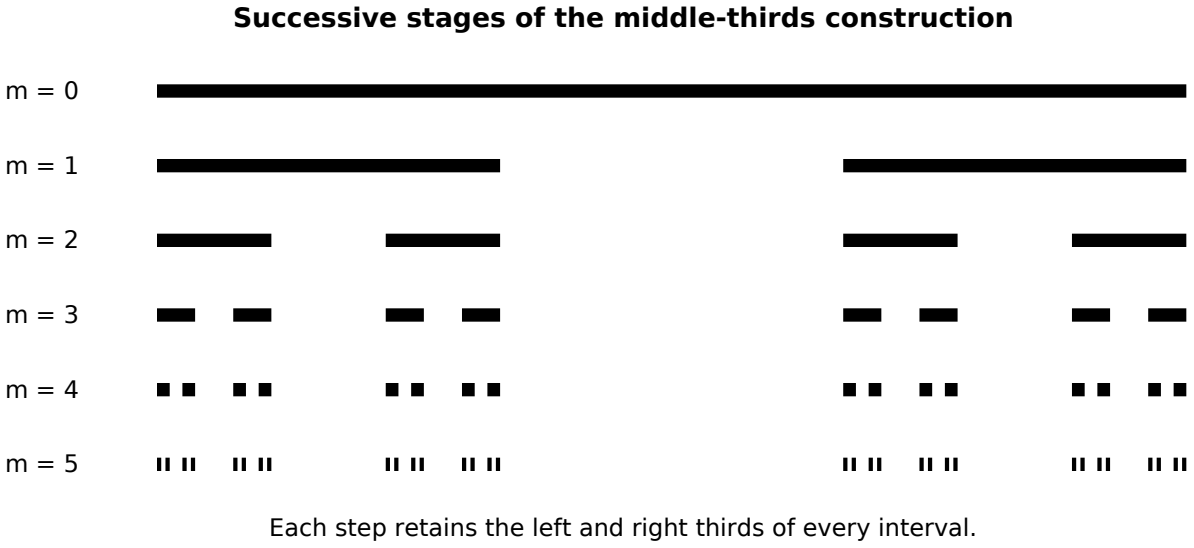


Figure 1. Six finite stages of the Cantor construction. The limiting set is defined by surviving every stage; no finite image displays all its detail.

An alternative description uses an infinite sequence of digits a_j, each equal to zero or one. Define:

$$c(a) = \Sigma_j \, 2a_j/3^j, \; j = 1, 2, 3, \ldots$$

The resulting values are exactly the points of C. Their ternary expansions use only the digits zero and two. The expansion restricted to these two digits is unique: if two such sequences first differ at index k, the difference at that index is $2/3^k$, while the entire possible later tail has magnitude at most $1/3^k$. Their values therefore cannot coincide.

At resolution 3^{-m}, the construction has 2^m separated surviving pieces. The ratio of the logarithm of this count to the logarithm of the inverse

scale is ln 2 / ln 3, approximately 0.63093. This gives its box-counting dimension; intermediate scales yield the same limit by bounding their counts between adjacent construction scales. Dimension is attached to a specified set and metric, rather than to a general impression of complexity.

Proposition C: Finite Digits Give a Controlled Approximation

If two allowed digit sequences agree through index m, then their represented points differ by at most 3^{-m}.

Proof. The coordinates through m cancel. The largest possible remaining difference is bounded by $\Sigma_{j>m}\, 2/3^{j}$. This geometric tail sums to 3^{-m}. Equality is attained when one remaining tail is all zeros and the other is all ones in the binary labeling used here.

The bound connects finite information with an infinitely specified object. It does not say that the finite prefix determines the entire point. It says exactly how much numerical uncertainty remains in this representation. Two descriptions can agree at one resolution while disagreeing at finer resolutions.

This distinction will recur in physics. A finite record can support a specified precision without exhausting its subject. The same bound also prevents a familiar exaggeration: infinite detail in the mathematical set does not mean that every finite observer can retrieve infinitely much information from one measurement.

An Iteration on the Fractal

Dropping the first digit of a sequence defines a dynamics on C. In terms of its real coordinate, write $D(u) = 3u$ on the left component $[0, 1/3] \cap C$ and $D(u) = 3u - 2$ on the right component $[2/3, 1] \cap C$. The branches are unambiguous because these two components are disjoint.

The operation shifts previously smaller-scale digits toward the beginning of the sequence. The discarded first digit cannot be reconstructed uniquely from the output. Every allowed output has two possible predecessors, obtained by prefixing either digit. This is a concrete example of information loss under an update.

It is also a reason to distinguish symbolic information from geometric appearance. A finite prefix can predict only a limited stretch of the future under this particular shift. More digits improve that prediction. The transformation specifies how access at one scale becomes relevant at a later iteration.

One System With Infinitely Many Interacting Fractals

The all-to-all interaction of Chapter 8 and the Cantor construction can now be combined in a single mathematical system. Take the state space C^N: countably many components x_i, each a point of C. Write $x_i = c(a(i, \cdot))$, so every component has infinitely many binary coordinates $a(i, j)$. The fractal is the space of states available to a component; a component's current state is a point of that space.

First enumerate the positive integer pairs (k, j) along successive diagonals. The pairing function is:

$$\pi(k, j) = (k + j - 2)(k + j - 1)/2 + k.$$

Its first pairs are (1, 1), (1, 2), (2, 1), (1, 3), (2, 2), and (3, 1). On the diagonal $k + j = s$, its values run consecutively from $(s - 2)(s - 1)/2 + 1$

to $s(s-1)/2$. These consecutive, nonoverlapping blocks cover every positive integer, so π is a bijection.

For every receiving component i, define the new binary digits b by:

$$b(i, \pi(k, j)) = a(k, j) \text{ XOR } a(i, j + 1).$$

The updated component is $F(x)_i = c(b(i, \cdot))$. XOR adds two bits modulo two. Each new digit combines one digit from a source component k with one digit of the receiving component i. Every source k contributes infinitely many such digits to every receiver i. There is no infinite XOR to evaluate: infinitely many separately defined two-input operations specify the complete next state.

Proposition D: An All-to-All Dynamics on Fractal States

The rule F is a well-defined continuous map from C^N to C^N in the product topology. Every complete output component depends essentially on every complete input component. Iterating F therefore gives a uniquely determined forward trajectory for every initial state.

Proof of existence. The restricted Cantor code is unique, and π assigns exactly one source pair to each output digit. Each XOR has a unique binary value. The sum encoding an output component converges absolutely, since its terms are bounded by $2/3^j$. It gives a member of C. Performing this definition for every i gives exactly one member of C^N, to which the same rule can be applied again.

Proof of all-to-all dependence. Fix a receiver i and a source k distinct from i. Changing only $a(k, 1)$ changes output digit $\pi(k, 1)$ for receiver i. If $k = i$, change only $a(i, 1)$; the digit at $\pi(i, 1)$ changes because its other input $a(i, 2)$ remains fixed. The uniqueness of the Cantor code means that each changed output digit changes the represented component. Thus no input component can be removed from the full update of any receiver.

Proof of continuity. A finite collection of output digits uses a finite collection of input digits. Specifying those input digits therefore fixes the chosen output digits. Finite digit specifications form a basis of neighborhoods for the Cantor set and its countable product. The inverse image of each such neighborhood is consequently open. Equivalently, to determine finitely many output components to a requested accuracy, sufficiently many input digits suffice.

Here all-to-all means essential dependence of each complete component on every input component. It does not mean that every individual output digit has infinitely many inputs, or that all sources contribute at the same resolution. The pairing places information from successively larger indices into finer output digits. This is one explicit form of universal interaction with a controlled finite approximation.

A Finite Calculation Inside the Infinite System

Let component 1 have a(1, 1) = 1 and every other digit be zero. All other components are zero. The receiver term a(i, j + 1) then vanishes for every i and j. Each output has exactly its first digit equal to one, because π(1, 1) = 1. After one update, every component is therefore 2/3. The input changed at one component and the effect appears in every component.

At the next update, a(k, 1) = 1 for every k while all later digits are zero. The nonzero output digits occur at positions π(k, 1) = k(k + 1)/2: positions 1, 3, 6, 10, and so on. Every component now has value Σ_k 2/3^[k(k + 1)/2]. The sum is again an element of C. This computation shows both collective feedback and the continued validity of the fractal state space.

A truncation has an exact one-step guarantee. Suppose the first m components are known through digit m + 1. For any receiver i ≤ m,

the first $L = m(m + 1)/2$ output digits involve only the first m diagonals of the pairing. All required source and receiver digits are then available. Proposition C bounds the remaining coordinate error by 3^{-L}, if the unknown output tail is filled with zeros.

Known Components m	Exact Output Digits L	Coordinate Error Bound
1	1	1/3
2	3	1/27
3	6	1/729
4	10	1/59049

The guarantee applies to receivers among the known components. To approximate several updates, trace the finite dependencies of the requested output digits backward through each update. A finite set has finitely many immediate digit inputs; after a fixed finite number of updates its required input set is still finite. This gives a finite procedure for any specified finite output question, although the amount of required information can grow.

The example now combines infinitely many components, infinitely many variables within each component, a fractal component state space, iteration, and direct all-to-all dependence. It establishes the compatibility of those mathematical requirements. It does not assign a spatial location, energy, duration, or measured coupling strength to the indices. Chapter 23 takes up the additional task of recovering physical quantities and laws.

Scale, Geometry, and Physical Interpretation

A fractal dimension is not automatically the dimension of physical space. The Cantor set above has a specified embedding in a line. A space of possible states can have a different dimension from the space in which a physical system is located. A fractal subset of a state space can organize trajectories without making a laboratory spatially fractal in the same sense.

The distinction matters for STOEM's cosmology. One proposal might assign fractal structure to distributions of matter over a range of scales. Another might assign it to an underlying state space. A third might concern patterns in temporal correlations. These proposals ask different questions and could have different physical consequences.

Exact self-similarity at arbitrarily small scales is a stronger claim than approximate scaling over a finite range. The mathematical examples establish the exact property for their stipulated sets. A claim about nature must identify its scale range and how the relevant pattern is inferred. The precision of the mathematical ideal supplies a comparison standard; it does not by itself extend an observed pattern to every scale.

The Role of Fractals in the Wider Metaphysics

Fractals demonstrate how a compact rule can organize distinctions across indefinitely many scales. The integrated map shows that such component spaces can participate in a single all-to-all dynamics. They can also display a difference between a concise generating description and a long list of generated details. Those are useful resources for thinking about intelligibility, compression, and the relation between a whole and its parts.

They do not decide why an instantiated universe exists, whether all physical laws arise from one iteration, or what makes a person capable of relationship with God. These are additional questions. Their continuing presence gives metaphysics work to do even after the fractal construction has been made exact.

The resulting position preserves the original ambition while clarifying its structure. STOEM can investigate a universe with fractal organization. Mathematical metaphysics asks which form of organization is being proposed, how its consequences follow, and how it becomes a description of a physical and personal world.

CHAPTER 10

A Mathematical Architecture of Description

The Ingredients

Let C be a finite, nonempty set of contexts. For each context i, let S_i be a nonempty set of locally admissible descriptions. Let E specify the pairs of contexts to be compared. For each (i, j) in E, let R_{ij} be a subset of $S_i \times S_j$. A pair belongs to R_{ij} when it satisfies the stated comparison between those contexts.

A complete assignment s selects one description s_i from each S_i. The set of complete assignments is the product $\prod_i S_i$. Let P be a finite collection of predicates on this product. Each predicate states an additional requirement that a complete assignment may satisfy or fail.

Write A = (C, S, E, R) for the descriptive structure. The admissible joint assignments are:

$$\Gamma(A; P) = \{ s \text{ in } \prod_i S_i : \text{all } R_{ij} \text{ and all } p \text{ in } P \text{ hold} \}.$$

The phrase all R_{ij} hold means that (s_i, s_j) belongs to R_{ij} for each specified comparison. The definition is entirely mathematical. It states what belongs to a set under selected conditions.

Symbol	Role
C	Contexts included in the account.
S_i	Descriptions available in context i.
E	Comparisons required between contexts.
R_{ij}	Allowed pairs under a comparison.
P	Additional requirements on joint assignments.
Γ(A; P)	Assignments satisfying all requirements.
J	The accompanying interpretation of these ingredients.

What Nonemptiness Establishes

If Γ(A; P) is nonempty, at least one assignment meets the requirements. If it is empty, no assignment meets them. Neither conclusion determines whether the intended subject exists. A nonexistent assignment can reveal a bad model, inconsistent premises, or an inappropriate demand for a particular kind of joint description.

The difference is especially important in quantum applications. A requirement that every potential measurement have a simultaneous context-independent value can be stronger than what the physical account warrants. An obstruction to that requirement does not establish that the experimental world lacks coherence.

Abramsky and Brandenburger provide an established mathematical treatment of contextuality through obstructions to global sections. Their work motivates care about which global object is being sought. The elementary construction here is a general constraint model, not a

replacement for their quantum framework and not itself a proof of contextuality.

The question addressed by Γ is exact but limited: are the selected descriptions jointly admissible under these comparison rules? Its usefulness depends on explaining why that question matters to the application.

Interpretation J

The letter J names the account connecting the formal ingredients with their subject. It explains what counts as a context, what the local descriptions refer to, and why the relations and predicates are relevant. It also specifies the scope in which a satisfying assignment would be informative.

J is not assumed to be completely captured by another formal object. Some parts can certainly be formalized. Units, measurement procedures, and reference tables can be described mathematically. The purpose of naming J is to prevent the semantic work from disappearing merely because the notation is precise.

A model of transportation can use an edge to mean that a road exists. A model of testimony can use an edge to mean that one record corroborates another. Both may be graphs. The shared graphical form does not turn roads into testimony or corroboration into travel. J distinguishes the applications.

This point connects with Newman's objection to a purely structural description of the external world. If the relevant relations are left unrestricted, a claim that some relations realize a structure can be much less informative than it appears. The application must identify which relations are meant and why they are the relevant ones.

Questions That a Description Must Preserve

Admissibility alone does not explain what a change of representation retains. We therefore add a family of questions. Let X be a set of descriptions under consideration, often Γ(A; P). A question q is represented by a function from X to a set Vq of answers.

A question might ask for a predicted probability, the color assigned to an object, or an identity label under a declared convention. Let K be the selected family of such questions. The interpretation explains why these questions matter and what their answers mean.

Now let f map X into a new set Y of representations. We say that f preserves q exactly when an answer function q* can be defined on f(X) such that:

$$q = q^* \circ f.$$

The equation says that the answer obtainable from x can be recovered from its new representation f(x). We call f faithful for K when it preserves every question in K.

This is a deliberately limited sense of faithfulness. A transformation can preserve every question in K while losing answers to questions outside K. Enlarging K can therefore change whether the same transformation counts as faithful.

A Small Example

Return to four cards a, b, c, and d. Let q ask for color, with a and b red and c and d blue. Let f retain only the color class. Then q can be recovered from f(x), so f preserves q.

Let r ask for shape: a and c are circles; b and d are squares. Since f(a) = f(b) while r(a) ≠ r(b), color cannot recover shape. Thus f preserves {q} but not {q, r}. The next chapter proves the general criterion.

The state space X can be instantiated by either iterative construction. A question can retain a coordinate, a weighted mean, or a finite prefix of a Cantor code. The finite context set records the comparisons being made, while the underlying state can remain infinite. This connects the formal architecture directly to finite access within STOEM.

CHAPTER 11

Preservation, Compression, and Loss

Result 1: Relabeling Preserves Admissibility

Suppose each local set S_i is replaced by a set S'_i through a bijection f_i. Retain the same contexts and comparison pairs. Transport the relations so that $(f_i(x), f_j(y))$ belongs to R'_{ij} exactly when (x, y) belongs to R_{ij}. Transport each predicate in P in the same way.

Let F take a complete assignment (s_i) to $(f_i(s_i))$. Then F is a bijection between $\Gamma(A; P)$ and $\Gamma(A'; P')$.

Proof. Every local bijection has an inverse. Their product therefore gives an invertible transformation of complete assignments. Each transported relation and predicate has the same truth value on an image assignment as the original condition had on its preimage. An assignment is admissible on one side exactly when its image is admissible on the other. Restricting F to the admissible sets gives the required bijection.

This result identifies a change that leaves the structure of the admissibility problem intact. If a purported relabeling changes whether an assignment is admissible, then some relation or requirement was changed rather than faithfully transported.

The metaphysical application remains conditional. The result does not decide which physical transformations should count as mere relabelings. It supplies a standard once the relevant correspondence has been justified.

Result 2: Additional Requirements Restrict Possibility

Let Q be another finite collection of predicates on the same complete assignments. Then:

$$\Gamma(A; P \cup Q) \subseteq \Gamma(A; P).$$

Proof. Any assignment satisfying all predicates in P and Q satisfies all predicates in P. Its comparison relations are unchanged. Hence it belongs to Γ(A; P).

The inclusion can be equality, a proper inclusion, or a reduction to the empty set. A condition may already follow from the existing requirements, may exclude some assignments, or may contradict every assignment.

This simple result has a useful philosophical consequence. A stronger set of assumptions is not automatically a better account. It makes a more demanding claim. Whether the additional demand is warranted must be argued in relation to the subject.

Result 3: Exact Preservation Has a Fiber Criterion

Let f map X into Y, and let q map X into an answer set V. There is a function q* on f(X) satisfying $q = q^* \circ f$ if and only if q is constant on every fiber of f. Equivalently:

$$f(x) = f(x') \text{ implies } q(x) = q(x').$$

Proof in the forward direction. If $q = q^* \circ f$ and $f(x) = f(x')$, then $q(x) = q^*(f(x)) = q^*(f(x')) = q(x')$.

Proof in the reverse direction. For y in f(X), define q*(y) to be q(x) for any x with f(x) = y. Such an x exists because y belongs to f(X). The fiber condition ensures that every eligible x gives the same answer, so the definition is unambiguous. It follows that $q^*(f(x)) = q(x)$.

The restriction to f(X) matters. The theorem requires recovery on representations that actually arise from X. It need not assign answers to unused elements of Y.

This result makes information loss visible. When f combines two descriptions to which q assigns different answers, the answer to q cannot be recovered from f alone. A later calculation cannot reconstruct that distinction without additional information.

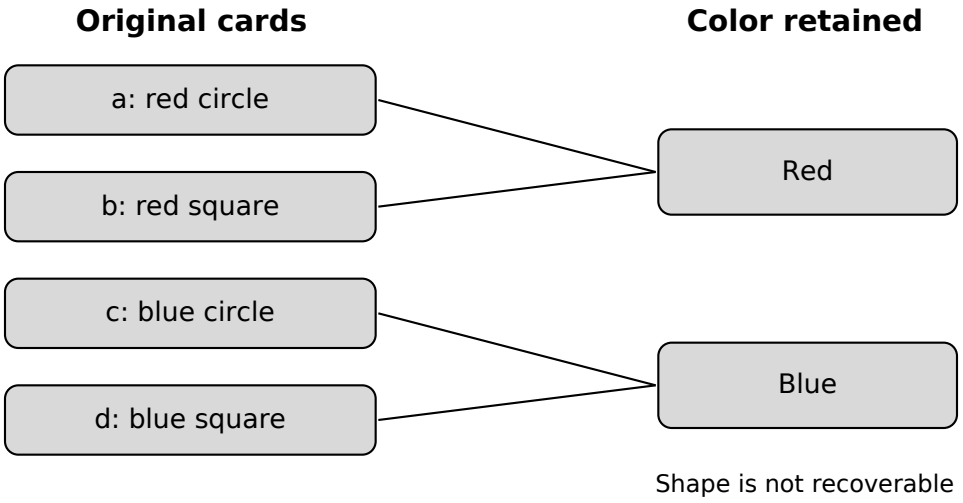


Figure 2. A color-only representation combines cards with different shapes. It preserves color and loses the shape question.

Result 4: The Canonical Quotient for a Family of Questions

Given a family K of questions on X, define x ~K x′ when q(x) = q(x′) for every q in K. Then ~K is an equivalence relation. The projection π from X to X/~K preserves every question in K.

Proof. Each equality of answers is reflexive, symmetric, and transitive. Requiring all those equalities retains the three properties. Every q is constant on each equivalence class by definition. Result 3 therefore supplies the answer function on the quotient.

The quotient removes exactly the distinctions that K does not detect. It is maximally coarse among representations preserving K, in the

following precise sense. If f preserves K and f(x) = f(x′), then every q in K gives equal answers on x and x′. Thus π(x) = π(x′), and there is a well-defined function h on f(X) with π = h ∘ f.

Every faithful representation can consequently be mapped to the canonical quotient. The theorem does not establish that the quotient is the subject itself. It identifies the information sufficient for the specified family of questions.

If K contains a question that uniquely identifies every element of X, the quotient has singleton classes. If K is empty and X is nonempty, all elements belong to one class. The meaning and importance of the representation depend on the questions selected.

Result 5: Preservation Composes

Suppose f maps X to Y and preserves a question q through q* on f(X). Suppose g maps f(X) to Z and preserves q* through q** on g(f(X)). Then g ∘ f preserves q.

Proof. The assumptions give q = q* ∘ f and q* = q** ∘ g. Substitution yields q = q** ∘ g ∘ f. The same argument applies to every question in a family.

This result permits a sequence of transformations to be assessed step by step. Its assumptions also identify the limitation. The second transformation must preserve the answer function induced by the first. A generic claim that both steps preserve something is insufficient if they preserve different things.

A useful contraposition follows from Result 3. If f already combines two descriptions that q distinguishes, no further function of f alone can recover q. The obstruction is not a shortage of cleverness in the later function. The later function receives the same input in both cases.

What These Results Accomplish

The results are elementary consequences of sets, functions, and stated predicates. They are not offered as new discoveries in abstract mathematics. Their purpose is to make several philosophical responsibilities explicit: identify what is preserved, specify the questions that define adequacy, and distinguish a change of notation from a change in content.

The constructions also expose a limitation of the method. K is supplied through an interpretation. Mathematics can reveal the consequences of selecting K, but it cannot establish the relevance of K merely by calculating the resulting quotient. The selection of meaningful questions remains part of the metaphysical and practical account.

CHAPTER 12

From Local Descriptions to Joint Accounts

Why Arrangement Matters

Knowing that every local state set is nonempty does not guarantee that a joint assignment exists. The relationships among the states impose further conditions. A description can be admissible by itself and incompatible with a neighboring description under the specified comparison.

Even compatibility across each individual edge can fail to produce compatibility across the whole network. The arrangement of comparisons matters. A cycle can bring a chain of requirements back to its starting point and create an obstruction that no isolated comparison reveals.

This is a mathematical fact about constraint systems. Its philosophical significance depends on what the constraints represent. It can describe incompatible records, inconsistent demands, or a failed proposal for combining contexts. The same formal pattern does not establish the same cause in every application.

Result 6: Extendability on a Finite Tree

Suppose the comparison graph is a finite tree. Choose a root and orient every edge from parent to child. If an original relation was written in the opposite direction, use its converse when orienting the edge. Assume that every state at a parent has at least one compatible state at each child. Assume there are no additional predicates in P.

Then Γ(A; P) is nonempty.

Proof. Choose any state at the root, which is possible because the root's state set is nonempty. For each child, choose a compatible state using the extendability assumption. Repeat at the next level. The tree is finite, so the process terminates. Each non-root vertex has exactly one parent, and no extra edge imposes a competing condition on a previously assigned vertex. Every required comparison is satisfied.

The hypotheses are important. Nonempty edge relations alone are weaker than the condition that every parent state extends. Additional global predicates can also destroy satisfiability. The result applies to the specified tree problem, not to every collection of locally successful accounts.

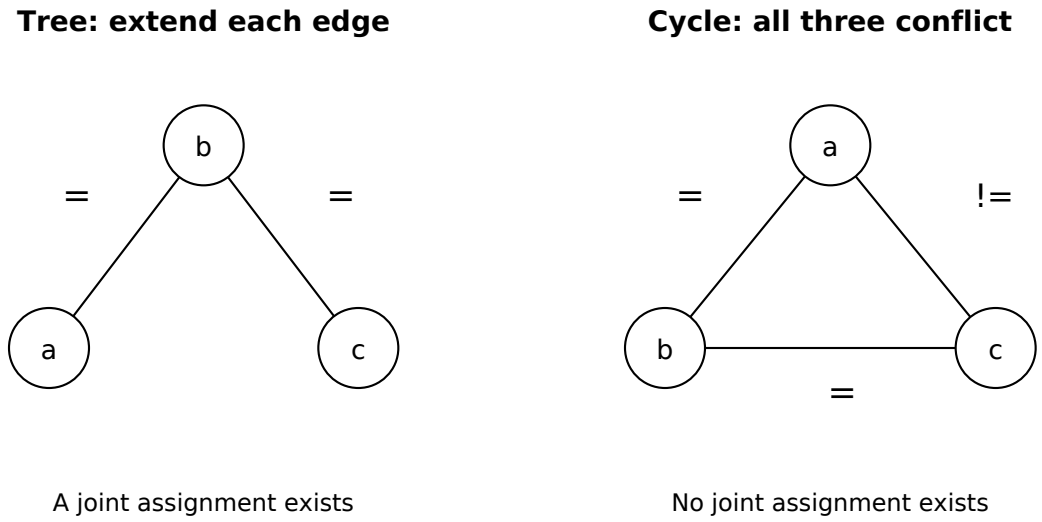


Figure 3. The finite-tree result depends on its assumptions. Adding a cycle can impose an incompatible comparison.

Result 7: A Cycle Can Obstruct Joint Admissibility

Let a, b, and c each take a value in {0, 1}. Require:

$$a = b,\ b = c,\ c \neq a.$$

Each edge permits every value at one end to extend to a value at the other. Any two requirements can be satisfied together. All three cannot.

Proof. The first two equalities imply a = c. The final requirement denies that equality. Hence no complete assignment satisfies the three requirements.

The eight assignments make the situation explicit. In the following table, 1 means that the column's requirement is satisfied and 0 means it is not.

a b c	a = b	b = c	c ≠ a
0 0 0	1	1	0
0 0 1	1	0	1
0 1 0	0	0	0
0 1 1	0	1	1
1 0 0	0	1	1
1 0 1	0	0	0
1 1 0	1	0	1
1 1 1	1	1	0

No row contains three ones. This finite obstruction is fully describable. The absence of a satisfying assignment does not make the network itself unintelligible.

Diagnosing an Obstruction

An empty admissible set does not identify which requirement should be revised. In the example, removing any one of the three conditions permits assignments. The mathematics establishes their joint incompatibility and the sufficiency of each single removal. It does not determine which condition was incorrectly imposed on an actual subject.

That determination requires evidence and interpretation. If the constraints represent witness reports, a contradiction may reflect an error, a difference in reference, or a difference in the events described. If they represent design requirements, the conflict may call for revising an objective. If they represent a quantum model, the form of the joint assignment may itself be the inappropriate assumption.

There is therefore a distinction between identifying an obstruction and explaining it. The first is often a mathematical achievement. The second requires an account of what generated the requirements and which ones remain justified.

Reconciliation and Its Price

A reconciliation can be achieved by changing a relation, weakening a requirement, or using a different representation. These moves should not be treated as equivalent. Changing the wording while transporting every condition faithfully leaves the obstruction intact by Result 1. Altering a requirement can remove the obstruction because it changes the problem.

This does not make revision illegitimate. It makes the price of revision visible. A useful reconciliation should state which claim was revised and why. Otherwise an account may appear to solve a conflict only because it has stopped preserving what made the conflict meaningful.

The same principle applies to agreement among perspectives. A common vocabulary can be obtained by retaining only what all perspectives share. That vocabulary may be useful, but it can also be too coarse to represent their important differences. The preservation framework asks which questions survive the reconciliation.

A common world can consequently sustain both successful comparisons and principled limits on combination. Unity consists in an intelligible account of those relationships. It does not require that every initially proposed set of constraints have a satisfying assignment.

Physical Wholeness and a Joint Description

STOEM's proposed whole should therefore not be identified with Γ for every chosen family of contexts. The universe can contain experiments whose outcomes fail a proposed joint-value constraint. What fails is that constraint under its interpretation; the experimental events still belong to the physical account.

An iterative model that aims to reproduce such experiments has a specific obligation. It must explain how preparations, settings, interactions, and recorded outcomes arise from its states without imposing the very joint assignments that the physical comparison excludes. Calling its variables universal does not discharge that obligation.

CHAPTER 13

Iteration, Change, and Physical Time

What an Index Supplies

The early STOEM papers proposed iteration as an alternative starting point for time. This proposal deserves to be separated into questions. Does iteration represent the order in which states occur? Does it define an underlying physical duration? Does it describe a computational procedure used by an investigator? The same sequence notation can appear in all three settings while carrying different meanings.

For an update $x_{n+1} = T(x_n)$, the integer n labels successive applications. The formula supplies an order and a next-state relation. It does not give each application a duration in seconds. A simulation may take an hour of laboratory time to represent one microsecond, or may evaluate a whole history without following the represented order of events.

STOEM's metaphysical proposal is that physical change can be understood through relationships of succession more basic than a clock reading. The clocks belong within the changing universe. This is a proposal about dependence: temporal measurement depends on physical processes whose ordered differences can function as records. It does not require that the universe be executed by a machine outside itself.

A Clock Is a Participating Process

Suppose a portion of a state describes a repeatable physical process together with a record of its cycles. A clock reading can then be represented by a function C on the relevant states, provided the physical interpretation explains how those values are recorded and compared. The function becomes a clock through that interpretation.

A periodic coordinate alone can be ambiguous. An oscillator returns to the same position many times. Reading its position without its direction of motion or cycle count does not determine how many periods have elapsed. A functioning clock preserves additional distinctions through a counter, a history, or another stable record.

One possible mathematical model assigns a positive duration $\delta(x)$ to each transition from state x. Along a trajectory, define:

$$\tau_n = \Sigma_{k=0}^{n-1} \delta(x_k),\ \tau_0 = 0.$$

The formula makes the cumulative clock parameter strictly increasing. Its physical use still depends on identifying the clock and the calibration that supplies δ. Different choices can give different elapsed durations for the same abstract sequence. Order alone leaves the metric of time unspecified.

If δ is a constant Δ, this reduces to $\tau_n = n\Delta$. That is an additional assumption. It may be convenient for a toy model, but it is not a consequence of writing down an iteration. A universal fixed duration would require a physical argument and an account of how internal observers recover it.

Why Planck's Constant Does Not Supply a Tick

The early Seed associated Planck's constant h with the transition from iterations to physical time. That association identifies a question worth developing, but h alone does not have units of duration. It has units of action, equivalently energy multiplied by time.

An expression h/E has units of time when E is a nonzero energy. Dimensional consistency permits such an expression but does not establish that it is the duration of a universal iteration. The choice of energy, the relevant dynamics, and the clock interpretation still require justification. A time scale in one physical process need not be a fundamental tick shared by all processes.

The same distinction applies to quantum uncertainty. Inserting h into an equation does not automatically turn a deterministic update into a stochastic law or a classical probability distribution into a quantum state. The mathematical role of the constant must be specified. The later quantum chapters show why amplitudes, probabilities, and measurement contexts cannot be interchanged by vocabulary alone.

This clarification strengthens the proposal by locating the missing bridge. STOEM needs an account of how a physically meaningful time parameter is recovered from its underlying description. A constant can participate in that bridge after the participating quantities and laws have been identified.

One Order or Many Compatible Orders?

A universal update number appears to place every component at the same iteration. Interpreted physically, that could amount to an underlying preferred ordering. Such a commitment is more substantial than a convenient order in which a calculation is performed. It must be examined in relation to relativistic physical descriptions.

There is another possibility. A system can specify which events depend on which others without declaring every pair simultaneous or temporally ordered. Different lists of updates can represent the same partial order. If two independent operations commute, exchanging their order leaves the final state unchanged. In that circumstance, a sequence used for calculation contains an ordering distinction that the result does not retain.

For a finite example, let A change the first coordinate of a pair and B change its second coordinate, with each operation depending only on the coordinate it changes. Then applying A followed by B has the same result as B followed by A. If A instead reads the second coordinate, commutation can fail. The dependency structure determines whether changing the schedule preserves the account.

This example does not derive relativistic spacetime. It identifies a design question for STOEM: which update orderings are part of the physical content, and which are removable features of its description? A global index, a partial order, and a spacetime history are different possible representations requiring different recovery arguments.

Relativity and the Recovery of Duration

Physical clocks moving along different worldlines can accumulate different proper times between shared encounters. The spacetime chapter gives a concrete Lorentz example. Any proposed underlying iteration must accommodate those physical comparisons if it is intended to recover relativistic phenomena.

It could do so by deriving different clock functions from the same underlying history, with each function reflecting the process and path of a particular clock. That is a possible mathematical strategy, not a completed derivation. Another strategy could make local dependencies more fundamental than a common update label. The

metaphysical preference for meaningful succession does not select the strategy by itself.

The distinction also bears on locality. An all-to-all coordinate update need not describe an instantaneous physical signal unless coordinates and interventions have been assigned a physical meaning. Once such a meaning is assigned, however, the model becomes responsible for the signalling relations it entails. The absence of a physical interpretation initially cannot be used to evade the consequences of a later one.

Becoming, Records, and the Open Future

A sequence of states can be read as a history of genuine becoming or as a representation of relations within a completed history. The notation alone does not settle that metaphysical disagreement. The two interpretations differ about what exists and what is meant by the future's openness.

Likewise, a deterministic update can be unknown to an observer without being indeterministic. A stochastic transition rule can specify probabilities without explaining agency. A many-to-one update can erase distinctions without establishing the thermodynamic arrow of time. Each inference requires its own account.

STOEM brings these questions into one setting because it includes the observers, their records, and the physical processes on which those records depend. Experienced succession is then part of the reality to be understood. Its presence cannot be established merely by adding a coordinate named consciousness to the state vector.

The constructive task is to relate ordered change, internal clocks, durable records, and personal experience. Success at one connection can guide the others while leaving their differences visible. A physical record of a clock reading, for example, can help explain the evidence

available to an observer without supplying a complete account of what experiencing time is like.

The Claim Adopted Here

The book adopts iteration as a productive representation of the ordered dependence of states and explores its possible role in a deeper account of physical change. It leaves the exact relation between that order and spacetime dynamics to a physical realization. This is a definite position with a definite remaining task.

Time is consequently neither eliminated nor fully explained by replacing t with n. The replacement changes what the description initially supplies. A successful mathematical metaphysics must then recover duration, temporal comparison, and the position of the experiencing person within the account. The physical chapters develop the requirements such a recovery must meet.

CHAPTER 14

Physical Being and the Scope of Physical Law

Physics Within Metaphysics

Metaphysics, as understood in this book, investigates being and its conditions of intelligibility. Physics investigates physical being: material systems, fields, spacetime, interactions, and the regularities of their change. In that order of inquiry, physics is a domain within metaphysics. It supplies a sustained investigation of a particular subject within the wider question of what exists and how it can be understood.

This inclusion gives physics a central place. The physical world contains laboratories, clocks, organisms, and the material circumstances of inquiry. A metaphysics that speaks extensively about descriptions while scarcely examining motion, energy, matter, or physical history leaves its own embodied inquirer insufficiently understood.

The inclusion does not settle whether physical being exhausts all being. That is the additional thesis associated with physicalism. Nor does it require that every question about physical being be answered by the same method. The evidence supporting an electromagnetic prediction differs from the argument supporting a claim about the ultimate ground of physical law. Both concern reality, but they ask different questions.

Mathematics enters throughout this hierarchy. It articulates structures in general metaphysics and in particular physical theories. The hierarchy therefore should not be pictured as though mathematics were a narrow subject located entirely inside physics.

Mathematical methods can express physical relations, logical conditions, biological assignments, and proposed relations of dependence. Their applications receive their subjects and meanings through interpretation.

What Makes a Description Physical?

A differential equation becomes part of a physical account when its variables, parameters, and solutions are connected to physical systems. That connection includes more than assigning familiar nouns to symbols. It concerns how quantities are measured, which preparations are possible, what counts as an interaction, and where the description is expected to apply.

Consider a mathematical oscillator. Its state can be represented by a displacement and a velocity. In one application, the displacement concerns a mass attached to a spring. In another, an analogous variable concerns an electrical circuit. The shared equation can reveal a common dynamical structure without establishing that the spring and the circuit are numerically identical objects.

The interpretation J must therefore specify the system and its relevant relations. The requirements P can include boundary conditions, constitutive assumptions, and the dynamical law. The question family K identifies the physical answers to be retained: an oscillation frequency, a measured voltage, or the response to a disturbance. This connects the earlier formal architecture directly to experimental physics.

A successful application may preserve only some of those answers. An approximation adequate for frequency can be inadequate for heating or material failure. The qualification is part of the physical content of the model. It tells the investigator which questions its success warrants answering.

Laws, States, and Conditions

A physical law and a physical state have different roles. A law constrains admissible behavior. A state specifies a condition within the space of behavior admitted by the law. Initial and boundary data select or restrict solutions; they are not usually supplied by the differential equation alone.

Even an exactly deterministic model generally requires suitable data before it yields a particular history. Conversely, a probabilistic model can specify precise distributions without selecting a single realized outcome. Determinism and precision are different properties. So are the existence of a mathematical solution and its physical realization.

Element of an Account	Physical Role
Dynamical law	Restricts how states or histories may be related.
State or preparation	Specifies the condition being investigated.
Boundary and initial data	Select or constrain solutions in a stated problem.
Measurement rule	Connects the account to possible records.
Domain of validity	States the conditions under which the account is intended to apply.

The table also prevents a metaphysical shortcut. A law that explains how a system evolves does not automatically explain why that system exists or why that law applies. These further questions remain available even when the physical calculation is complete for its intended purpose.

What Kind of Authority Does a Law Have?

Several metaphysical readings of law remain possible. A law might summarize the most systematic regularities of events. It might express powers or dispositions belonging to physical entities. It might constrain which histories can occur. These readings can share equations while differing about what makes those equations authoritative.

The disagreement matters when one asks about unobserved cases. A recorded sequence of regular events is compatible with many continuations. A claim of law reaches beyond the record by licensing expectations or counterfactual conclusions. The metaphysical question concerns what warrants that reach and what kind of necessity it expresses.

The present book does not resolve this dispute by defining law as whatever makes a model work. That would conceal the issue. It requires an account to distinguish the empirical support for a proposed regularity from its interpretation as a feature of physical being.

This is a place where physics disciplines metaphysics from within. If an account claims that a particular kind of interaction is impossible, reproducible evidence for that interaction challenges the account. The generality of metaphysics does not exempt its claims about physical beings from the evidence those beings supply.

Explanation Across Physical Scales

A physical system can support several descriptions. A solid may be studied through elastic deformation, atomic arrangements, or electronic behavior. Each description makes different distinctions available. Their coexistence is informative when the account explains how they are connected and what is preserved between them.

Reduction asks how one description can be obtained from another under stated conditions. Emergence asks how organized behavior becomes intelligible at a level where its useful variables may differ from those of its constituents. Neither word by itself determines whether the resulting properties are ontologically independent, approximately autonomous, or merely convenient labels.

In this book, the first demand is a comparison with an explicit purpose. Which quantities must agree? Over what range? Which details may be discarded? An appeal to a more microscopic account is incomplete until it explains how the macroscopic questions are recovered. A successful macroscopic account is incomplete if it claims universal authority while ignoring the conditions of its own construction.

Physics thus becomes a major testing ground for mathematical metaphysics. It supplies cases in which representation succeeds, fails, changes scale, and encounters limits. The next chapters investigate those cases through symmetry, matter, spacetime, thermodynamics, quantum measurement, and cosmology.

The physical chapters develop the quantities a STOEM realization would need to recover: conservation laws, fields, clock comparisons, records, quantum probabilities, and cosmic histories. Chapter 23 joins these tasks through a recovery equation: describing an updated underlying state should agree with updating its retained physical description.

CHAPTER 15

Symmetry, Energy, and Conservation

A Transformation That Leaves Something Unchanged

Symmetry concerns a transformation and an invariant. A claim that a system is symmetric is incomplete until it specifies both. Rotating a geometrical figure, translating an experimental arrangement, and changing the coordinates used to describe it are different operations. They can preserve different things.

A change of notation can leave the represented situation unchanged. A physical transformation can instead compare distinct situations governed by the same law. Moving an isolated apparatus to another location is not simply renaming its position. The claim of spatial symmetry concerns what remains the same in the physical account when that displacement is made under the stated conditions.

This distinction connects directly with preservation. The family K may contain distances, transition probabilities, or conserved quantities. A transformation is faithful for K if those answers are recoverable. A symmetry makes a stronger, more particular claim about how the structure or law is carried into itself.

Symmetry also distinguishes a law from a state. A rotationally symmetric law need not produce a rotationally symmetric arrangement in every solution. An individual orbit can have an orientation. The absence of symmetry in a chosen state therefore does not immediately show that the governing law lacks it.

Noether's Connection

Noether's first theorem connects continuous variational symmetries with conservation laws. Its application requires a specified action, suitable differentiability, and the relevant equations of motion. Time translation, spatial translation, and rotation lead in familiar mechanical settings to conservation of energy, momentum, and angular momentum.

The result is conditional and physically powerful. Once an action and its interpretation are accepted, the symmetry restricts possible behavior. A metaphysical preference cannot preserve all the assumptions while canceling the consequence. An apparent violation instead calls for examining the system boundary, the symmetry, the dynamics, or the evidence.

An action principle assigns a mathematical quantity to a candidate history. The physical trajectory is stationary under the allowed variations. Stationary does not always mean a global minimum, and the mathematical comparison of histories does not by itself establish that a physical system deliberates about future alternatives.

A Worked Oscillator

Consider an ideal undamped spring system with positive mass m and stiffness k. Let x(t) be displacement from equilibrium and let v(t) = dx/dt. The dynamical assumption is:

$$m\,d^2x/dt^2 = -k\,x.$$

Define its mechanical energy by:

$$E = m\,v^2/2 + k\,x^2/2.$$

Differentiate along a solution. The ordinary product and chain rules give:

$$dE/dt = m\,v\,dv/dt + k\,x\,v.$$

Substituting $m\,dv/dt = -k\,x$ makes the two terms cancel. Thus $dE/dt = 0$. The calculation establishes conservation for the stated ideal dynamics. It does not assume that every object called a spring satisfies those dynamics under every condition.

For a particularly transparent example, choose units in which $m = k = 1$ and take $x(t) = \cos t$, so $v(t) = -\sin t$. Then:

$$E = (\sin^2 t + \cos^2 t)/2 = 1/2.$$

At $t = 0$, the displacement is 1 and the velocity is 0. At $t = \pi/2$, the displacement is 0 and the velocity is -1. The state has changed while the total energy has not. Conservation therefore does not mean that nothing happens. It identifies a quantity that remains invariant through a specified class of changes.

This is a concrete example of the distinction between identity and preservation. Retaining only E combines many different phase-space states. It preserves the energy question and loses the separate position and velocity questions. Two states can have the same energy while supporting different predictions about what the system will do next.

Open Systems and Apparent Loss

Now introduce a positive damping coefficient b and replace the equation by:

$$m\,d^2x/dt^2 = -k\,x - b\,v.$$

The same calculation yields:

$$dE/dt = -b\,v^2 \leq 0.$$

The modeled mechanical energy decreases except where v is zero. This result does not by itself show that energy has disappeared from a larger physical world. The damping term represents a coupling whose other degrees of freedom have been omitted. A fuller account must investigate heating, radiation, or other transfers as appropriate to the mechanism.

The distinction is ontological as well as numerical. What has been designated the system? Which interactions cross its boundary? Does E describe every relevant form of energy or only the mechanical contribution? A disagreement about conservation can originate in an unnoticed change to one of those questions.

The undamped and damped equations are also different requirements P. Their consequences cannot be mixed freely. Keeping the damping term while insisting that the same mechanical energy remain constant would generally impose incompatible conditions. The original metaphysical framework helps identify the source of that conflict.

Energy Without a Universal Substance

Energy is a physical quantity with precise roles in physical theories. Treating it as a universal material from which everything is made adds an ontological assertion that the conservation calculation does not establish. The oscillator has energy; the equation does not identify energy with the oscillator's entire being.

Relativity further requires care about scope. In a general curved spacetime, a globally conserved total energy need not be available in the same form as it is for an isolated mechanical system with time translation symmetry. Local stress-energy conservation is a covariant statement. Additional symmetries or suitable boundary conditions can support global conserved quantities.

Consequently, a cosmological argument cannot simply borrow a laboratory energy formula and assume that every feature survives unchanged at the scale of the whole universe. The relevant invariant, the geometry, and the conditions of comparison must be supplied.

The metaphysical significance of conservation is nevertheless substantial. It exhibits lawful persistence through change and connects that persistence to a specified invariance. It shows how the same world can sustain difference in state and sameness in a selected quantity. Mathematics makes that relationship exact; physics establishes where it applies; metaphysics examines the being and dependence expressed by the account.

Conservation in an Iterative Universe

For an iterative state map T, conservation of a quantity Q has the exact form Q(T(x)) = Q(x) for the states under consideration. Infinite feedback does not entail this identity. It must follow from the chosen map or be imposed as a requirement on it.

STOEM can therefore use conservation to discriminate among proposed dynamics. A candidate that changes a supposedly conserved quantity without accounting for an interaction or a change in scope has a definite explanatory problem. The finite averaging example developed later illustrates one conserved mean, while leaving the physical interpretation of that mean to be supplied.

CHAPTER 16

Matter, Fields, and Physical Interaction

From Bodies to Distributed Quantities

A familiar object has an apparent boundary. One can point to a stone, weigh it, and follow its motion. A field description introduces a different form of organization: quantities are assigned throughout a region of space or spacetime, with relations governing their variation and interaction.

The shift matters metaphysically because it changes the apparent units of the account. An interaction need not be represented solely as one bounded body directly pushing another. An electromagnetic field can be part of the physical description through which an interaction is transmitted and measured.

Yet a field should not be identified with an arbitrary list of numbers at locations. The type of quantity, its transformation rules, its equations, and its relation to measurement contribute to its physical meaning. A temperature field and an electric field are both distributed descriptions; they do not thereby have the same dimensions, dynamics, or role.

The question of being remains. Is a particular field treated as fundamental, as an effective description of collective behavior, or as one representation within a wider theory? The mathematical format alone does not select the answer. An account must connect the field to the physical distinctions it preserves.

Interaction as a Physical Relation

An interaction is more than the observation that two numerical records correlate. A physical theory specifies how systems can affect one another or jointly evolve, often through a coupling in its dynamical equations. Predictions depend on the coupling, the states, and the conditions under which the account is applied.

This gives relational metaphysics a substantive task. Naming a relation is only a beginning. The account must explain what bears the relation, what it permits, and how its physical consequences can be distinguished from those of another proposed relation.

A spring connecting two masses supplies a simple case. The restoring forces depend on their relative displacement. Removing the coupling changes the possible motions. Treating the masses as two independent oscillators can therefore lose collective behavior that the coupled description preserves.

The lesson extends to more advanced theories without requiring that every interaction resemble a spring. A shared mathematical operation does not make electromagnetic, gravitational, and biological relations interchangeable. Their physical interpretations and their domains of validity remain essential.

Gauge Freedom and Redundant Description

Classical electromagnetism provides a precise example of several descriptions representing the same fields. On a suitable smooth region, write the magnetic field B and electric field E using a vector potential A and scalar potential φ:

$$B = \nabla \times A, \quad E = -\nabla \varphi - \partial A/\partial t.$$

Here ∇ is the spatial differential operator, $\times$ denotes the vector cross product in the curl, and $\partial/\partial t$ is differentiation with respect to time. Let χ be a sufficiently smooth scalar function. Change the potentials by:

$$A' = A + \nabla\chi,\ \varphi' = \varphi - \partial\chi/\partial t.$$

The magnetic field is unchanged because the curl of a gradient vanishes. The electric field is unchanged because the added spatial and time derivative terms cancel. This calculation uses ordinary smooth derivatives that commute in the stated setting.

Thus, if K asks only for the resulting E and B fields, these changes preserve K. The potentials contain representational freedom relative to those questions. The conclusion concerns the displayed transformation and its stated domain; global topology, boundary conditions, and coupling to quantum matter require additional care.

This example gives content to the phrase same physical description. There is a transformation, a family of physical quantities, and a demonstration of invariance. Merely asserting that two verbal pictures are equivalent would not supply those elements.

It also reveals a limit of inference. Redundant potentials do not establish that electromagnetic phenomena are unreal. A failure of uniqueness in the representation is compatible with a definite physical subject. The metaphysical task is to identify the equivalence that the theory warrants, rather than demanding that each symbol name a separate observable entity.

Particles in a Field Account

Quantum field theory connects particle descriptions with quantum states of fields. In appropriate regimes, excitations can be organized as particles with characteristic properties and interactions. The field framework also treats creation and annihilation processes that exceed a fixed inventory of enduring classical particles.

The relation between a particle and a field is therefore part of the theory's ontology and interpretation. The vocabulary of little objects following paths cannot simply be transferred into every quantum-field calculation. Nor should a useful particle account be discarded merely because a broader theory gives it a qualified domain.

The Standard Model organizes known elementary matter and the electromagnetic, weak, and strong interactions. Gravity is not included in that model. Its predictive successes do not make it an established complete inventory of all physical being, and its limitations do not erase the evidence supporting its successful applications.

For the present book, this distinction is crucial. A metaphysics must be able to recognize a powerful physical account without converting its present scope into a decree about everything that can exist. It must also resist treating incompleteness as permission to attribute any desired property to the unknown remainder.

A Vacuum Is a Physical State

Within quantum field theory, a vacuum is a state specified relative to a physical framework. In familiar stationary settings it is associated with lowest energy. It retains field structure and can have nontrivial correlations. It is not simply the absence of every entity, relation, law, or condition.

This makes the phrase something from nothing ambiguous when the starting point is a quantum vacuum. A physical process beginning in such a state already assumes a theory and a space of possibilities. It may explain the appearance of excitations without answering why the framework itself exists.

The distinction is not a reason to dismiss vacuum physics. It places its explanations at the level they actually address. The metaphysical question of absolute absence differs from the physical question of which phenomena can arise from a specified state with no particles in a particular description.

Effective Reality and Emergence

Many physical descriptions are effective: their variables and laws are designed for a range of scales or conditions. A sound wave is intelligible through pressure and motion without requiring a separate trajectory for every constituent of the medium. The collective description has a subject because the organized medium behaves in ways it can capture.

The issue is not whether a coarser description omits details. It does. The issue is whether it preserves the questions needed for its purpose and whether the relation to the finer description can be justified. Approximate preservation may be the appropriate standard, with errors stated rather than hidden.

This gives emergence a disciplined place in the book. A higher-level pattern can be physically real and explanatory even when it depends on underlying constituents. Dependence does not make the pattern an illusion. It does require an account of the conditions that sustain it and the circumstances under which its description breaks down.

For STOEM, a useful next step is to assign field quantities to classes of underlying states. Gauge-related states must give the same retained field answers. This makes a proposed fractal pattern physically significant only insofar as the pattern survives the relevant physical equivalences; a pattern confined to redundant labels would describe the representation instead.

CHAPTER 17

Spacetime, Gravitation, and Physical Order

Events and Their Coordinates

A physical event and its coordinate label should be distinguished. Different coordinate systems can assign different numbers to the same event. A meaningful comparison must identify what the transformation preserves and how each coordinate system is tied to physical procedures.

Relativity makes this distinction especially visible. An observer's time coordinate is not automatically a universal time shared by all observers. Yet the failure of a universal coordinate does not eliminate measurable relations between events. It changes the structure within which those relations are expressed.

The philosophical temptation is to move too quickly from relativity of description to arbitrariness of reality. The mathematics supports a more definite conclusion. Some quantities vary with the frame, while others remain invariant under the specified transformations. The physical account explains which are which.

A Worked Lorentz Transformation

Consider one spatial dimension in flat spacetime. Measure time in seconds and distance in light-seconds, so the speed of light is represented by 1. The relative velocity parameter v is therefore a dimensionless fraction of the speed of light. For two events, let t and x denote their coordinate differences in one inertial frame.

For a frame moving at v, with $|v| < 1$, define:

$$\gamma = 1/\sqrt{(1 - v^2)}.$$

The Lorentz transformation of the differences is:

$$t' = \gamma(t - v\,x),\ x' = \gamma(x - v\,t).$$

Substituting and expanding gives:

$$t'^2 - x'^2 = \gamma^2(1 - v^2)(t^2 - x^2) = t^2 - x^2.$$

The cross terms cancel. Thus the interval $I = t^2 - x^2$ is invariant in these units and with this sign convention. It is a particular preserved relation between the events, not a claim that all their coordinate values remain unchanged.

Choose $v = 3/5$, so $\gamma = 5/4$. The following examples display the difference:

Separation	t, x	t′, x′	Interval I
Timelike	5, 3	4, 0	16
Lightlike	2, 2	1, 1	0
Spacelike	0, 5	−15/4, 25/4	−25

For the timelike example, the transformed frame places both events at the same spatial coordinate. A clock moving uniformly between them records an elapsed proper time of 4 seconds. The original frame assigns a coordinate time difference of 5 seconds to the same pair.

The spacelike example begins with simultaneous events in the first frame and a nonzero time difference in the second. Simultaneity for separated events is therefore not preserved by this transformation. Their spacelike separation is preserved. The change in temporal

ordering does not establish that a controllable influence has traveled backward in time between them.

What a Causal Structure Supplies

The sign of the interval distinguishes timelike, lightlike, and spacelike separations in this example. The distinction is part of the causal structure used by relativistic physics. It constrains which event pairs can be connected by ordinary material motion or light propagation under the adopted theory.

This structure should not be conflated with every philosophical use of cause. A causal explanation may also involve interventions, mechanisms, statistical dependencies, or boundary conditions. Whether a proposed explanation respects the physical causal structure is a substantive question, but the words need to be connected explicitly.

The distinction prepares the later discussion of Bell correlations. A correlation, a mediation rule, and a controllable signal are not one undifferentiated relation. The geometry gives precise content to spacelike separation. It does not allow the remaining assumptions of a quantum argument to remain unnamed.

Gravity and a Dynamical Geometry

In general relativity, the metric is part of the physical account of gravitation. It determines interval structure and is constrained by field equations involving stress-energy and, when included, a cosmological constant. Geometry consequently participates in the dynamics rather than serving only as an unchanging external stage.

Free fall is described through spacetime geometry, with ideal test bodies following appropriate geodesics. The test-body qualification matters: backreaction, extended structure, and nongravitational

forces require more elaborate treatment. The geometrical account has conditions of use, just as the spring model does.

This changes a metaphysical picture in which matter consists solely of occupants placed inside a wholly passive container. It does not by itself decide every dispute between relational and substantival accounts of spacetime. Both the empirical content and the interpretation of the metric require attention.

Coordinate freedom remains a representational issue. A different coordinate description of the same geometrical situation is not necessarily a different physical world. Conversely, a change in curvature cannot generally be erased by merely choosing new coordinate names. Representation and physical structure must be distinguished without separating them beyond explanation.

Motion, History, and the Present

Relativity places constraints on any account that treats a single distant present as an immediately given physical structure. It does not automatically settle all metaphysical questions about becoming, the reality of the past, or the status of the future. Those positions must state how they accommodate the physical relationships the theory describes.

An account of a complete spacetime history can encode temporal relations without representing change as a succession of updates to an external universal clock. That formal possibility is important. It still requires an interpretation of what the history represents and how the experiences of beings within it relate to the whole.

The present book therefore distinguishes the mathematics of a spacetime model, its physical interpretation, and a metaphysics of time. None can be substituted silently for another. A block-like mathematical representation is not, by its shape alone, a proof that

every philosophical question about temporal existence has been answered.

Boundaries of the Description

Horizons and singular behavior require the same care. A horizon limits specified forms of communication or access within a spacetime account. It is not automatically the boundary of all being. A singularity theorem establishes a result under particular geometrical and physical hypotheses; it does not by itself describe a material object of infinite density existing at a point in an otherwise complete theory.

Where a physical account reaches a limit, metaphysics has work to do: which assumptions have become questionable, and what kind of explanation is still being sought? The limit is not evidence for every possible replacement. It is a demand to identify the unresolved question accurately.

Spacetime physics thus strengthens the book's central claim. Numbers become physically informative through a structure of relations, interpretation, and measurement. Their invariants can be exact even where familiar intuitions about space and time must be revised.

The interval calculation supplies a definite recovery target. An interpretation of STOEM indices as events must preserve that interval where special relativity applies. Internal clock processes or a recovered geometry may provide the connection. The universal iteration index alone supplies an order, while the physical construction must supply the clock and interval relations.

CHAPTER 18

Thermodynamics, Information, and the Arrow of Time

The Physical Setting of Knowledge

A record requires a physical realization. A mark on paper, a stored voltage, and a molecular configuration persist under particular conditions and can be altered by particular interactions. The possibility of knowledge therefore includes questions about stability, noise, dissipation, and the maintenance of distinctions.

Thermodynamics gives these questions a physical setting. It concerns energy transfers, temperature, entropy, and the constraints on processes. Statistical mechanics connects such descriptions to accounts of microscopic states and their distributions. A metaphysics of intelligibility should examine both the meaningful record and the physical conditions under which it survives.

This does not identify meaning with entropy. A damaged manuscript and a shuffled set of unrelated marks require different semantic judgments even if some statistical measure assigns them the same value. Thermodynamic and informational quantities become relevant through the questions being asked, not through the unrestricted use of the word information.

Microstates and Macrostates

A macrostate description groups microscopic possibilities according to selected quantities. Different microscopic arrangements can share a temperature, a density, or a particle count in a specified region. That grouping is another example of equivalence relative to a family K.

For a finite illustration, consider four distinguishable markers, each placed on the left or right. There are 16 complete assignments. Let n be the number on the left. The map retaining only n has the following fibers:

Number on Left n	Assignments Ω(n)	Fraction of All Assignments
0	1	1/16
1	4	4/16
2	6	6/16
3	4	4/16
4	1	1/16

The assignment LLRR and the assignment LRLR both give n = 2. The count question is preserved, while the individual location questions are lost. The table follows by enumerating the subsets of markers placed on the left.

If these microstates are assigned equal statistical weight, the fractions become probabilities. That condition is an assumption about the ensemble. Counting alone does not establish that an actual system visits all assignments equally often, or that its time evolution is irreversible.

The Boltzmann form S = kB ln Ω associates an entropy with a macrostate containing Ω accessible microstates in an appropriate statistical setting. In the illustration, the count-two macrostate has S = kB ln 6, while either all-on-one-side macrostate has S = 0. These values concern the stipulated finite grouping; they are not a complete model of gas thermodynamics.

Probability and Physical Irreversibility

The second law cannot be obtained merely by noticing that a many-to-one map discards distinctions. The map is an operation on a description. A physical process requires a dynamical account of the system and its surroundings, together with the conditions under which macroscopic behavior is inferred.

This distinction matters because reversible microscopic dynamics can preserve a fine-grained description while a coarse account exhibits increasing entropy. Initial conditions, statistical assumptions, correlations, and the selected macroscopic variables contribute to the explanation. Calling the final description less detailed does not by itself supply that explanation.

The familiar temporal direction of records therefore presents a substantive problem. Why are the relevant conditions such that records accumulate in the direction we call the future? How do stable traces arise within a world that also contains fluctuations and irreversible processes? A general account of time must address the physical production of records as well as their ordering.

Low-entropy boundary conditions play an important role in discussions of the thermodynamic arrow. Their invocation relocates a question: why those conditions, and what warrants the statistical treatment built around them? It does not make the arrow an arbitrary convention that can be reversed by changing the name of a coordinate.

Erasure and Its Physical Cost

Consider resetting an unbiased binary memory to one logical state. At the logical level, both possible inputs have the same output. The previous bit is no longer recoverable from that output alone. The operation fails the fiber criterion for preserving the input-bit question.

A physical implementation adds assumptions absent from this abstract map. In the standard thermodynamic idealization, the memory states are equivalent in their internal thermodynamic structure, the surroundings include a thermal reservoir at temperature T, there is no usable retained side information about the input, and the control apparatus completes a cycle. The familiar Landauer bound for the heat Q delivered to the reservoir is:

$$Q \geq k_B T \ln 2.$$

Using $k_B = 1.380649 \times 10^{-23}$ joules per kelvin and T = 300 kelvin gives approximately:

$$k_B T \ln 2 = 2.87 \times 10^{-21} \text{ joules}.$$

This is a limiting thermodynamic scale under the specified conditions. A real device can dissipate more. Correlations, finite reservoirs, and other changes to the setup require a more careful statement; Reeb and Wolf give a precise quantum-statistical treatment with finite-size corrections.

The distinction between logical and physical erasure is essential. Forgetting a word in a mathematical description does not automatically dissipate this amount of heat. Nor does the bound mean that every computation must lose one bit of information per operation. The process, its physical implementation, and the information available elsewhere must be specified.

Persistence Requires Conditions

A memory can be stable enough for one task and inadequate for another. A thermal fluctuation irrelevant to a macroscopic inscription may matter to a small device. A biological record can depend on repair and continuous exchange with its environment. Persistence is consequently a physical achievement with a domain of validity.

The metaphysical account gains something from this dependence. Identity through time need not mean unchanging microscopic composition. A maintained organization can preserve the questions relevant to a record even while its constituent states change. The claim must identify that organization and the physical conditions sustaining it.

There is also a cost to treating every distinction as equally important. A useful memory stabilizes some differences while tolerating variation in others. This is preservation relative to K embodied in matter. The selection of K may express a purpose, but the feasibility of the resulting device is constrained by physical behavior.

Entropy and Existential Interpretation

Thermodynamic irreversibility bears on aging, repair, work, and the endurance of artifacts. Those connections give it existential significance. They do not turn the second law into a complete theory of moral decline, historical collapse, or the fate of meaning.

Such extensions require new arguments. A physical entropy increase and the loss of an intelligible tradition are different kinds of claim, even when one can influence the other. The physical conditions of interpretation belong within metaphysics; the meanings interpreted cannot simply be read from an entropy formula.

The contracting map in Chapter 8 erases initial differences, but a thermodynamic reading would additionally need energy, temperature, and a reservoir. That distinction gives a specific next task: identify which retained STOEM quantities support the entropy and energy balances used in this chapter.

CHAPTER 19

Quantum Interpretation as a Metaphysical Choice

Rules and Their Meaning

A physical theory combines mathematical rules with an account of how those rules connect to preparation, measurement, and prediction. A philosophical interpretation addresses further questions about what the state represents, what a measurement establishes, and how the theory relates to actuality.

The distinction is useful even when the boundaries are contested. Two accounts can agree on a probability calculation while disagreeing about what the calculated state signifies. An altered dynamical proposal can make different predictions as well as different ontological claims. The comparison must identify which kind of difference is present.

Our framework therefore records three elements separately: the mathematical and operational rules, the interpretation of those rules, and additional metaphysical commitments. This does not force interpretation into a detachable verbal appendix. An interpretation can guide which models are developed and which questions are considered meaningful. It makes those consequences visible.

Measurement as a Physical Interaction

A measuring device is a physical system. An idealized quantum measurement correlates distinguishable states of that system with states of what is measured. Describing this interaction gives physical content to the word observation before a philosophical interpretation is selected.

Let e0 and e1 be orthonormal states of the measured system, and let R be a ready state of the apparatus. Suppose the interaction carries e0 ⊗ R to e0 ⊗ A0 and e1 ⊗ R to e1 ⊗ A1, where A0 and A1 represent distinguishable records. The symbol ⊗ combines the state spaces of the two systems.

By linearity, an input superposition with amplitudes α and β leads to the joint state:

$$\alpha\, e0 \otimes A0 + \beta\, e1 \otimes A1.$$

The apparatus has entered the physical description. The displayed state nevertheless does not, simply by being written, select one recorded result. The relationship between the joint quantum account and a definite outcome is part of the measurement problem.

Decoherence and Stable Records

Interactions with additional environmental degrees of freedom can suppress interference in a reduced description of the system and apparatus. Environmental states correlated with different records become effectively distinguishable. The account can then predict approximately classical probabilities for selected accessible questions while retaining a quantum description of the larger system.

Zurek's decoherence program examines how environmental interactions favor stable records and effective classical behavior. Schlosshauer analyzes the relationship between these results and competing interpretations of measurement. This physics explains important features of records without making every interpretive question disappear.

The distinction can be stated in the book's vocabulary. Reducing a description to selected degrees of freedom changes which questions are retained. Agreement with a classical mixture for those questions

does not automatically establish that the full quantum state has become a single member of that mixture.

A collapse account, an account retaining unitary evolution, and a relational account can therefore face the same physical calculation while assigning it different ontological significance. Their commitments must remain explicit. The interpretation concerns what the physical account says about actuality, including the status of the records it predicts.

Copenhagen as an Available Orientation

Copenhagen is a historical family of positions rather than a uniquely specified doctrine. Howard's study emphasizes significant differences between Bohr's complementarity and views later presented as a unified Copenhagen interpretation. Selecting the label therefore leaves work to be done: which commitments about measurement and physical description are being adopted?

A Copenhagen-oriented account can emphasize classical descriptions of arrangements and outcomes, complementarity, and limits on assertions about properties outside a specified context. Heisenberg's discussion also assigns probabilities both an objective aspect, as tendencies, and an aspect concerning knowledge. This is more nuanced than a claim that everything depends on private human awareness.

Within the present metaphysics, such an orientation is an explicit philosophical choice. Where its emphasis is epistemological, that emphasis should be retained. Treating it as a subject of metaphysical examination does not require attributing a positive ontology to every version of the view.

The choice can be reasoned. It can be assessed for coherence, its connection with successful predictions, and its treatment of the

measurement problem. Its historical prominence is relevant to the history of physics but does not eliminate the need to state its commitments.

Relational Description

Rovelli's relational quantum mechanics questions an observer-independent state and describes physical systems relative to other systems. It does not reserve the role of observer for a conscious human being. This provides another way of organizing the relationship between state descriptions and physical interactions.

A relational metaphysics can learn from this proposal without treating every relation in philosophy as a quantum relation. The relation between a word and its reference, or between a person and a promise, does not become an instance of a particular quantum interpretation merely through the shared vocabulary.

The distinction is especially important in a general book. Quantum theory can sharpen questions about observation and description. It should not become an unexplained metaphor that automatically settles identity, consciousness, or value.

Comparing State and Outcome Commitments

The following comparison uses representative formulations. Each family contains internal differences. The Copenhagen entry records the contextual orientation discussed above; the Everett entry includes the later decoherence-based account of branches. Bohm's additional configuration, GRW-type objective collapse, and QBism make further alternatives explicit.

Approach	Quantum State	Measurement Outcome
Copenhagen	Predictive state tied to specified arrangements; ontological readings vary.	Definite recorded result; its classical description has a privileged role.
Everettian	Universal state retained as the quantum account.	Different records persist in branches; definiteness is relative to a branch.
Bohmian	Wave function together with an actual configuration, ordinarily particle positions.	One recorded result determined by the actual configuration and its motion.
Objective collapse	Physical state subject to spontaneous stochastic localization.	Localization suppresses macroscopic alternatives and supports definite records.
Relational	State and physical facts specified relative to interacting systems.	A fact established relative to a system in an interaction.
QBist	An agent's personal probability assignments for consequences of actions.	An experience for the agent undertaking the action.

The measurement interaction earlier in this chapter now yields distinct readings. An Everettian account retains the joint superposition and explains why records are stable within branches. Bohmian mechanics adds an actual configuration that selects the recorded result without requiring a physical collapse of the universal wave function. An objective-collapse theory changes the dynamics so that localization is a physical process. These are distinct commitments about what makes an outcome actual.

Relational quantum mechanics makes interaction, rather than consciousness, central to relative facts. QBism assigns a different role to the agent: the state expresses that agent's expectations, and the

Born rule constrains those expectations. Neither position should be assimilated to the claim that a human mind arbitrarily creates the entire external world. Their accounts of probabilities and facts differ from each other.

Comparing Observer and Dynamics Commitments

Approach	Observer's Role	Dynamics
Copenhagen	Describes arrangements and communicates outcomes; consciousness need not trigger collapse.	Unitary evolution and measurement updating; the meaning of reduction varies.
Everettian	Physical participant with records relative to a branch.	Universal unitary evolution; no fundamental collapse.
Bohmian	Physical subsystem whose records depend on configuration.	Wave evolution plus a guiding law; standard particle dynamics is nonlocal.
Objective collapse	Ordinary physical system; observation does not cause localization.	Modified stochastic law, with collapse parameters open to experiment.
Relational	Any interacting physical system can serve as the reference system.	Ordinary quantum transition rules with relational attribution of facts.
QBist	Agent acts on the world and updates personal expectations.	Quantum rules guide probability assignments and updating; collapse is not a new physical force.

The comparative costs matter. Everettian approaches must explain the status of branches and the role of Born probabilities in a universally unitary account. Bohmian approaches give a clear configuration ontology but must connect their guiding dynamics with

relativistic physics. Collapse models make outcome selection dynamical and incur empirical constraints on their additional parameters. These costs follow from the respective solutions each approach attempts.

Copenhagen-oriented views owe a clear account of the status of classical description and the boundary used in applying measurement rules. Relational views owe an account of how facts relative to different systems support comparisons between their records. QBism owes an account of how agents coordinate experience and scientific practice while treating state assignments personally. These are questions for assessment, not a declaration that all six approaches fail.

STOEM can now make a reasoned selection by stating which explanatory demand it gives priority: a single actual configuration, universal unitary dynamics, physical outcome selection, relative facts, or an agent-centered probability account. Preserving established probabilities is a shared constraint; additional empirical predictions must also be tested. The spiritual relationship by itself does not settle this comparison. Choosing an interpretation becomes substantive when the account accepts the commitments and explanatory costs listed here.

What Would Count as Choosing?

To adopt an interpretation is to make commitments that affect what the account says. If an interpretation regards a state as complete in a specified sense, the account should say what completeness excludes. If it treats a state as relative to a system, it should explain the relevant relation. If it includes additional variables, it should explain their role and how the empirical predictions arise.

A declaration of neutrality can also conceal commitments. A model that silently assigns values to every possible measurement has already made an assumption about joint description. A model that treats recorded outcomes as the only meaningful content has also taken a philosophical position. Explicitness is preferable to allowing either position to enter through the notation without discussion.

The present framework does not make all choices equally good. It makes the criteria of assessment visible. An interpretation may be internally inconsistent, fail to account for known results, or leave its central terms unclear. It may also solve one problem while creating another. Comparing those consequences is part of metaphysics.

The Role of Preservation

The preservation model supplies a limited but useful question: which answers remain available under a proposed change of description? If two interpretations agree on the predictions for a specified set of experiments, that agreement concerns a family K of operational questions. It does not automatically show agreement on every ontological question.

Conversely, a difference in vocabulary does not by itself establish a difference in content. A translation may preserve the relevant assertions. Formal work on theoretical equivalence examines several ways of making such claims precise. The definitions chosen for equivalence matter because different criteria can classify theories differently.

For our purposes, the important discipline is to state what the comparison preserves. A claim of physical equivalence, logical equivalence, or identity of ontology requires its own account. The word interpretation should make that responsibility explicit rather than provide a reason to avoid it.

CHAPTER 20

Locality, Time, and Causal Dependence

Several Questions Hidden in One Choice

A discussion framed as a choice between locality and causality can conceal several distinct commitments. Where are interactions mediated? Which variables depend on which settings? Can a controllable signal be sent outside a light cone? What explains the temporal order of records? Each question concerns a different feature of an account.

The distinctions do not make every combination possible. A model must satisfy the mathematical and experimental constraints relevant to its commitments. The purpose of separating the questions is to identify what is being assumed and what a result actually excludes.

In a standard Bell-model setting, outcomes a and b depend probabilistically on settings x and y and an explanatory variable λ. One relevant condition is factorization:

$$P(a, b \mid x, y, \lambda) = P(a \mid x, \lambda)\, P(b \mid y, \lambda).$$

A separate condition concerns whether the distribution of λ depends on the settings. These assumptions, with the relevant probabilistic setup, constrain attainable correlations. Their meanings and combinations should be examined explicitly. Bell-type reasoning does not reduce to a choice between two undefined everyday words.

Mediation and Temporal Direction

Wharton and Argaman discuss locally mediated models that relax a customary arrow-of-time assumption. In these accounts, mediating parameters can depend on later measurement settings. Their survey distinguishes such proposals from an exclusively initial-value picture and identifies the extension to a comprehensive account of quantum phenomena as a further challenge.

This supplies a useful lesson for metaphysics: local mediation and dependence only on earlier inputs are separate assumptions. A functional dependence on later settings must be interpreted carefully. It does not automatically describe a controllable message sent to the past.

The same distinction arises more generally when a mathematical problem is specified by boundary conditions. Solving a trajectory with conditions at two endpoints differs from calculating a trajectory from an initial state. The endpoint specification alone does not establish which endpoint causes the other. A causal interpretation requires additional commitments.

Histories and Processes

A history can be represented as a whole, as a sequence of states, or through relations among events. These representations can be useful for different questions. A whole-history account makes global constraints visible. A sequential account makes a particular form of update visible.

Neither representation alone settles whether becoming is fundamental, whether the future is open in a metaphysical sense, or how deliberation relates to action. Those conclusions require an interpretation of time and possibility. A mathematical index labeled t does not supply that interpretation by itself.

This point can be made without denying the physical success of temporal models. A clock reading is an operational fact within a specified practice. A theory relates readings to processes with considerable precision. The further question concerns the kind of temporal reality that the account attributes to those relationships.

Counterfactual Questions

Causal reasoning often asks what would happen if a condition were changed. Such a question requires a rule identifying which other features remain fixed and which respond. Without that rule, a comparison of two configurations need not be a comparison of interventions.

Suppose a room's light is on whenever a switch is in one position. The correlation is compatible with several arrangements. The switch may control the light, a hidden device may control both, or the relation may have been recorded under restricted circumstances. An intervention claim needs a model of how manipulating the switch enters the system.

Our preservation language applies here as well. A proposed intervention comparison preserves some background conditions while varying a selected feature. Its meaning depends on the preserved conditions. Holding too much fixed can make the intervention impossible. Holding too little fixed can make the comparison uninformative.

The philosophical question is therefore not exhausted by drawing an arrow between variables. The account must explain what the arrow represents, which alternatives are admitted, and why the counterfactual comparison is warranted.

An Explicit Table of Commitments

Issue	Question for the Account
Spatial mediation	What intermediate physical relationships are proposed?
Temporal dependence	Which earlier or later conditions enter the description?
Signalling	Which controllable changes can alter accessible outcomes?
Settings	How are experimental settings related to explanatory variables?
Intervention	What changes, and what remains fixed, in a counterfactual comparison?
Temporal experience	How do records, memory, and experienced succession enter?

In Chapter 9, every component affects every other, yet a finite output prefix involves only finitely many input bits. This mathematical distinction does not establish relativistic locality. It does identify the questions a physical reading must answer: which bits can be controlled, which can be read, and how update order relates to physical duration. Those choices determine whether the underlying dependence permits an observable signal.

CHAPTER 21

A Worked Quantum Example

A Two-Dimensional State Space

Consider a two-dimensional complex vector space with orthonormal basis vectors e0 = (1, 0) and e1 = (0, 1). A normalized pure-state vector has the form ψ = (α, β), where α and β are complex numbers satisfying:

$$|\alpha|^2 + |\beta|^2 = 1.$$

For this example, adopt the standard projective measurement rule: the probability of a basis outcome is the squared magnitude of the corresponding inner product. This is a physical rule being assumed for the application, not a consequence of the general metaphysics.

Define a Z measurement by the basis e0, e1. Its outcome probabilities are $|\alpha|^2$ and $|\beta|^2$. Define an X measurement by the basis $e+ = (e0 + e1)/\sqrt{2}$ and $e- = (e0 - e1)/\sqrt{2}$. Its probabilities are:

$$pX+ = |\alpha + \beta|^2/2,\ pX- = |\alpha - \beta|^2/2.$$

The formulas follow by taking inner products with the two X basis vectors. Normalization ensures that each pair of probabilities sums to one. The different measurement contexts ask different questions of the state.

Two States Indistinguishable in One Context

Take ψ+ = e+ and ψ− = e−. In both states, the magnitudes of the two Z components are $1/\sqrt{2}$. Therefore both give Z probabilities (1/2, 1/2).

For the X measurement, ψ^+ gives probabilities (1, 0), whereas ψ^- gives (0, 1). The two states are indistinguishable under the Z probability question and distinguishable under the X probability question.

Let X0 be the set containing ψ^+ and ψ^-. Let f record only the Z probability distribution, and let q record the probability of X^+. Then $f(\psi^+) = f(\psi^-)$, while $q(\psi^+) = 1$ and $q(\psi^-) = 0$.

Result 3 applies immediately. No function of the retained Z distribution alone can recover q on both inputs. The obstruction is a precise loss of information relative to the enlarged family of questions. It is not a failure of the underlying quantum account to describe the two states.

A Mixture and a Coherent State

Now consider an equal classical mixture of preparations e0 and e1. For any measurement in this example, the predicted distribution is the average of the distributions associated with the two preparations. Its Z probabilities are (1/2, 1/2), and its X probabilities are also (1/2, 1/2).

The mixture therefore agrees with ψ^+ in the Z context while disagreeing in the X context. The comparison distinguishes a coherent superposition from that particular mixture through a specified measurement. The use of the word coherent is justified here by the physical and mathematical state account, not by a general philosophical preference for unity.

Preparation	Z0	Z1	X+	X−
e0	1	0	1/2	1/2
ψ+	1/2	1/2	1	0
ψ−	1/2	1/2	0	1
Equal mixture of e0 and e1	1/2	1/2	1/2	1/2

The table concerns predictions for the stated preparations and measurements. A single observed outcome does not identify the preparation with certainty in every case. The probabilities describe the statistical account, and inference about an unknown preparation requires an appropriate experimental procedure.

What Is Preserved by a State Description?

Multiplying a pure-state vector by a complex phase of magnitude one leaves all projective measurement probabilities unchanged. If $\psi' = \exp(i\theta)\psi$, then an inner product with a measurement vector acquires the same phase, whose squared magnitude is one. The probabilities are therefore identical.

Under a family of questions consisting of such measurement probabilities, a description retaining that global phase includes a distinction the questions do not detect. A quotient by global phase can preserve those probabilities. This is a different situation from discarding the relative sign between $\psi+$ and $\psi-$, which changes the X predictions.

The example gives two transformations with different statuses. Removing a global phase preserves the stated measurement questions. Retaining only the Z distribution fails to preserve the larger family containing X predictions. Both transformations simplify a description, but they simplify it in different ways.

This is precisely why the preservation requirement must be stated. The word simplification does not tell us whether an experimentally relevant distinction has been removed.

What the Example Does Not Establish

The calculation does not choose between Copenhagen and relational quantum mechanics. It establishes predictions under the adopted measurement rules. Interpretations that reproduce those rules can agree on the table while differing in their account of states and outcomes.

Nor is this a proof of Bell nonlocality or a general contextuality theorem. A single-qubit example with these two measurement choices demonstrates that different measurements distinguish different state descriptions. Stronger conclusions about the impossibility of particular hidden-variable accounts require their own assumptions and arguments.

These limitations sharpen the example's contribution. It shows exactly how an operational family K determines an equivalence relation on descriptions, and how enlarging K can distinguish cases previously grouped together. The mathematics of preservation identifies the difference without turning the result into a declaration about every question of quantum ontology.

The Metaphysical Lesson

The subject constrains the useful questions. An investigator cannot demand that retaining only the Z probabilities also retain every X prediction when the calculation shows otherwise. Equally, the success of the probability rules does not by itself answer every question about the reality represented by ψ.

The worked table gives STOEM a precise quantum target: preserve the two coherent preparations' agreement for Z while retaining their difference for X. A classical probability distribution for Z alone loses the required distinction. A successful recovered account must therefore retain the phase-sensitive information that the two measurement contexts detect.

CHAPTER 22

Cosmology and the Conditions of a Physical Universe

A Physical Account of the Whole

Cosmology extends physical inquiry to the large-scale structure and history of the universe. The subject includes the systems in which observers and laboratories arise. That inclusion gives cosmology a special place in metaphysics, but it does not remove the need to distinguish a physical model of the universe from an exhaustive account of all being.

The observations available to cosmology come from within the world being described. Light, matter, clocks, and instruments participate in the same physical history that the account seeks to reconstruct. Inference therefore depends on laws of propagation and interaction, together with assumptions about the conditions between the observed source and the observer.

This is not unique to cosmology: every observation has physical conditions. Its scale makes those conditions especially consequential. A signal arriving now can constrain a much earlier state, while its interpretation depends on the model connecting emission and reception.

The book's framework applies directly. The evidence supplies constraints; a cosmological model specifies histories; interpretation connects the variables to physical quantities. The question family K includes the observations and inferred relationships an acceptable comparison must preserve.

Expansion and What the Scale Factor Means

In a homogeneous and isotropic cosmological model, a scale factor a(t) describes how spatial distances between suitable comoving locations vary with cosmic time. It is part of an idealized geometrical account. It is not a claim that every bound object expands in the same way.

Define the expansion rate H by:

$$H(t) = (1/a)\, da/dt.$$

Multiplying a by a positive constant does not change H. The normalization of the scale factor is therefore distinct from its fractional rate of change. One can conventionally set a to 1 at a selected reference time while retaining the model's expansion history.

That is a simple application of preservation. If K contains H and ratios of scale factors, a constant rescaling preserves those questions. It does not change the inferred physical history merely by assigning a different normalization to the same description.

For comoving emission and reception in this model, cosmological redshift satisfies:

$$1 + z = a(\text{reception})/a(\text{emission}).$$

If the ratio is 2, then z = 1. The emitted wavelength is received with twice its original value in the idealized comparison. Additional motions and gravitational effects require appropriate treatment when interpreting a particular observed spectrum.

The equation links a measurable spectral relationship to a model of spacetime evolution. It does not identify redshift with a metaphysical expansion of every category of being. The physical interpretation supplies both the force and the limit of the inference.

Conditions Select Different Histories

An elementary idealization makes the dependence on assumptions visible. Consider a spatially flat, expanding Friedmann cosmology containing pressureless matter, with negligible radiation and no cosmological constant. Its equations permit a scale factor proportional to t raised to the power 2/3, where t is measured from the idealized initial boundary.

Differentiate that expression to obtain:

$$H(t) = 2/(3t).$$

This relation is exact within the stated idealization. It is not a universal formula for the age of the actual universe. Changing the matter content, curvature, or cosmological term changes the problem and can change the relation between age and expansion rate.

Here the requirements P carry substantial physical content. The words flat, pressureless, and negligible are not decorative qualifications. They determine which solutions are admitted and which conclusions follow. Dropping them while retaining the numerical conclusion would change a conditional physical result into an unsupported general assertion.

The example also distinguishes the existence of a model from its adequacy. One can solve the equations and then ask which observations the solution preserves. Mathematical solvability alone does not establish that the model describes the actual universe.

Observation and the Status of a Model

The Planck collaboration's 2018 analysis compared cosmic microwave background temperature, polarization, and lensing information with a specified family of cosmological models. Its reported success for a six-parameter ΛCDM framework is evidence within an articulated physical and statistical account. Parameter inferences depend on the model assumptions used in the analysis.

This makes cosmology a strong example of disciplined inference. Multiple forms of evidence can constrain a common account. Agreement is significant because the account risks failure against those constraints. Its success need not mean that every component has received a complete microscopic explanation.

Dark matter and dark energy name roles within cosmological accounts that still call for deeper physical explanation. Their inclusion should not be represented as a complete identification of their underlying nature. Equally, the absence of that complete identification does not make the observational constraints disappear.

An adequate metaphysics can recognize both points. Evidence narrows admissible accounts, while the remaining explanatory questions remain real. The unknown is neither an empty space for arbitrary invention nor a reason to pretend that the present description is final.

An Early Boundary Is Not Yet an Ultimate Ground

A model of an earlier, hotter, denser physical universe addresses physical history. Whether a particular extrapolation has an initial boundary, an extension, or a regime requiring another theory is a further issue. A boundary of a model and an ontological emergence from absolute nothing are not automatically the same claim.

An infinite physical past would not by itself explain why there is a physical universe. A finite physical past would not by itself identify its ultimate ground. Duration and dependence are different questions. The later chapter on actuality examines their relationship without making a cosmological equation carry an argument it has not supplied.

The same caution applies to a quantum proposal for cosmic origins. If the proposal begins with a state space, a dynamical prescription, or a rule for probabilities, it has specified conditions. Whether those conditions are necessary, contingent, derived, or unexplained remains available for metaphysical examination.

Constants, Alternatives, and Actuality

Physical theories contain quantities and structures whose values or forms may be treated as inputs. Asking why those inputs have the character they do can be a scientific question, a metaphysical question, or both, depending on the proposed explanation.

A comparison of alternative universes needs more than an assertion that parameters could differ. It needs a specified space of alternatives, an interpretation of possibility, and, if probabilities are claimed, a measure and a reason for using it. Without those elements, statements that a universe is overwhelmingly unlikely are not yet determinate calculations.

This does not prohibit reasoning about fine-tuning or an ultimate source of physical order. It requires the argument to identify its premises and its remaining obligations. Neither a theistic conclusion nor a non-theistic conclusion can be obtained simply by leaving the probability model unstated.

A Fractal Cosmology and Its Obligations

The proposal that the universe has fractal organization can now be located within these questions. It must specify whether the structure concerns physical distributions, an underlying state space, or the organization of possible histories. It must also explain how the observed geometrical and dynamical relationships arise within its range of application.

An early STOEM thought experiment imagined the same constellations being available throughout the universe. This is not the usual cosmological homogeneity assumption. Homogeneity concerns the relevant statistical or geometrical description; it does not say that identified nearby stars appear in the same angular arrangement from every location. The present synthesis retains the inquiry into cosmic order without adopting that picture as a physical principle.

The theological claim of a created order is likewise distinguished from a particular spatial pattern. It does not require the universe to repeat a familiar arrangement for every observer. Its relation to physical cosmology concerns the existence and dependence of the order whose history and geometry cosmology investigates.

CHAPTER 23

From an Iterative Model to Physical Law

The Meaning of Emergence

STOEM proposes that familiar physical organization can arise within a deeper interacting system. To make that proposal informative, we must identify both the deeper description and the physical description to be recovered. A richer state space does not automatically produce the particular laws found in nature.

Let X be an underlying state space and T its update. Let p map X into a space Y of selected descriptions. For a physical application, p might retain a density, a field configuration, or a family of measurement predictions. The interpretation must explain what those outputs mean and why these are the appropriate quantities.

A closed reduced dynamics would be a function S on p(X) satisfying:

$$p \circ T = S \circ p.$$

The equation says that two routes agree. We may first update the detailed state and then describe it coarsely, or first describe it coarsely and then use the reduced law. It is an exact recovery condition for one update. The desired agreement is a mathematical question that can be answered under stated assumptions.

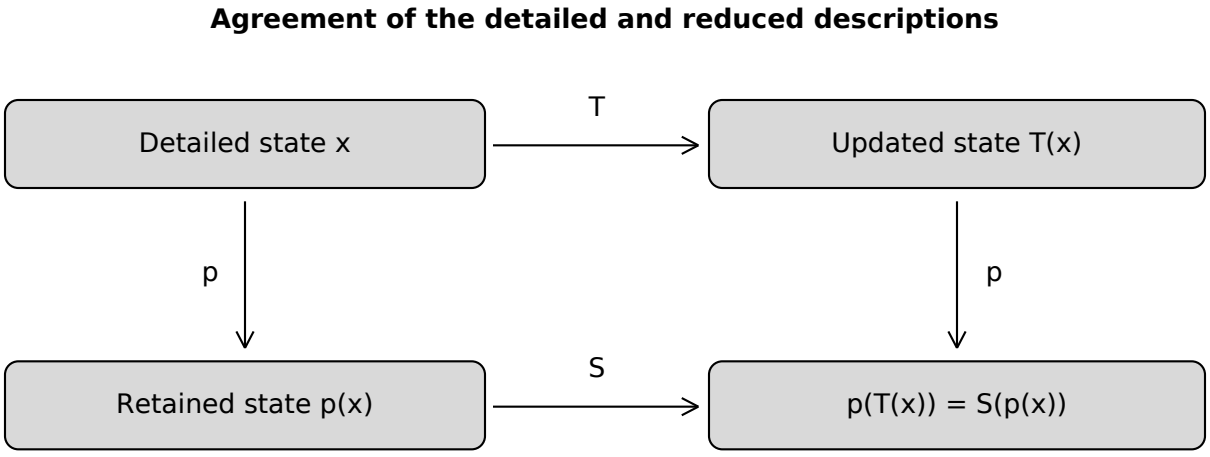


Figure 4. The recovery equation compares updating a detailed state before retaining selected information with updating the retained information directly.

The condition is often called a semiconjugacy when used in dynamical systems. Here its meaning is supplied directly. The map p need not preserve every detail, but it must preserve enough information for the selected future to be determined by the retained present.

Proposition E: When a Coarse Dynamics Exists

There exists a function S on p(X) satisfying the recovery equation if and only if p(x) = p(x′) implies p(T(x)) = p(T(x′)) for every pair of underlying states.

Proof in the forward direction. If the two present descriptions coincide, applying S to them gives the same output. The recovery equation identifies those outputs with the two descriptions after T, so they coincide as required.

Proof in the reverse direction. Given y in p(X), choose any x with p(x) = y and define S(y) = p(T(x)). The hypothesis says that every such

choice gives the same value. Hence S is well-defined. Its values belong to p(X), and it satisfies the recovery equation by construction.

Induction then gives $p(T^n(x)) = S^n(p(x))$ for every nonnegative integer n. Exact one-step recovery therefore supplies exact recovery at every finite number of updates. This follows from the algebra of the stated maps, not from a general expectation that large systems should become simple.

The proposition is an application of the earlier fiber criterion to the future question $p \circ T$. Its importance lies in the requirement it exposes. A representation may preserve the present quantity of interest yet fail to preserve the information required to predict its next value.

A Failure That Reveals Missing Information

Take $X = \{0, 1\}^2$ and $T(a, b) = (b, a)$. Let $p(a, b) = a$. The states (0, 0) and (0, 1) have the same retained present, zero. Their retained futures are zero and one. Therefore no function S of the first coordinate alone can recover this update on all four states.

The underlying system is deterministic. The difficulty arises because the chosen description omits a variable relevant to its future. Introducing a probability rule at the coarse level would require a distribution over the omitted coordinate or another statistical assumption. The deterministic model alone does not supply that distribution.

An alternative description retains a XOR b. Swapping the coordinates preserves this value, so its reduced dynamics is the identity. Different question families therefore support different answers about closure within the same underlying model.

Memory can also repair a description. In the swap example, two consecutive readings of the first coordinate reveal both coordinates of the current state in a known order. Enlarging the description to include history can restore prediction. This is one reason effective physical accounts may require extra variables or memory terms.

An Exactly Solvable Interacting Example

Consider two real coordinates a and b in [0, 1], updated by:

$$a' = (3a + b)/4,\ b' = (a + 3b)/4.$$

Both outputs remain in [0, 1]. Define the mean $\mu = (a + b)/2$ and the half-difference $d = (a - b)/2$. Direct substitution gives $\mu' = \mu$ and $d' = d/2$. Therefore:

$$a_n = \mu + 2^{-n}\, d_0,\ b_n = \mu - 2^{-n}\, d_0.$$

The mean has an autonomous reduced law: it remains constant. The difference decays at a definite rate. The paired description (μ, d) retains the complete state because $a = \mu + d$ and $b = \mu - d$. Keeping only μ loses the difference while preserving this particular invariant.

For the initial state (1, 0), the first updates are:

Iteration n	a_n	b_n	Mean
0	1	0	1/2
1	3/4	1/4	1/2
2	5/8	3/8	1/2
3	9/16	7/16	1/2
4	17/32	15/32	1/2

This example exhibits collective persistence and the decay of a distinction within one dynamical model. Assigning it a physical meaning requires further work. If a and b represent temperatures, for instance, the physical assumptions making their arithmetic mean conserved must be justified. The calculation itself has not derived heat flow in arbitrary materials.

Proposition F: Approximate Recovery Has a Horizon

Exact closure may be too strong for a physical approximation. Suppose Y has a distance dY, S maps p(X) into itself, and every underlying state satisfies $dY(p(T(x)), S(p(x))) \leq \varepsilon$. Suppose also that S is Lipschitz with constant $L \geq 0$, meaning it expands distances by at most a factor L.

Start the reduced and detailed accounts from matching descriptions, $y_0 = p(x_0)$, and evolve each by its own rule. Let $e_n = dY(p(T^n(x_0)), S^n(y_0))$. The triangle inequality gives:

$$e_{n+1} \leq \varepsilon + L\, e_n,\ e_0 = 0.$$

Proof. Insert the intermediate point $S(p(T^n(x_0)))$ between the two descriptions at step $n + 1$. The first distance is at most ε by the one-step assumption. The second is at most L times the previous discrepancy. Iterating this inequality proves $e_n \leq \varepsilon \sum_{k=0}^{n-1} L^k$.

When $L = 1$, the bound is $n\varepsilon$. When L differs from one, it is $\varepsilon(L^n - 1)/(L - 1)$. For $L < 1$, it is bounded above by $\varepsilon/(1 - L)$. For $L > 1$, even a small one-step error may give a weak bound at long horizons. The formula makes the time range of an approximation part of its content.

If the one-step estimate holds only in a limited region, the conclusion applies only while the required states remain in that region. An approximation cannot inherit a global guarantee from a bound checked along a few selected trajectories.

What Physical Recovery Would Require

These results give STOEM a concrete task. Recovering gravitation requires an interpreted geometry and dynamics that reproduce the relevant gravitational relationships. Recovering quantum theory requires the appropriate probabilities, compositions of systems, transformations, and measurement relations. Recovering thermodynamics requires statistical and physical conditions for macroscopic behavior.

A proposed fundamental system may use variables quite different from those appearing in the recovered description. The relationship p explains how those variables acquire a physical interpretation at the relevant scale. Units, symmetries, boundary conditions, and error tolerances belong in that explanation.

The existence of one conserved average is consequently a limited achievement. It establishes one exact reduction within an example. It does not derive the observed spectrum of particles, the Einstein equations, or quantum measurement rules. Those are distinct recovery problems with much richer content.

This places physics within STOEM without reducing physics to a list of names. The physical theories examined in the preceding chapters supply demanding targets. The underlying metaphysics motivates an investigation of their common ground. The mathematical recovery conditions state what success would mean in a given construction.

A Research Program With Identifiable Advances

Progress can now be described precisely. A proposed update may become well-defined on a larger state space. A physical quantity may acquire an interpretation. A reduced law may be derived exactly or with a controlled error. An apparent obstruction may identify information that a proposed description must retain.

Each advance improves a specified connection between levels. This is the kind of mathematical work through which STOEM can develop without asking one symbolic expression to explain everything at once. The metaphysical proposal remains broad, while the steps that articulate it can be exact and independently examined.

CHAPTER 24

A Worked Genetic-Code Example

The Coding Table as an Interpreted Function

The standard genetic-code table maps RNA codons to amino-acid assignments or a termination category. Let B = {U, C, A, G}, and let C0 = B^3 be the set of 64 triplets. Let A0 contain the 20 standard amino-acid labels together with a separate termination label, Stop. Define g from C0 to A0 by the standard code.

This is a deliberately restricted representation. It describes codon assignments in the standard table. It does not claim that the full biological process of translation is exhausted by a lookup function. The National Center for Biotechnology Information's code tables provide the reference assignments and document the existence of alternative genetic codes.

The interpretation matters at the first step. If Stop is counted among the output categories, there are 21 categories. That does not make termination an amino acid. The mathematical codomain can contain different kinds of output under an explicitly stated convention.

Synonymy as an Equivalence Relation

Define c ~g d when g(c) = g(d). This is an equivalence relation because equality of outputs is reflexive, symmetric, and transitive. Its classes are the fibers of the coding function.

The projection π from C0 to C0/~g preserves the coding output. There is a function $\bar{g}$ on the quotient with g = $\bar{g}$ ∘ π. This is an instance of Result 3, and the quotient is the canonical representation for the single question asking what output the standard code assigns.

The relation establishes functional equivalence under that question. It does not establish numerical identity of the codons. UUU and UUC are distinct triplets even though they receive the same amino-acid assignment. The distinction is elementary, but it is exactly the kind of distinction that can disappear when the word same is left unqualified.

The Size of the Fibers

The standard table gives the following distribution of fiber sizes:

Codons per Output Category	Number of Categories	Codons Accounted For
1	2	2
2	9	18
3	2	6
4	5	20
6	3	18
Total	21	64

The two categories with three codons are isoleucine and Stop. The three categories with six codons are leucine, serine, and arginine. The table groups by cardinality, so categories with different biological roles can appear in the same row.

Counting verifies the arithmetic: 2 + 18 + 6 + 20 + 18 = 64. The category count is 2 + 9 + 2 + 5 + 3 = 21. These counts concern the declared standard-code map, including the separate termination category.

A Question That the Quotient Loses

Let q3 ask for the third nucleotide of a codon. The coding quotient does not preserve q3. For example, g(UUU) = g(UUC), but q3(UUU) = U and q3(UUC) = C.

By Result 3, there is no function on amino-acid output categories alone that reconstructs the third nucleotide for every codon. This is not a limitation of a particular reconstruction algorithm. The quotient has identified inputs requiring different answers.

If the required family is K = {g, q3}, the appropriate equivalence relation becomes equality of both answers. The resulting quotient is finer than the coding quotient. If the family includes the entire codon identity, the canonical quotient has 64 singleton classes.

The example makes the hierarchy of representations explicit. An amino-acid sequence can preserve a particular translated output while omitting distinctions present in the nucleotide sequence. The omitted distinctions cannot be recovered uniquely from the output alone.

Refinement by First and Second Position

Let q12 record the first two nucleotides. Consider K = {g, q12}. Two codons are now equivalent only if they lie in the same first-two-position box and receive the same coding output.

For the standard code, the 16 boxes separate into 25 classes under this combined question. Some boxes remain one class. Others split into two or three because their codons have different assignments. This number can be obtained directly by counting the distinct outputs within each box.

For example, the UU box has two classes: UUU and UUC share one output, while UUA and UUG share another. The UG box has three

classes: UGU and UGC share cysteine, UGA is Stop, and UGG is tryptophan. The AU box has two classes: AUU, AUC, and AUA share isoleucine, while AUG is methionine under the coding-output convention used here.

The refinement preserves more of the triplet structure while still identifying selected third-position differences. Its meaning comes from the chosen questions. The number 25 does not automatically identify a biological module or a mechanism in the ribosome.

From Table to Mechanism

A lookup table and a physical decoder answer different questions. The table records assignments. A mechanistic account must explain how molecules, interactions, and cellular conditions realize the process. A formal partition can guide questions about mechanism, but its existence is not sufficient evidence that the mechanism implements the same partition as a separate physical step.

This distinction is useful in research. A proposed decomposition of the code can suggest an experimental hypothesis. It can identify patterns that a mechanistic account should explain. The hypothesis becomes stronger when its proposed intermediate states, interventions, and discriminations are specified.

The present book does not infer a decoding mechanism from the quotient alone. Its exact result concerns which questions a given representation preserves. That result remains valuable because it identifies the additional evidence needed to connect a mathematical organization with a biological process.

Why This Example Belongs in Metaphysics

The coding example is finite and familiar, yet it displays the same issues encountered in more abstract discussions of reality. There is a subject, an interpreted function, a criterion of sameness, a loss of information, and a question about physical realization.

It also shows that mathematical metaphysics need not depend on exotic phenomena. Ordinary scientific representations already contain choices about identity and admissibility. Making those choices explicit can improve both philosophical clarity and scientific reasoning.

The distinction between preserving an output and preserving an input is exact. The further significance of that distinction depends on the purpose of the account. This is the central method of the book in a setting where its mathematical consequences can be checked exhaustively.

Life Within an Interacting Universe

The genetic code places the STOEM inquiry in an embodied setting. Translation is realized through molecular systems whose organization and interactions matter. A universe described as infinitely interacting must still explain how relatively stable biological distinctions and mechanisms become possible within it.

The coding quotient shows one form of stable functional equivalence. It does not explain the origin of the code, the evolution of translation, or the spiritual status of the organism. Those remain different questions. The example is useful because it specifies one relationship between mathematical representation and living organization that the wider metaphysics can preserve.

CHAPTER 25

Logic and the Account's Own Conditions

The Logic We Are Using

The proofs in this book use ordinary classical reasoning about sets and functions. They assume familiar meanings for equality, membership, conjunction, and negation. Those commitments are part of the mathematical setting, and the philosophical argument should make them visible.

A general metaphysics can examine those commitments without pretending to begin from a position containing no distinctions or inferential standards. To state a question already distinguishes it from other questions. To offer a reason already depends on some relation between premises and conclusions. A purported account that abolishes every standard of assertion would also abolish the means of stating itself.

This does not establish that every feature of classical logic is metaphysically compulsory. It establishes a practical requirement of the present inquiry: enough inferential continuity must remain for its claims to be intelligible. An alternative logic must identify what changes and how reasoning in the alternative proceeds.

The Difference Between Logic and Notation

Renaming True and False while consistently translating their roles changes notation. Introducing an additional truth value changes a formal structure, but the significance of that change depends on how connectives and consequence are defined. Neither move can be assessed solely by counting the names of truth values.

Likewise, a partial description is not automatically an assertion with a third truth value. It may simply fail to answer a question. A probability of one half is not the same thing as half truth. These distinctions can be formalized in different ways, but the chosen way needs an interpretation.

The preservation results help separate the cases. A bijective renaming that transports all operations and relations retains the corresponding structure. A many-to-one recoding can discard distinctions. A new semantics can support different inferences. Each is a definite mathematical operation with a different philosophical meaning.

A Simple Warning About Unrestricted Formation

Suppose an account assumes that every condition defines a set of all things satisfying it. Apply that assumption to the condition that a set is not a member of itself. Let R be the alleged set of all sets that are not members of themselves.

Asking whether R belongs to R produces a difficulty. By the defining condition, R belongs to R exactly when it does not belong to R. Under classical reasoning, the unrestricted formation assumption cannot be retained in that form.

The lesson is specific. A plausible-sounding rule for constructing objects can be inconsistent. The conclusion is not that all sets, all distinctions, or all mathematics are impossible. It is that the formation principle needs restriction or a different formal treatment.

A metaphysical construction can encounter a similar problem if it introduces an unrestricted totality and then assumes every feature of the totality is represented internally in exactly the same way. The appropriate response is to examine the assumptions. The word totality does not guarantee a coherent model of everything it is intended to contain.

Self-Reference and Embedded Inquiry

A theory of reality occurs within reality. Can it explain the existence and operation of the inquirer? This question does not require every account to contain a complete internal duplicate of itself. Self-reference alone does not yield a final theory.

A weather model can explain conditions affecting its computer without representing every computation. A theory encompassing both processes has broader scope, but still requires interpretation.

The same discipline applies to metaphysics. Including the observer can improve the account. Naming a self-referential structure does not by itself explain consciousness, truth, or existence. Those claims require arguments connecting the structure to the phenomena.

Can Meaning Be Formalized All the Way Down?

Parts of meaning can be formalized. A dictionary can record conventions. A semantic theory can relate expressions to interpretations. A model can represent the conditions under which a statement is true. These achievements are substantial.

The remaining question is what gives the formal account its intended application. One answer may appeal to use, another to causal contact, another to inferential role, and another to a metaphysical relation of intentionality. The book does not assume that no formal account can ever address these issues. It requires that the connection be supplied rather than presumed from the syntax alone.

The same limit applies to K. Mathematics determines which answer functions a transformation preserves. Labeling their values meanings still requires an account of why they mean what they do.

Infinity Without an Unrestricted Totality

The set X in Chapter 8 is constructed as a specified space of arrays. It is not the set of every conceivable thing. Its index set, coordinate range, and operations supply boundaries within which the proofs proceed. This is enough to express a definite infinite model without claiming that all reality has been captured as one unrestricted mathematical collection.

The distinction also applies to God. In the theological account, divine grounding is not introduced by adding one more coordinate to X and calling the enlarged set complete. That would be a new mathematical representation whose interpretation still needed explanation. Metaphysical comprehensiveness concerns the scope of the questions addressed; it does not license unrestricted set formation.

CHAPTER 26

Experience and Personal Identity

The Person Conducting the Inquiry

A metaphysics of the world must have a place for the person who asks what the world is. Human inquiry includes sensations, memories, purposes, and encounters with other people. These are aspects of the subject matter, even when the inquiry represents them through physical or informational descriptions.

The position adopted here is that experience belongs to reality and must be accommodated by a comprehensive account. At this stage it identifies what an adequate explanation must address. STOEM then adds a theological commitment: the embodied person also stands in a God-Man relationship. The assertion of that relationship does not by itself provide a complete explanation of how consciousness arises or persists.

A model that predicts a report has explained something about reporting. A model that classifies a neural pattern has explained something about the classification and its relation to the data. Connecting either achievement with the occurrence of experience requires an explicit account. The distinction does not prove that such a connection is impossible.

This approach keeps the inquiry open without treating experience as an optional ornament added to a world already described completely. The person who sees, remembers, and acts is among the beings whose existence the metaphysics concerns.

Several Questions of Persistence

A person can be described through bodily continuity, memory, social recognition, legal identity, relationships, and first-person experience. These descriptions overlap, but they answer different questions. Their agreement can support an account of persistence without making them identical criteria.

Consider a person whose appearance changes over many years. Visual similarity decreases, yet historical and bodily continuities support identification. A purely visual classifier may fail while other descriptions succeed. The failure concerns the classifier's preserved questions, not necessarily the person's persistence.

Memory presents another issue. Two records can contain the same autobiographical statements without thereby being one person's experience. A copied diary preserves words and perhaps information about a life. Its copying does not transfer the numerical identity of the writer to the copy.

The distinction between preserving content and preserving a person is consequently essential. A functional description may support a useful comparison between systems. A claim of personal identity demands an account of why that comparison preserves the relevant being.

Why a Score Is Not a Person

Suppose a representation assigns each person a numerical score. Many people can receive the same score. The function therefore creates fibers containing distinct persons. Any question that distinguishes members of one fiber cannot be recovered from the score alone.

This is an immediate application of Result 3. If the question asks for a person's name, history, or particular commitments, a score that combines different answers loses that information. The statement remains true even if the score predicts some outcome accurately.

The example does not show that all scoring systems are useless. It shows that their authority is limited by the information they preserve. A model adequate for one decision can be inadequate for another. Treating its output as the full identity of its subject exceeds what the representation contains.

The metaphysical point is broader than a criticism of quantification. A verbal label can compress people just as aggressively. The relevant issue is what the description retains and whether its intended use requires distinctions it discards.

Recognition as a Relationship

Personal identity also appears in relationships of recognition. To recognize someone is to connect an encounter with a continuing person. The connection can involve memory, testimony, bodily evidence, and shared history. It is both epistemic and practical: recognition changes how people respond to one another.

A mistaken recognition can be corrected, which shows that the relation is answerable to more than an immediate feeling. At the same time, the evidential basis of recognition is not exhausted by one numerical criterion. A complete account must explain how several kinds of evidence and continuity are integrated.

This provides an existential example of a common world. Persons encounter one another through perspectives that are partial yet corrigible. The possibility of correction depends on retained relationships of reference and evidence. The world of persons is

therefore relational in its intelligibility even if one has not adopted an ontology consisting solely of relations.

The Scope of a Mathematical Account

The preservation framework can clarify which questions about a person a representation answers. It can identify compression, distinguish equivalence from identity, and reveal incompatible requirements. It does not derive an exhaustive theory of personhood from those operations.

The book's positive commitment is to explanatory adequacy: a theory claiming to account for persons must address the features it claims to include, and it must state the criteria through which a representation concerns the same person. This demand applies to physical, informational, psychological, and theological accounts alike.

The issue is not whether persons are beyond every possible formal description. It is whether a particular description has earned the authority attributed to it. Mathematical metaphysics serves the existential inquiry by making that question exact where exactness is possible and explicit where further argument is required.

Persistence and Spiritual Significance

The STOEM synthesis adds a question that the preservation of a physical pattern alone does not answer: what grounds the enduring significance of this person? Its theological answer concerns recognition and relationship with God. That significance is not measured by how many distinctions an observer can currently recover from the person's behavior.

This has an implication for the criteria we use. A decline in memory or an inability to communicate can alter access to a person's experience without establishing the disappearance of that person's worth. The metaphysical and ethical account must state what it recognizes through bodily and personal continuity. The next chapter develops the spiritual relationship within that embodied understanding.

CHAPTER 27

The Orthogonal God-Man Relationship

The Person Within and Beyond Physical Description

The God-Man relationship is a constitutive commitment of STOEM. A human being participates bodily in the physical universe and stands in a spiritual relationship with God. The word orthogonal marks the claim that the latter relationship is not exhausted by the physical description of the former. Man here means humanity, including every human person.

This commitment concerns one embodied person. It need not divide human life into a physically complete individual and an additional object attached elsewhere. The person who eats, works, remembers, suffers, and responds to others is also the person whose life can be understood in relation to God. The proposed distinction concerns aspects of existence and explanation.

The relationship's mechanism remains undefined. The book does not introduce a measurable spiritual variable and then treat its numerical behavior as discovered. It instead develops what is being affirmed by the relationship: dependence, personal address, recognition, responsibility, and the possibility that human significance exceeds every inventory of physical functions.

The distinction between meaning and mechanism is familiar in less comprehensive settings. Understanding a promise involves the physical production of words and also the commitment made through them. An account of the sound waves can contribute to explaining the event while leaving the commitment unarticulated. This example does not prove a spiritual ontology. It shows why different kinds of explanatory question can concern the same embodied event.

What Orthogonal Means Here

In an inner-product space, orthogonality has a precise definition: the inner product of the relevant vectors is zero. To apply that definition literally to body and spirit would require a space, an inner product, representations of both terms, and an argument that the mathematical relation has the intended significance. Those ingredients have not been supplied by the original metaphor.

Even literal orthogonality would not automatically establish causal independence. Two orthogonal coordinate directions can be coupled by a dynamical law. Conversely, two variables can be statistically independent without being vectors at all. Orthogonality, independence, and absence of interaction should therefore remain distinct.

In this book, orthogonal has a stated metaphysical use: the God-Man relationship belongs to an explanatory dimension that a physical account alone is not assumed to exhaust. It is not an extra direction in the spacetime metric. The account's positive meaning comes from the relation between God and the person, rather than from importing the properties of perpendicular lines.

This clarification also limits what can be inferred. The word does not specify an instantaneous transmission process. Direct relationship, in the theological sense used here, concerns the person's relation to God without requiring a physical relay between locations. Applying a speed to that relationship would first require a physical signal model that has not been proposed.

Why the Human-Spirit Product Is Reconsidered

The June 2023 Seed represented the human-spirit state through a product of symbols for physical humanity and spirit. It also used such a product to express a proposed Complexity Force. These expressions drew attention to a connection, but their algebraic interpretation remained unspecified.

Ordinary multiplication can have many roles: multiplying real quantities, composing operators, or building a product structure. Each role comes with different meanings and consequences. Without domains, units, and an operation, multiplying a symbol for a person by a symbol for spirit does not establish a law of consciousness or a physical force.

The present account therefore retains the intended relationship while declining to make the product carry an argument it does not contain. A richer description of a person can include physical, experiential, interpersonal, and theological claims. Their unity must be explained through the being of the person and the relations in which that person stands.

The same applies to complexity. A system may become more complex under one measure and less complex under another. Complexity alone does not identify intelligence, moral worth, or spiritual standing. A supposed force producing complexity would require a defined quantity and a physical or metaphysical account of its action. It cannot be obtained by treating an evocative name as a completed derivation.

Positive Content of the Relationship

The first positive claim is dependence. In the theistic development of STOEM, the person exists within a created order whose ground is God. This includes the physical conditions of life rather than beginning only where those conditions become difficult to explain. Biology remains a study of real processes within that order.

The second claim is personal significance. The person can be addressed, recognized, and held responsible. The theological proposal places that significance within relationship with God. It does not make a person's standing proportional to intelligence, productivity, memory, or success. Those measurable capacities describe important aspects of life without supplying its entire value.

The third claim concerns response. Human life includes possibilities of trust, refusal, gratitude, repentance, and love. In a Christian interpretation, these can be understood as responses within a relationship with God. Their theological meaning is not identical to the numerical complexity of the physical activity accompanying them.

These commitments give the account content without supplying a mechanism. They also create responsibilities for the wider metaphysics. It must explain how an embodied agent can participate in these relations, how its claims connect with experience and testimony, and how it can distinguish theological interpretation from an arbitrary attribution of divine purpose to any event.

Prayer, Action, and Embodiment

Prayer is a useful example because it can be physically described and personally understood. There may be spoken words, posture, attention, memory, and changes in subsequent action. A physiological study can investigate some of those processes. The theological act of addressing God concerns what the person is doing through them.

This does not entail that every outcome following prayer was produced by a special intervention. A causal attribution about a particular physical result still requires grounds. Nor does a complete physiological account, if available, automatically decide whether the act has a genuine addressee. That disagreement concerns ontology and interpretation as well as bodily mechanism.

STOEM's orthogonal relationship gives the second question a place within its account. It affirms a divine addressee as part of its theological orientation while retaining the ordinary responsibility to examine claims about particular events. The two responsibilities can coexist because they are not identical claims.

Embodiment also prevents the spiritual relationship from becoming an excuse to neglect physical conditions. Illness, hunger, exhaustion, and material deprivation affect persons. Care for those conditions belongs to care for the person. The theological dimension adds meaning and obligation to embodied life; it does not make bodily vulnerability irrelevant.

Persons, Copies, and Artificial Systems

The early STOEM dialogue distinguished the human participant from the artificial system by the proposed spiritual relationship. The distinction reflected a theological commitment. It was not a conclusion of an experiment capable of detecting a spiritual coordinate.

The present framework preserves a narrower, defensible conclusion about representation. Matching a person's words or selected functions does not by itself establish numerical identity with that person. The preservation proofs explain why equivalence relative to a question family is weaker than identity of the subject. They do not decide every possible question about the consciousness or moral standing of artificial systems.

A theological account may affirm a distinctive human vocation while leaving those further questions open. What it should avoid is claiming that a behavioral benchmark has measured the presence or absence of relationship with God. The benchmark has an operational meaning; the theological attribution has another source of warrant.

This restraint protects the human claim as well. A person whose observable capacities change does not thereby become a different quantity of spiritual relationship. The book's account of personal significance rests on personhood and relation, rather than on a performance threshold.

A Boundary With Philosophical Consequences

Leaving the relationship undefined does not mean that nothing can be said about it. It means that its asserted reality is not accompanied by a complete account of how it operates. The distinction is meaningful only if the positive commitments are stated and the limits of the explanation remain visible.

The proposal can be questioned. A critic may dispute divine existence, the irreducibility of personal relation, or the distinction between physical completeness and metaphysical completeness. Those are substantive disagreements. They should be addressed at the level of the actual claims rather than settled by demanding a coordinate that the account never introduced.

For STOEM, the orthogonal relationship marks the limit of treating the physical model as an exhaustive inventory of being. Mathematics can describe the physical organization of the embodied person and clarify relations among descriptions. The metaphysical inquiry also asks who the person is, what grounds that person's existence, and what relationship with God means. The wider account is responsible for keeping all of those questions in view.

CHAPTER 28

Agency, Value, and Responsibility

Acting Within the World

An agent acts from within a world of conditions and relationships. Deliberation, intention, bodily action, and consequences belong to the situation the agent inhabits. A metaphysical account of agency must explain how these features are related.

A mathematical representation can describe available actions, constraints, and outcomes. It can predict behavior under assumptions. It can also represent a whole history containing an agent's decisions. None of these operations by itself settles which conception of freedom is appropriate.

The question of freedom depends on what alternatives count as available, how reasons enter the explanation, and what kind of dependence is compatible with agency. An initial-value model and a whole-history model can each be given more than one philosophical interpretation. Their notation alone does not decide the issue.

The working commitment of this book is that reasons and deliberation are legitimate subjects of explanation. An account that describes their physical realization still owes an account of their role in action. An account that treats them as irreducible must explain how they participate in the same world as physical processes.

Objectives Enter From Somewhere

Optimization begins after an objective and a feasible set have been specified. A maximum is a result relative to those choices. The mathematics can determine which option best serves the objective; it does not by itself establish why that objective should govern the decision.

Consider two hypothetical plans for maintaining a public collection. Plan A preserves more original objects but provides access to fewer visitors. Plan B provides wider access but replaces some original objects with reproductions. A score can combine preservation and access, but its ranking depends on the chosen weights and on how the two features are measured.

The tradeoff is not dissolved by expressing both features numerically. The decision still involves judgments about the value of originals, the importance of access, and the obligations of the institution. Different defensible priorities can produce different rankings.

The example parallels the preservation framework. Selecting K determines which questions a representation must retain. Selecting an objective determines what a decision procedure seeks. In both cases, the formal result depends on a prior commitment whose justification belongs to the application.

Descriptive and Normative Requirements

A physical impossibility differs from an ethical prohibition. Both can exclude an option from a feasible set, but they exclude it for different reasons. The distinction matters when a requirement is challenged.

If a material cannot support a load, changing an institutional preference will not alter that physical fact. If a policy forbids an action, discovering that the action is physically possible does not

settle whether the prohibition should remain. A model that combines the two exclusions should retain their sources.

The same applies to personal identity. A system may be able to treat two people as interchangeable for a narrow calculation. Whether it ought to do so depends on the purpose and the ethical commitments of the practice. Mathematical equivalence under a selected question is not a complete account of moral standing.

The normative position proposed here gives value to truthful inquiry, the recognition of persons, and accountability for interventions. These are declared ethical commitments. They are supported by the role that trust, recognition, and responsibility play in shared life, but they are not deduced from the existence of a set or a graph.

Intervention and Understanding

The capacity to produce an outcome can exceed the completeness of an explanation. A technique may work under specified conditions while important features of its mechanism remain uncertain. This is a familiar reason to distinguish practical control from exhaustive understanding.

The distinction does not establish that every intervention has unlimited or hidden consequences. It establishes an obligation to state the scope of the model supporting the intervention. Which relationships are included? Which outcomes are monitored? Which questions remain unanswered?

The preservation framework suggests a concrete discipline. When a model is simplified for a decision, identify which questions the simplification preserves. If the decision depends on a question outside that family, the simplified model is insufficient on its own. Additional information or a richer representation is required.

This is a general principle of responsible reasoning. It applies to scientific experiments, technological systems, and institutional decisions without treating their risks as identical. The significance of an omitted distinction depends on the subject and the action contemplated.

Accountability for Abstraction

An abstraction can become socially consequential when an institution acts on it. A category determines eligibility, a score influences access, or a model guides resource allocation. At that point, the representation's limits are part of the ethical situation.

Accountability requires more than reporting that the calculation was correct. It requires explaining why the chosen variables, constraints, and objectives were appropriate. A correct optimization can still optimize the wrong objective for the human purpose it is meant to serve.

Mathematical metaphysics contributes by making those commitments visible. It does not replace ethical judgment with a formula. It identifies the points at which a formula depends on judgments about identity, value, and what counts as a relevant difference.

Agency thus belongs in the book for a precise reason. The beings who represent the world also change it. Their descriptions become instruments of action, and the conditions of adequate representation become part of their responsibility to others.

Responsibility in the God-Man Relationship

STOEM situates agency within a created order and a relationship of personal significance. Freedom is therefore not equated with a random term in an update. An unpredictable fluctuation can occur without deliberation, and a predictable act can still express a reason. The relevant question concerns how the person participates in action and responds to reasons.

The theological orientation adds accountability before God and concern for other persons. These commitments guide the interpretation of action; they are not deductions from fractal dimension or computational complexity. They also prevent the value of persons from being reduced to their contribution to an aggregate objective. A mathematically optimal outcome can remain ethically unacceptable if the objective discards obligations the account ought to preserve.

CHAPTER 29

Actuality and the Ground of Existence

What a Possible World Leaves Open

A mathematical structure can describe a possible arrangement without explaining why that arrangement is actual. A catalog of structures can enlarge the range of possibilities while leaving the selection of actuality untouched. The question why this world exists remains different from the question what follows from a proposed description of it.

This distinction does not prove that a mathematical explanation of existence is impossible. It identifies a burden for any such explanation. The account must explain how its formal existence claim connects with the existence of the represented world.

In mathematics, to establish that an object exists ordinarily means to establish existence under the relevant axioms and semantics. In physical inquiry, an existence claim concerns an instantiated process or entity. In metaphysics, the meaning of existence may itself be part of the investigation. Moving between these uses requires an argument.

The present book therefore keeps actuality in view as a question that admissibility alone does not settle. The set Γ can identify descriptions allowed by a model. It does not contain a special mark of physical actuality unless an interpretation and further information supply one.

Three Broad Positions

One position treats the existence of reality as a primitive fact. Explanations occur within reality, while the existence of the whole is accepted without a further ground. The position owes clarity about why the demand for explanation stops at that point and what explanatory work remains possible within the whole.

A second position seeks a necessary ground. It argues that contingent beings or facts depend on something whose existence or nature is not contingent in the same way. Such an argument must explain the relevant necessity, the dependence relation, and why the proposed ground has the required status.

A third position gives a theological account of the ground. In a theistic development, God is understood as the ground of created existence and its intelligibility. This introduces commitments about the character of the ground that go beyond identifying a formal source in a dependence diagram.

These positions are not merely different names for one conclusion. They offer different accounts of explanation and different answers to the question of what must ultimately be accepted. A mathematical metaphysics should make their commitments available for examination.

The Limits of a Dependence Diagram

A directed graph can represent an assumed pattern of dependence. If the graph is finite, nonempty, and acyclic, it has at least one vertex with no incoming edge. To see this, suppose every vertex has a predecessor. Following predecessors in a finite set must eventually revisit a vertex, producing a cycle. That contradicts acyclicity.

The argument is exact. It does not prove that reality has a first ground. The finiteness, the absence of cycles, and the adequacy of the graphical representation are assumptions of the model. An account of infinite dependence requires different analysis. An account in which grounding is not represented adequately by pairwise edges requires a different formalism.

Even within the finite model, a source vertex is a structural role. Identifying it with a necessary being, a divine creator, or a brute fact requires additional philosophical claims. The graph distinguishes a pattern of dependence; it does not establish the full nature of what occupies each role.

This is a useful example of the hierarchy defended throughout the book. Mathematics makes consequences of a proposed structure explicit. Metaphysical interpretation explains what the structure means and why it applies.

Intelligibility and a Ground

The question of ground also concerns why reality is intelligible. A world can exhibit stable relationships through which measurement, memory, and correction become possible. One can ask whether that order is a primitive feature, a consequence of deeper necessities, or an expression of a theological ground.

A successful mathematical description supplies evidence of some order in the domain it describes. The further inference to an ultimate source of order requires an argument. The inference cannot be made automatic by the beauty of an equation or by the emotional force of a pattern.

At the same time, the question is philosophically legitimate. Explaining how a particular calculation works does not eliminate the broader question of why there is a world in which such calculations

apply. A general metaphysics can ask both questions while distinguishing the kinds of answer they require.

The Theistic Development Adopted Here

The STOEM synthesis adopts a theistic interpretation of the ultimate ground. It understands the interacting physical order as created and sustained by God, and the human person as standing in a spiritual relationship with that ground. This is a substantive metaphysical and theological commitment. The argument should make clear what supports it and what the mathematics alone leaves undecided.

One line of reasoning begins with the distinction between a conditional account of behavior and an account of actuality. An update describes what follows if a state and a law obtain. Adding further updates can extend that explanation without yet explaining why there is an instantiated order to which the condition applies. If contingent actuality calls for a ground beyond a succession of conditional descriptions, a purely dynamical account remains incomplete with respect to that question.

The qualification matters. A critic can reject the demand for such a ground or accept the whole as a primitive fact. The preceding argument identifies the place of disagreement. It does not make the critic contradict a theorem about state spaces. The theistic development accepts the explanatory demand and proposes a ground whose existence is not borrowed from another physical state.

Identifying that ground with God adds further content. A necessary mathematical object, if admitted, is not automatically a personal creator. STOEM's God-Man relationship therefore participates in the theological interpretation rather than emerging as a corollary of necessity alone. Creation, intelligibility, and personal relationship are

considered together in an account that remains open to philosophical criticism.

The strength of this position is its independence from a temporary gap in a physical explanation. It concerns an order that might possess increasingly complete internal laws. The responsibility of the position is to explain what divine grounding means and how it contributes to understanding, rather than merely repeating the word ground. The following chapter develops creation as dependence and examines its relation to physical beginnings.

Why Prefer a Personal Ground?

The positive case is comparative. It joins three features of the subject: contingent actuality, a world accessible to reason, and persons who deliberate about truth and obligation. The proposal of a personal ground is preferred here because it brings those features into one account of dependence. This preference requires more than the observation that physics has unfinished work.

The first premise is that contingent actuality warrants an explanation of its dependence. This is a restricted explanatory principle: where an existing order could have been otherwise, asking why it obtains is legitimate even if its internal transitions are fully described. The book accepts this demand rather than treating the whole order as the stopping point. A critic who accepts a brute world rejects that premise, not an arithmetical consequence.

The second premise is that a complete ground of that order cannot consist solely of further conditions whose actuality is borrowed in the same unexplained way. Adding another contingent state may explain a transition while leaving the dependence of the total account unresolved. The proposal therefore seeks a ground whose existence is not derived from a prior physical state. This is a claim about

dependence, not a deduction that the physical history must be temporally finite.

The third premise concerns what needs grounding. Rational judgment is answerable to truth, and personal action is answerable to reasons that are not identical to immediate advantage. The present metaphysics treats these as real aspects of persons. A causal account of how a belief arose and an assessment of whether its reasons are good answer related but different questions. Explaining the causal history does not by itself eliminate the second question.

The comparative inference is that a ground capable of intellect, intention, and relationship offers a unified account of an intelligible world containing rational and responsible persons. It places their capacity for reason within a reality whose source is itself rational. It places personal address and response within a reality for which relationship has ultimate significance. Personal here denotes these capacities by analogy; it does not posit a larger biological organism alongside the universe.

The inference is abductive: it favors an explanatory interpretation, rather than deducing personality from the existence of a first term. Its force depends on accepting the preceding premises and finding the unification substantial. The advantage claimed for theism is not a shorter equation. It is that the same ground explains why there is an order, why intelligibility belongs to it, and why persons have significance within it.

Alternatives and the Cost of the Preference

The brute-fact position has an economy of commitments: it accepts a world with its laws and inhabitants without an additional divine ground. Its cost, under the explanatory standard adopted here, is that actuality and the place of rational and moral significance receive no common ultimate explanation. This is a philosophical cost only for a reader who accepts the demand for such an explanation. It is not an internal contradiction in the alternative.

An impersonal necessary order answers part of the demand. It may explain why some structure must obtain and how persons emerge within it. The present objection is not that impersonal processes cannot produce cognition; that would prejudge the explanatory work of biology and neuroscience. The question is whether necessity alone accounts for this actual order and for the authority of reasons and obligations. A developed impersonal account could address that question. Merely naming a necessary structure does not yet do so.

A personal ground also incurs obligations. Intention must contribute more than a label for whichever world happened to exist. The account must distinguish purposeful explanation from arbitrary preference and must explain why freedom does not make every proposed history equally intelligible. It must also confront suffering and divine hiddenness. A world containing severe suffering and ambiguous access to God places pressure on claims of perfect goodness and a desire for personal relationship.

The Christian development responds by making love, responsibility, reconciliation, and hope central to the relationship; Chapter 30 examines this response. These commitments supply a direction for explanation and action. They do not establish that every instance of suffering has received an adequate explanation. The personal-ground

proposal is stronger when this difficulty is allowed to count against it, rather than being redescribed as automatic confirmation.

The question who grounds God also deserves a direct answer. The proposal does not exempt one contingent object from a rule imposed on all the others. It distinguishes a ground understood as underived from the dependent order. Whether such a ground is coherent and whether personality can belong to it are substantive questions. Declaring God necessary is a statement of the proposed answer, not a proof of it. The cumulative case concerns whether that answer explains the subject better than the alternatives.

STOEM adopts the personal account because it takes actuality, intelligibility, and personal significance to be connected features of reality. Its warrant is the proposed explanatory unity together with the theological reasons developed next. The infinite fractal construction gives the physical proposal a mathematical form; it neither adds a probability to the existence of God nor substitutes for this philosophical comparison.

The Existence of the Interpreter

The question also returns to the person asking it. An inquirer is physically situated and capable of forming judgments about the order to which that inquirer belongs. STOEM interprets this situation as both participation in a created world and the possibility of personal relationship with God.

The proposal does not erase the work of explaining cognition. It situates that work within a wider question about truth and personal significance. A successful account of neural processes would explain real conditions of human inquiry. The theological interpretation would still ask what grounds the world and the person whose inquiry those processes sustain.

The metaphysical claim is therefore an order of dependence and meaning. Being supplies the actuality of the subject. Its relationships make structured descriptions possible. Mathematics articulates those descriptions and constrains their consequences. Within the theistic development, the existence and intelligibility of that order are interpreted through divine grounding. This is the positive position that the objections must now assess.

CHAPTER 30

Creation, Dependence, and the Physical Beginning

A First Iteration and a Ground

The early STOEM formulation associated divine creation with the first iteration of the universe. The image is understandable: a sequence begins, and the beginning invites a question about its source. Yet the image joins two claims that can be separated. One concerns a first state in a physical history. The other concerns the ground of that history's existence.

A sequence indexed by the nonnegative integers has a first member by construction. A sequence indexed by all integers does not. Both can be represented mathematically. Neither indexing choice, by itself, identifies what makes the represented universe actual. A change of labels does not settle a doctrine of creation.

The theological development in this book understands creation as dependence of the physical order on God. A physical beginning may be part of that order, but the dependence claim is not restricted to explaining a first event. It concerns every existing process and the order within which processes can occur.

This interpretation gives STOEM a more durable theological structure. A revision in physical cosmology need not automatically revise the meaning of creation. What would have to be revised depends on whether the theological claim concerned a particular physical boundary or the existence of the order containing it.

The Christian Setting

The opening of John's Gospel places the origin of created things in relation to the Word. The passage in Colossians concerning the supremacy of the Son connects creation with the continuing coherence of what exists. Read theologically, these passages make dependence more comprehensive than an initial mechanical impulse.

This book draws on that Christian understanding. The Logos is not introduced as a numerical parameter inside the iterative map. The theological claim concerns the ground and significance of the created order, including its intelligibility and the person's relationship with God. Its justification draws on theological interpretation and faith as well as metaphysical reasoning.

Calling a mathematical update Logos would not establish that identification. The formal map and the theological claim have different roles. A mathematical construction can articulate an ordered world; Christian theology interprets creation through the Word and the personal relation disclosed in Christ. Connecting the two requires an argument about dependence and meaning.

This is also why the book does not derive Christian doctrine from a fractal. A fractal can display repeated structure, but repeated structure alone does not imply incarnation, reconciliation, or divine love. The Christian orientation supplies substantive commitments whose sources and content should remain explicit.

Why the Christian Identification Matters

The step from a personal ground to the God disclosed in Christ requires additional premises. General reasoning about dependence does not identify the incarnation or resurrection. The Christian form of STOEM accepts the apostolic witness to Christ as a source of knowledge about the ground's character. This is the point at which a general theistic argument receives a particular theological commitment.

Three features make that commitment relevant to the account developed here. First, John's account identifies the Word with the source of created existence and also with a life entering human history. Intelligibility and personal presence belong to one theological subject. The Logos therefore connects the order investigated by reason with the possibility of address and recognition. The comparison is one of meaning; no mathematical map is being identified with Christ.

Second, the passage in Colossians places the coherence of creation and reconciliation in relation to the same Son. The created order is understood as sustained, while the relationship concerns restoration and peace. This gives the God-Man relation more content than coexistence between two independent domains. It concerns how an embodied person belongs to an order that is both intelligible and the setting of reconciliation.

Third, the Christian claim places suffering within the account through the crucified Christ, and hope within it through the resurrection. Paul makes the resurrection central to the warrant of Christian faith. These claims expose the theological interpretation to historical and testimonial assessment. They cannot be guaranteed by an attractive metaphysical scheme. Their acceptance in this book is explicit, so the reader can distinguish that acceptance from the mathematical results.

The reason for preferring this identification, within the adopted faith, is that it joins origin, intelligibility, personal encounter, suffering, and hope in one account. Its central image of divine relation is not simply control over a system. It includes participation in human vulnerability and the promise of restoration. That changes the meaning of orthogonality: the spiritual relation is distinct in explanatory kind while concerning the very person who suffers and acts physically.

A reader who questions the apostolic testimony may accept the general metaphysics while rejecting this identification. The disagreement then concerns evidence and theological interpretation, not the convergence of the infinite model. A full historical defense of that testimony lies beyond the present mathematical argument. The references identify the passages on which this theological reading depends, rather than concealing them inside a claim of mathematical necessity.

Creation and Scientific Explanation

A scientific explanation can identify how a physical state develops from another state, how structures form, or how several laws are recovered from a deeper physical account. Such explanations concern real features of the created order within the theological reading adopted here.

The completeness of a physical explanation for its intended question does not eliminate the question of why the order exists. Conversely, the assertion of divine grounding does not calculate a particle mass or replace a dynamical law. The two accounts connect through their subjects while answering different explanatory demands.

This relationship permits physical inquiry to advance. If a phenomenon previously regarded as unexplained receives a reliable

physical explanation, the metaphysical dependence claim need not retreat to another unexplained phenomenon. It concerned the existing order throughout. God was not defined as the name of the current gap.

The same point places a discipline on theological language. Claiming that God grounds the order is not enough to explain any chosen physical event as a special act with a particular purpose. That further attribution requires further reasons. A comprehensive theological orientation cannot make every local interpretation equally warranted.

Divine Freedom and the Choice of a World

STOEM's infinitely interacting universe is a proposed created order. Why that order rather than another? One answer appeals to divine freedom and purpose. That answer changes the kind of explanation being offered: an intentional account is proposed rather than a necessity derived from a recurrence relation.

The book does not claim to calculate the divine intention from observed complexity. A broad range of mathematical systems can generate elaborate structures. Complexity therefore cannot uniquely identify a purpose. Nor does the sheer number of imaginable alternatives supply a probability for this world unless a measure and sampling account have been justified.

A theological account can nevertheless argue that an intelligible world capable of sustaining personal relationship has significance. The argument concerns the suitability of an order for forms of life and relationship. It should state where interpretation enters and how alternatives are being compared.

The distinction between necessity and gift also matters. A model can make some consequences necessary once its assumptions are

adopted. It does not follow that its own actuality is necessary. The theological account can understand the actual world as freely created while accepting mathematical necessity within a description of that world.

Suffering and the Limits of a Beautiful Model

An account of created order must include vulnerability, conflict, and suffering. A visually beautiful fractal does not explain why a person suffers. An elegant update rule does not establish the goodness of every history it permits. The difference between formal beauty and moral justification must remain visible.

Within STOEM, an interacting universe explains why processes are mutually implicated, but mutual implication alone supplies no complete theodicy. The presence of lawfulness can support life and also permit injury. The theological question concerns how a good ground of existence is related to those realities and what response is owed to persons who endure them.

The Christian orientation brings resources of lament, responsibility, compassion, and hope to that question. The present construction does not turn those resources into a proof that every particular suffering has been explained. Its immediate ethical consequence is that the person remains a subject of care rather than becoming a disposable detail of a supposedly perfect mathematical pattern.

This is a substantial boundary on cosmological metaphysics. A theory of everything that erases the significance of particular persons to protect its image of the whole has changed the subject. STOEM's God-Man relationship keeps that significance within the architecture of the account.

The Created Order and Its Mathematical Description

The mature proposal can now be stated. The physical universe is investigated as an interacting order whose organization may admit the fractal and iterative structures proposed by STOEM. Its physical laws require explicit realization and recovery. Its existence is interpreted theologically as dependent on God. The embodied person participates in that order and in a God-Man relationship whose mechanism remains undefined.

Mathematics articulates structures within this account and exposes their consequences. Metaphysics examines what it means for those structures to be instantiated and for their subjects to stand in relationships of dependence. Theology supplies a definite interpretation of the ultimate ground and of the personal relationship.

The result has a clear direction without claiming that every question has been resolved. Creation concerns the existing order as a whole and the particular beings within it. Its meaning cannot be confined to the first line of a simulation, and its implications cannot be assessed solely by the appearance of the final image.

CHAPTER 31

Objections and Replies

Is the Hierarchy Circular?

The first objection is that the book uses mathematics to argue that mathematics depends on metaphysics. If the argument already relies on logic and mathematical objects, perhaps it cannot establish any priority over them.

The reply is that explanatory priority need not be a claim to perform the inquiry without the resources being examined. A theory of measurement can use measurements. A study of language can be written in language. The relevant requirement is that the account acknowledge its commitments and avoid deriving a conclusion through an assumption equivalent to that conclusion.

Our formal results do not prove the entire metaphysical thesis. They establish limited facts about preservation and admissibility. The thesis is supported by examining what is required for those facts to concern a subject. Readers can accept the mathematical results while disputing the proposed account of explanatory priority.

That possibility is a strength of the exposition. It makes the point of disagreement identifiable rather than embedding the entire ontology in an opaque formal definition.

Could Interpretation Be Another Mathematical Structure?

A second objection is that J, the interpretation, could itself be formalized. A richer mathematical structure might include the model, its users, their conventions, and their relation to the world. If so, the argument that interpretation requires something further may appear to generate only a sequence of increasingly comprehensive models.

The book allows that possibility. It does not prove that interpretation is beyond every mathematical representation. Its claim, argued through the four premises in Chapter 3, is that an account of actuality requires a warranted identification of its subject. A richer model can represent that identification without making its truth follow solely from internal consistency. Adding a formalized J can be progress if it explains a connection that had previously been left implicit.

The remaining question is whether the enlarged model has established the identity of formal structure and reality, or has supplied a more informative representation. That question must be answered by argument. The existence of a richer model does not settle it merely by construction.

This reply limits the strength of the book's thesis. It defends the need for metaphysical examination of mathematical application. It does not establish a universal theorem that reality is nonmathematical.

Are the Mathematics and Terminology Too Elementary?

The mathematical results are elementary. Relabeling invariance, monotonicity under added constraints, factorization through fibers, quotient constructions, composition, and the behavior of trees and cycles are familiar forms of reasoning.

Their role is to keep the philosophical claims inspectable. A reader can see exactly what follows and where interpretation enters. The infinite constructions and dynamical recovery results introduced for STOEM extend the range of examples. Their proofs remain accessible and their assumptions explicit. More advanced mathematics is justified where it addresses a concrete limitation, such as a physical field limit, probabilistic reduction, or higher-order compatibility.

The book's contribution should therefore be assessed as a philosophical organization of these tools and their applications. The

seven numbered preservation and compatibility results, together with Propositions A through F, are not advertised as previously unknown theorems. A future mathematical advance would need its own comparison with the relevant literature and its own proof.

This standard prevents mathematical sophistication from becoming a substitute for an argument. The goal is to use the mathematics needed to make the issue exact.

Does Choosing K Make Meaning Arbitrary?

The question family K determines the distinctions a faithful representation must preserve. An objection follows immediately: if K is chosen by the investigator, cannot any preferred result be obtained by choosing convenient questions?

An arbitrary choice can indeed produce an uninformative account. The framework makes that vulnerability explicit. It does not claim that every K is justified. The questions must be connected to the subject, the purpose, and the claims being made for the representation.

In the quantum example, adding the X probability question is justified when the account claims to preserve predictions for an X measurement. In the genetic-code example, adding codon identity is justified when the task requires reconstruction of the original triplet. The relevance comes from the stated application, not from the quotient calculation itself.

The most difficult cases concern disputed purposes or disputed ontologies. There the framework identifies the dispute rather than automatically resolving it. That is still progress when a disagreement previously appeared to be a purely numerical one.

Does a Common World Require a Global Assignment?

No. The book explicitly distinguishes a common reality from a particular joint-assignment model. An empty Γ can show that the selected conditions cannot be combined. It does not establish that no coherent account of the subject exists.

The cycle example makes this visible. The network and its obstruction have a complete mathematical description even though no assignment satisfies all three constraints. A quantum theory can likewise provide a coherent account without supplying the specific context-independent assignments demanded by another model.

The metaphysical commitment to a common world requires an account of relationships among contexts. It does not require every possible proposal for their unification to succeed. Recognizing the difference prevents the mathematics of obstruction from being used as a metaphor for universal incoherence.

Does Operational Agreement Settle Ontology?

Agreement on predictions establishes agreement relative to the specified operational questions. It does not automatically settle whether two theories make the same claims about what exists. Equally, different ontological vocabulary does not automatically establish different content if a faithful translation is available.

Theoretical equivalence is therefore a substantive problem. The book's question-family approach provides one limited criterion. It should be compared with richer logical and categorical criteria when the application requires them. It does not claim that every dispute between interpretations can be resolved by calculating a quotient of a finite set.

The useful discipline is to state the comparison standard. If the aim is agreement on experimental distributions, say so. If the aim is preservation of reference or ontology, explain what that means and how it is assessed.

Does an Infinite Model Explain Every Physical Process?

No such conclusion follows from the size of a state space. The infinite constructions give definite updates and selected consequences. A physical theory also requires an interpretation, a measurement account, and recovery of the relevant physical relationships. Increasing the number of coordinates can increase expressive capacity without supplying any of those connections.

The distinction gives the STOEM program a concrete direction. Its proposal of infinite interaction can guide constructions, while the recovery conditions identify what a successful connection with physics would require. The remaining work is substantial. Calling the proposal a theory of everything identifies its intended scope rather than certifying that every physical law has already been derived.

Does the Fractal Proposal Add Anything Essential?

The broad metaphysics could survive the replacement of a fractal realization by another structure, as the early dialogue itself contemplated. That flexibility does not make the fractal proposal empty. It identifies a stronger candidate account within the broader metaphysics, one that must specify where self-similarity or other fractal properties occur.

Proposition D combines countably many Cantor-state components with direct all-to-all dependence in one update. Its proof and finite-prefix bound establish that these mathematical requirements can coexist. The open task is their physical realization: neither the

pairing of digits nor the topology of C^N supplies spacetime, energy, or quantum probabilities by itself. The stronger physical proposal consequently remains a claim that must connect this architecture with the recovery requirements in Chapter 23.

Is the God-Man Relationship Only an Undefined Label?

It would be an empty label if nothing were asserted through it. The book instead gives it positive content: created dependence, personal significance, address, response, and responsibility. It leaves the mechanism undefined. The distinction between asserting a relation and completely explaining its operation is central to the proposal.

A critic can dispute those positive claims. The appropriate disagreement concerns their metaphysical and theological warrant. A missing equation does not by itself settle that disagreement, just as the addition of an equation would not automatically establish a divine relationship. Chapter 29 offers the positive comparison with a brute world and an impersonal necessary order. Chapter 30 identifies the additional testimonial and theological commitments behind the Christian reading. These are the grounds on which the assertion should be assessed.

Does Divine Grounding Make Physics Optional?

The opposite follows from the account's scope. If the physical universe is part of the created order, its processes remain subjects of understanding. Claims about those processes are answerable to physical inquiry. Divine grounding is not a substitute for a missing coupling law or a recovery map.

The early gravity sketches do not become derivations of general relativity through theological affirmation. Nor does a future physical derivation automatically make the ground of actuality a solved

problem. The distinction preserves the explanatory responsibilities of both inquiries within one account of reality.

Does Mathematical Order Prove Divine Goodness?

A coherent or beautiful structure does not by itself establish moral goodness. The creation chapter therefore includes suffering and vulnerability within the subject matter. A theological interpretation owes an account of their significance; a visual pattern supplies no automatic justification for them.

The God-Man relationship also keeps the particular person within view. An account of the whole is inadequate if it makes personal suffering insignificant merely because the aggregate model is elegant. The book's ethical commitments arise through its understanding of persons and responsibility, rather than through an inference from numerical complexity.

What Remains Open?

The book adopts STOEM as a metaphysical proposal with a theistic development. It supplies exact constructions and conditional results while leaving the final physical realization, the full account of consciousness, and the mechanism of the spiritual relationship unresolved. These are identified tasks and limits of explanation.

The commitments are nevertheless substantial: an interacting physical order, the explanatory priority of being and interpretation, the distinction between mathematical representation and actuality, and a God-Man relationship that physical description does not exhaust. The book's adequacy depends on the reasons offered for these commitments and on the coherence of their connections. Its openness does not remove those points of assessment.

CHAPTER 32

A Discipline of Intelligibility

The Unity Achieved

The synthesis began with STOEM's picture of an infinitely interacting universe and an orthogonal God-Man relationship. It joined that picture to an inquiry into being, identity, interpretation, and the preservation of meaningful questions. The resulting account has a definite architecture: the physical order is situated within a wider metaphysics, while mathematical representations articulate selected structures of that order.

The universe proposed by STOEM includes the processes through which observers encounter it. An observer's description is one event within the world and also a claim about the world. That dual position makes interpretation indispensable. The physical existence of a record and the truth of its content are related questions whose distinction the account must preserve.

The God-Man relationship extends the inquiry to personal significance and divine grounding. Its mechanism remains undefined, but its place is explicit. The embodied person participates in the interacting world and in a proposed spiritual relationship that is not exhausted by the world's physical description. The theological development identifies creation as continuing dependence, rather than restricting it to a first update.

What the Mathematics Contributes

The infinite interaction construction shows how infinitely many contributions can produce a defined aggregate and a unique next state. Its truncation estimate connects finite approximation with a declared horizon. The contracting example proves that infinite interaction alone does not entail chaos. The fractal construction supplies a precise account of self-similarity and of the uncertainty retained by a finite prefix. Proposition D then combines fractal component states and all-to-all interaction in one map, with a finite calculation and a truncation guarantee.

The preservation results make a second contribution. They distinguish faithful relabeling from a change of requirements, identify when information is lost, and state the coarsest description adequate for a chosen question family. They show why local compatibility need not yield a joint assignment and why an obstruction must be interpreted before it is attributed to the world.

The recovery results connect these ideas to dynamics. A retained present supports an autonomous next-state law only when all detailed states sharing that present also share the retained future. Approximate agreement carries an error bound whose usefulness depends on the stability and duration of the comparison. This is a precise way to ask how physical laws might arise within a deeper iterative description.

These contributions are conditional. Their philosophical value lies in making dependencies visible. A proof can show what an account must give up when a proposed preservation fails. It can also identify which part of a larger ambition has actually been accomplished.

Physics Gives the Proposal Substance

Physics supplies the world of matter, fields, clocks, interactions, and records in which the metaphysical inquiry is embodied. Conservation makes a relation between change and persistence exact. Gauge freedom distinguishes physical content from redundant description. Relativity clarifies what temporal and spatial comparisons preserve. Thermodynamics explains conditions under which records and organized systems endure.

Quantum theory brings the relationship between state, measurement, and actuality into focus. The worked example distinguishes a coherent state from a mixture through a specified measurement. The interpretation comparison examines Copenhagen-oriented, Everettian, Bohmian, collapse, relational, and QBist commitments across state, outcome, observer, and dynamics. The locality discussion separates mediation, signalling, settings, and temporal dependence, allowing a choice to be assessed through its actual assumptions.

Cosmology extends the inquiry to the physical history containing those processes. Its models connect observations with geometrical and dynamical accounts. STOEM's proposal must engage those connections if it is to develop into a physical realization. The general metaphysics gives that undertaking a setting; the laws and measurements supply its concrete obligations.

The Person Is Included in the Account

A comprehensive metaphysics concerns the beings who live within the order it describes. The genetic code provides one example of a mathematically expressible relationship embodied in biological processes. Consciousness, personal identity, and agency then raise further questions about what it means for an embodied person to know and act.

A preserved output is not automatically a preserved person. A numerical score is not a complete history. A behavioral resemblance does not establish numerical identity. These distinctions have ethical consequences when abstractions guide decisions affecting human lives. The authority of a calculation depends in part on whether it retains the distinctions the decision requires.

STOEM's theological development adds recognition before God and responsibility within a created world. These commitments do not emerge from maximizing complexity. They concern the being and relationships of persons. The mathematical discipline can clarify their representations without claiming to generate their entire meaning.

A Way of Continuing

Further work can proceed through identifiable questions. A candidate coupling can be defined and examined. A proposed physical quantity can acquire a recovery map. A coarse law can be proved under assumptions or bounded over a specified range. A theological interpretation can clarify a relation of dependence or answer an objection about personal significance.

The method also permits revision. If a particular fractal realization fails, the failure has a scope. If a physical prediction conflicts with evidence, the relevant physical commitment must be reconsidered. If an argument for divine grounding is disputed, the disagreement should be addressed through its premises and meaning. Progress consists in improving these connections while making their assumptions visible.

Mathematical metaphysics consequently places mathematics under an inquiry into being without weakening mathematical consequence. It asks what the symbols concern, which relations are actual, what a transformation preserves, and what grounds the existence of the whole. STOEM gives that inquiry a proposed universe and a spiritual orientation. The work of understanding is to make their relationship intelligible, from the precise structure of an iteration to the existence of the person who asks what it means.

Glossary and Notation

Principal Terms

Actuality. The instantiation of a being, event, or configuration, distinguished from its admissibility within a model.

Admissibility. Satisfaction of specified requirements within a representation. The term is conditional on the requirements and their interpretation.

Agency. Action understood through an account of deliberation, reasons, intention, and the conditions in which action occurs.

Common world. A reality in which different beings and perspectives can be related through an intelligible account. It need not admit every proposed joint assignment of values.

Context. A specified setting of access and description, including relevant procedures, vocabulary, scale, or purpose.

Decoherence. Suppression of interference in a reduced quantum description through interactions and correlations with unobserved degrees of freedom.

Effective description. An account intended to preserve selected physical answers over a stated range of scales or conditions.

Entropy. A quantity defined within a specified thermodynamic or statistical account; its meaning and units depend on that account.

Equivalence relation. A reflexive, symmetric, and transitive relation. Its use in an application requires a criterion explaining the kind of sameness represented.

Exact preservation. The recoverability of a selected answer after a transformation of representation. It is expressed by the factorization $q = q^* \circ f$.

Explanatory priority. A proposed order of dependence in an explanation. It need not coincide with temporal order or the order in which knowledge is acquired.

Faithfulness for K. Preservation of every question in the specified family K. Faithfulness is relative to that family.

Fiber. The set of inputs mapped by a function to one output. Combining inputs in a fiber can discard distinctions between them.

Formal admissibility. Satisfaction of the conditions of a mathematical model, distinguished from metaphysical possibility, physical realizability, and actuality.

Gauge freedom. A redundancy in a representation under transformations that preserve the relevant physical content, with boundary and global conditions specified.

Ground. What is proposed to account for a specified existence, identity, or truth. A particular account must state the relevant dependence relation.

Identity. Sameness of a specified kind. Numerical identity, functional equivalence, and observational indistinguishability must be distinguished.

Intelligibility. The availability of warranted distinctions, meaningful reference, and sufficient continuity for comparison or correction.

Interpretation. The account connecting a formal representation with its subject and explaining the meaning and scope of its ingredients.

Invariant. A quantity or relation unchanged by a specified transformation.

Joint assignment. A choice of one local description in each context included in a model.

Locality. A term requiring specification of the relevant spatial or spacetime relationships. Local mediation, no-signalling, and particular probabilistic conditions are distinct commitments.

Macrostate. A description grouping microscopic possibilities according to selected macroscopic quantities.

Metaphysics. The general inquiry into being and its conditions of intelligibility adopted in this book.

Numerical identity. The claim that two references concern one entity, distinguished from similarity or equivalence between distinct entities.

Operational question. A question tied to specified preparations, measurements, observations, or other procedures.

Physical law. A principle constraining physical behavior whose empirical support and metaphysical interpretation must both be examined.

Physics. The investigation of physical being, treated here as a domain within metaphysics.

Preservation requirement. A statement of which answers or distinctions a transformation must retain to serve the intended purpose.

Proper time. Elapsed time along a timelike worldline as determined by the spacetime metric.

Quotient. The set of equivalence classes under a specified equivalence relation. It removes differences the relation does not retain.

Reference. The relation through which an expression or description concerns its intended subject.

Relabeling. A change of names or coordinates. A faithful relabeling transports the relevant relations, operations, and requirements.

Representation. A structure used to describe a subject under an interpretation. Its mathematical existence does not alone establish the physical existence of its subject.

Signalling. Controllable influence on accessible outcomes through a change of setting or action, under an explicitly specified operational account.

Symmetry. A specified transformation preserving a structure or law in a stated sense.

Theoretical equivalence. A relation between theories under a chosen standard of preserved content. Different standards can concern predictions, logical structure, or other commitments.

Vacuum. A state within a physical field theory, not the metaphysical absence of every being or condition.

Symbols Used in the Formal Chapters

Symbol	Meaning
C	Finite nonempty set of contexts.
S_i	Nonempty set of descriptions in context i.
E	Set of specified comparisons.
R_{ij}	Allowed pairs for a comparison.
P, Q	Collections of predicates on complete assignments.
A	The descriptive structure (C, S, E, R).
$\Gamma(A; P)$	The set of joint assignments satisfying all conditions.
J	The interpretation accompanying the model.
X	A set of descriptions under consideration.
K	A family of questions to be preserved.
q	A function assigning an answer to each description.
f	A transformation into another representation.
$f(X)$	The outputs actually attained by f.
q^*	An answer function on the transformed representations.
$\sim_K$	Equality of answers to every question in K.
π	Projection to an equivalence-class representation.

In equations, $\circ$ denotes function composition, $\cup$ denotes union, and $\subseteq$ denotes inclusion. A prime marks a transported or transformed version of an object. Subscripts distinguish contexts or named

measurement outcomes. These conventions are local to the book and do not carry an additional ontology by themselves.

Physical notation is defined within each worked example. In particular, E denotes energy or the electric field in the relevant physical chapter, as distinct from the comparison set E in the formal architecture. A likewise denotes a vector potential in electromagnetism and the descriptive structure in the formal chapters. The symbol kB denotes Boltzmann's constant; H denotes the cosmological expansion rate in Chapter 22.

Terms Introduced in the STOEM Synthesis

All-to-all coupling. A defined dependence in which every component can contribute to every output. It need not have a physical interpretation as instantaneous signalling.

Contraction. A map that decreases distances by a uniform factor smaller than one. The example in Chapter 8 converges to its unique fixed point by a direct estimate.

Created dependence. The theological claim that the existing order depends on God. It is distinguished from a claim that a physical history has a first indexed state.

Fractal realization. A model that specifies where a fractal structure occurs and how it participates in the proposed account. The Cantor construction supplies one exact example.

God-Man relationship. STOEM's proposed spiritual relation between God and the embodied human person. Man is used inclusively for humanity; the mechanism of the relationship remains undefined.

Iteration. Repeated application of an operation. An iteration index supplies an order of updates without automatically supplying physical duration.

Lipschitz bound. A bound on how much a function can expand distances relative to the distances between its inputs.

Orthogonal, metaphysical use. The distinction by which the God-Man relationship is not assumed to be exhausted by physical description. This use does not assert inner-product orthogonality or a new spatial direction.

Recovery map. A function p connecting detailed states with selected descriptions. Its physical application requires an interpretation of the quantities retained.

Reduced dynamics. An update S on retained descriptions. It is exactly recovered when $p \circ T = S \circ p$ holds for the detailed update T.

Self-similarity. A specified relation between a structure and rescaled copies of portions of that structure. Its exact mathematical form is distinguished from approximate scaling over a physical range.

STOEM. Spiritual Theory of Everything Model. In this book, the originating proposal of infinitely interacting fractals, iteration, divine grounding, and an orthogonal God-Man relationship, developed through a distinction between metaphysical commitments and particular mathematical realizations.

Truncation. Replacement of an infinite description by a specified finite one. An error bound states which comparisons the replacement preserves to a declared tolerance.

Guide to the Explicit Results

Result	Location and Content
Proposition A	Chapter 8: the all-to-all update is defined and has a finite-step stability bound.
Proposition B	Chapter 8: an infinitely coupled contraction converges to a unique constant state.
Proposition C	Chapter 9: a shared fractal prefix controls the remaining coordinate difference.
Proposition D	Chapter 9: an all-to-all dynamics on infinitely many fractal component state spaces.
Results 1-5	Chapter 11: relabeling, added requirements, fiber criterion, question quotient, and composition.
Results 6-7	Chapter 12: extension on a finite tree and an obstruction on a cycle.
Proposition E	Chapter 23: the criterion for an exact reduced dynamics.
Proposition F	Chapter 23: propagation of an approximate recovery error.

The state symbols, distances, and updates are defined locally in their respective constructions. X can denote the infinite array space or a general descriptive state space. T denotes the relevant detailed update; U is the contracting example; F is the integrated Cantor-state update in Chapter 9; p and S denote a recovery map and its proposed reduced dynamics. Their repeated use does not identify the separate examples as one physical model.

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The references identify the principal historical comparisons, formal precedents, and physical or biological conventions used in the book. The preservation results, infinite constructions, and dynamical recovery arguments are given in the text. Historical comparisons are limited to the issues explicitly discussed and do not purport to summarize an author's complete philosophy. The STOEM documents are primary sources for the development of the proposal; their inclusion does not endorse every exploratory statement they contain.

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Theological Source and Reading Notes

The Holy Bible, New International Version. 2011 text. Biblica. The Christian account of creation discussed in Chapter 30 refers particularly to John 1:1-18, Colossians 1:15-20, and 1 Corinthians 15:3-19. The passages concern the Word and incarnation, creation and reconciliation, and the resurrection as central to Christian testimony. These passages are discussed without extended quotation. Text consulted through Bible Gateway.

Chapters 1, 8, 9, 13, 23, 27, and 30 reconstruct the early STOEM questions rather than reproducing the original dialogues. The update maps, proofs, approximation bounds, and finite recovery examples in those chapters are explicit constructions for this book. They do not purport to be governing equations already established in the 2023 papers.

The Cantor set and the diagonal pairing of positive integer pairs belong to standard mathematics. Proposition D combines Cantor coding with a specified XOR update to realize all-to-all component dependence. The construction is introduced for this book; no priority claim is made for this class of symbolic systems. The real-coordinate infinite coupling examples use elementary bounds on convergent series. The recovery propositions are applications of factorization through a function and iteration of an error inequality. Their purpose is an inspectable metaphysical and physical interpretation, with no claim that these mathematical forms were first discovered here.