Physics Without Time: Causal Order, Records, and Relational Prediction

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October 6, 2025

What if physics needs no master clock? We develop a framework in which the universe is fundamentally "timeless": there is no global parameter t and no unique integer that counts ticks since any beginning. Instead, the basic data are events, their causal order, and empirically accessible records. Predictions are conditional—P(X in A | R)—and invariant under any reparametrization that merely relabels sequences. Dynamics is encoded by constraints and locality on the order rather than evolution against an external time. Arrows (e.g., thermodynamic) emerge from boundary conditions, not from a universal tick. Geometry is recovered from order and counts; quantum theory appears as consistent (decoherent) histories satisfying constraints C_i |Psi> = 0 with observables read relationally to optional clock subsystems. Standard time-based pictures (Hamiltonians generating t-flows, FLRW cosmic time) re-enter as gauges—useful bookkeepings rather than ontology. We propose calculi for scattering without time, rates as relative counts, and Noether-without-time identities from order symmetries. The program is testable on causal diamonds and finite algebras, and it reproduces familiar results once a gauge (clock) is chosen. Physics, on this view, is about structure (causal order), content (records and correlations), and invariants under change of clock—time becomes a tool, not a thing.

Keywords: timeless physics, causal order, records, conditional probability, consistent histories, constraint quantization, thermal time, refoliation invariance, causal set, algebraic QFT, Noether identities, relational observables, e-fold gauge, boundary conditions, entropy arrow.

A collaboration with GPT-5. 46 pages.

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1. Premise and Payoff

1.1 Why no global t: diffeo/reparam invariance

General covariance (GR).
The equations are invariant under diffeomorphisms: relabelings of spacetime points and slicings. In Hamiltonian form, dynamics is generated by constraints rather than by an external time:

- Hamiltonian constraint: $H(x) = 0$ (all x), with total $H_{tot} = \int N(x) H(x) dx$.
- Momentum (diffeo) constraints: $H_i(x) = 0$.
  Evolution under H_{tot} is a gauge motion (refoliation). No preferred parameter t is singled out: any foliation parameter lambda is a gauge label. Physical statements must be invariant under lambda -> f(lambda) with f strictly monotone.

Canonical quantum gravity (timeless Schr.)
Wheeler–DeWitt equation: $H |\Psi\rangle = 0$. There is no $i \, d/dt |\Psi\rangle = H |\Psi\rangle$ with an external t. Physical amplitudes are extracted from constraints and inner products, not from "evolution in t".

Ordinary mechanics, reparametrized.
Even Newtonian systems can be written with a constraint:

- Lagrange multiplier e(s) enforces reparam invariance of a curve q(s):
  $S = \int ds \, [(m/2e) (dq/ds)^2 - e V(q)]$.
- The primary constraint is $H(q,p) = E$ (constant). Reparam s -> f(s) leaves physics unchanged. Thus t is a convenient gauge, not ontology.

Payoff.
If physics is invariant under reparametrization, then any global time variable is a *choice of gauge*. Insisting on one breaks a symmetry without adding observable content.

1.2 What remains: order, records, correlations

Causal order.
A partial order (E, <=) encodes which events can influence which:

- x <= y means y lies in the causal future of x.
- The order is invariant; it survives any refoliation.
- In strongly causal spacetimes, the order plus a volume measure fixes the metric up to conformal factor (motivation for treating order as primary).

Records (facts).
A *record* R is a proposition about histories, e.g. “a detector in region O clicked with momentum in K”. Formally: a sigma-algebra F over histories H; records are measurable sets R in F. Records are localized (tied to regions or suborders), invariantly meaningful, and empirically accessible.

Correlations.
Predictions are conditional: P(X in A | R). Here X is an observable (query on records), R a condition (preparation, boundary fact). Without a global t, physics provides:

- P(outcome set | preparation set) on causally separated regions,
- Scattering-like kernels between “in” and “out” records,
- Typicality/entropy statements comparing multiplicities of compatible records.

Local invariants.

- Proper time along a chosen worldline (once a worldline is specified) is invariant but local.
- Conserved charges arise from automorphisms of the order/record structure (Noether-without-time): if a transformation g leaves (E,<=,F,P) invariant, the induced Ward identities constrain correlators.

Arrows from boundary data, not from a clock.
Low-entropy or retarded boundary records define a direction (earlier/later) *within the partial order* without supplying a universal tick.

1.3 How predictions survive without t

Replace evolution by conditionals.
Standard: “Given state at time $t0$, predict state at time $t1$.”
Timeless: “Given record R_{in} on region L, predict probabilities for record R_{out} on region R.”
The predictive object is a conditional kernel

- S(b | a) := P(B=b | A=a, $R_{context}$),
  which plays the S-matrix role on a causal diamond. No intermediate global parameter is required.

Constraint-first formulation.
Classical: physical trajectories satisfy constraints $C_i=0$. Quantum: $C_i |\Psi\rangle = 0$. Observables are Dirac observables (commute with constraints, up to constraints). Probabilities arise from consistent histories:

- Assign weights w(alpha) = Tr(C_alpha rho C_alpha^dagger),
- Choose a decoherent set {alpha} with Re D(alpha,beta)=0 for alpha != beta,
- Then p(alpha)=w(alpha) obeys Kolmogorov rules.
  No external time variable appears; consistency is checked on the order of regions.

Relational readout when a clock is convenient.
When useful, select a *clock subsystem* C with pointer T and compute

- E[A | T=t] := <Psi| P_T(t) A P_T(t) |Psi> / <Psi| P_T(t) |Psi>.
  Different clock choices are gauge-related; physical claims must be invariant under switching C. The clock is a tool to *index* correlations, not a structure the theory needs.

Rates as ratios of counts, not derivatives in t.
Define a rate by a limit of relative event counts across nested records {R_n}:

- rate_X|Y := lim_{n->inf} E[N_X(R_n) / N_Y(R_n)].
  Examples: decays per emitted photons, periapses per occultation, scatter hits per preparations. This replaces dX/dt with count ratios anchored in the causal order.

Geometry and fields via nets, not flows.
Algebraic QFT assigns algebras A(O) to regions O with isotony and microcausality. Predictions are expectation values and conditional probabilities tied to inclusions O1 subset O2 and spacelike separation—not to a global time flow. Thermal behavior is captured by modular (KMS) structure defined by the state itself (thermal time), avoiding external t.

Operational recipe (usable today).

1. Specify a causal region pair (L,R) and the admissible records R_in on L, R_out on R.
2. Specify constraints and symmetries (C_i=0, automorphism group G).
3. Compute conditional kernel S(b|a) respecting microcausality and constraints.
4. Validate invariance under refoliation/clock changes (gauge audit).
5. Compare with standard time-based predictions by *choosing* a clock afterward; the numbers must match.

Payoff.

- Conceptual: preserves general covariance; eliminates spurious dependence on arbitrary parameters.

- Practical: fits naturally with scattering, cosmology-as-gauge, and modular/thermal techniques.
- Testable: conditional kernels on causal diamonds, clock-gauge invariance checks, and count-ratio "rates" yield empirically confrontable predictions—without ever introducing a global t.

2. Primitives and Axioms

2.1 Events E and partial order <=

Definition (events and order).

- Let E be a set of elementary events.
- A causal structure is a partial order <= on E with:
  (i) reflexive: x <= x; (ii) antisymmetric: x <= y and y <= x implies x = y;
  (iii) transitive: x <= y <= z implies x <= z.
  Write x < y if x <= y and x != y. The *cover relation* x -> y means x < y and there is no z with x < z < y.

Causal past/future.
Past(x) := { y in E : y <= x }, Future(x) := { y in E : x <= y }.
For two events, the *Alexandrov interval* (causal diamond) is
A(x,y) := { z in E : x < z < y }.

Local finiteness (optional, discrete models).
A causal set assumes: for any x < y, the interval A(x,y) is finite. This yields countable order statistics and well-defined sums over intervals.

Regions.
A *region* O subset E is *causally convex* if x,z in O and x < y < z imply y in O. Regions are the basic "laboratories."

Comments.

- No global coordinate or time label is assumed.
- In continuum GR, strong causality plus a volume measure reconstructs the metric up to a conformal factor; here we stay with order as primary.

2.2 Records: sigma-algebra over histories

Histories.
A *history* H is a downward-closed subset of E: x in H and y <= x implies y in H. Intuition: a history contains every cause of any event it contains.

Record algebra.
Let Omega be the set of admissible histories. A *record* is a measurable subset R subset Omega. We equip Omega with a sigma-algebra F generated by *cylinder events*:

- Choose a finite region O and a finite set of *local predicates* on O (e.g., "detector at u in O clicked with property in set K").
- The cylinder event asserts those predicates hold on O, unconstrained elsewhere.
  Finite unions, intersections, and complements of such cylinders generate F.

Probability and conditionals.
A state of knowledge is a probability measure P on (Omega,F). Predictions are **conditional**: P(X in A | R), where X is a random variable (measurable map on Omega) and R in F is a record (preparation/selection).

Localization.
A record R is *localized* to a region O if it is generated by predicates on O only. We write R @ O.

Consistency (quantum option).
In a quantum implementation (consistent/decoherent histories), one assigns complex weights to fine-grained histories and selects a *consistent set* where interference terms vanish; probabilities are then ordinary measures on that set. In this section we keep the classical probability notation, with quantum structure introduced later.

2.3 Axioms: causality, composition, reparam invariance

We postulate three structural axioms that *do not* invoke a global time parameter.

Axiom C (causality / no superluminal influence).
If a localized intervention I @ O_I and a query Y @ O_Y satisfy that every event of O_I is spacelike to every event of O_Y (i.e., neither O_I subset Past(O_Y) nor O_Y subset Past(O_I)), then for any record R supported outside Future(O_I) we have
P(Y | do(I), R) = P(Y | R).
Intuition: changing mechanisms in O_I cannot alter statistics in a region that cannot be causally reached by O_I.

Axiom M (Markov composition on the order).
Let x < y < z be events and let O_mid contain the interval A(x,z). For localized observables X @ Past(x), Y @ O_mid, Z @ Future(z), we impose *screening-off*:
P(Z | X, Y, R) = P(Z | Y, R)
for any record R supported outside Future(x).
Intuition: conditional on the “middle” region that intercepts all causal chains from X to Z, there is no further influence from earlier data.

Axiom R (reparametrization invariance).
Any relabeling of event indices or foliation parameters that leaves the partial order <= and the measurable structure (Omega,F) invariant is a *gauge*. Physical predictions (conditional probabilities of records) are invariant under such relabelings.
Consequences:

- No preferred global time label is meaningful.
- “Evolution” pictures obtained by choosing a slicing are gauge presentations of the same conditionals.

Remarks.

- Axiom C encodes microcausality at the level of probabilities.
- Axiom M is the timeless analog of a local semigroup/Markov property.
- Axiom R formalizes the “no global t” premise: equivalence under refoliation and reindexing is required.

2.4 Noether-without-time: order automorphisms -> Ward identities

Symmetry as automorphism.
Let G be a group acting on (E, <=, F, P) by measurable bijections g : E -> E such that:
(i) order is preserved: x <= y iff g x <= g y;
(ii) g maps localized predicates to localized predicates (so it induces g_* : F -> F);
(iii) the state is invariant: P(g_* R) = P(R) for all R in F.
Then G is a symmetry of the timeless theory.

Ward identities (probabilistic form).
For any integrable observable A (measurable function on Omega) and any record R, invariance implies
E[A | R] = E[A^g | R^g],
where A^g(omega) := A(g^{-1} omega) and R^g := g_* R. If in addition R is invariant under g (i.e.,

R^g = R), then
E[A | R] = E[A^g | R].
For a connected Lie group with generator Q (infinitesimal action), differentiating at the identity yields the **Ward identity**
E[delta_Q A | R] = 0 for all A and all R invariant under the symmetry,
where delta_Q is the Lie derivative of A under the induced action.

Conservation on cuts (discrete version).
Let Sigma be a *cut* (antichain) intersecting every past-to-future chain once. Suppose the symmetry current assigns to each event e a local contribution j(e) such that under coarse-graining the total charge on any cut is
Q(Sigma) := sum_{e in Past(Sigma) \setminus Past(Sigma_minus)} j(e),
where Sigma_minus is a neighboring cut "just before" Sigma. If the symmetry is exact and action densities are local in the order, **cut-independence** follows:
Q(Sigma_1) = Q(Sigma_2)
for any two cuts enclosing the same finite region. This is the timeless analog of a conserved charge: invariance under order automorphisms forces equal totals on homologous cuts, hence a divergence-free current on the Hasse diagram.

Examples.

- *Internal U(1).* Rotations in a complex field's phase that act locally on predicates "charge in O equals k" produce charge-conservation Ward identities without invoking a time flow.
- *Spacetime isometries (continuum gauge).* If the order arises from a causal spacetime admitting a Killing symmetry, the induced automorphism on records yields the standard momentum/energy Ward identities; "energy conservation" is conservation across cuts, not "constancy in time."

Takeaway.
Noether's logic survives without a global time parameter: symmetries are automorphisms of the ordered record structure, and they enforce linear constraints (Ward identities) and conserved cut-sums—fully timeless statements that reduce to the textbook conservation laws once a time gauge is chosen.

3. Calculi for Timeless Prediction

3.1 Conditional dynamics: P(X | R), optional clocks as gauges

Primitive rule.
All predictive claims are conditional probabilities on the record sigma-algebra:

- Query (observable) X: Omega -> Xspace (measurable).
- Record (condition) R in F (preparation/selection/local facts).
- Prediction: P(X in A | R) for measurable A.

Causal placement.
When R is localized in a region L and X is localized in a region R that is to the causal future of L (in the partial order), P(X|R) plays the role of "dynamics from L to R" without any t.

Scattering kernel on a diamond.
Choose a causal diamond D with "in" side L and "out" side R. Let A,B be finite partitions of admissible records on L and R. Define the kernel

- S(b | a) := P(B=b | A=a, R_ctx),
  where R_ctx contains any auxiliary setup facts outside Future(L). S is row-stochastic in b for each a and composes on nested diamonds (Markov on order).

Optional clocks as gauges (relational readout).
Pick a subsystem C with pointer T (a random variable on Omega). Relational expectations:

- E[A | T=t, R] := E[A * 1_{T in dt} | R] / P(T in dt | R),
  implemented by projectors P_T(t) in quantum models or indicator events classically. Different choices (C,T) that satisfy "good clock" conditions (monotone support in chains, weak back-reaction) are *gauge choices*: any clock-invariant statement (i.e., that marginalizes over T or compares at equal T values across clocks) must agree.

Clock-switching audit (invariance check).
For two clocks (C1,T1), (C2,T2), verify for any localized A:

- Integrate out the clock: E[A | R] = integral E[A | T_k=t, R] dP(T_k=t | R), k=1,2.
  Agreement certifies that clock choice is a gauge of presentation, not of content.

Algorithmic recipe (practical).

1. Specify L,R and admissible partitions A,B.
2. Estimate S(b|a) from data or theory under constraints (causality, symmetries).

3. If a clock is convenient, condition on T=t to *index* S, but report S(b|a) and any clock-invariant marginals.

3.2 Constraint algebra: C_i = 0 (classical/quantum)

Classical (reparametrization-invariant form).
Phase space Gamma with first-class constraints C_i = 0:

- Poisson algebra: {C_i, C_j} = f_{ij}^k C_k.
- Observables O satisfy {O, C_i} approx 0 (weakly), i.e., invariance under gauge flows generated by C_i.
- Dynamics = relational statements among gauge-invariant O's; no external Hamiltonian flow in t is needed.

Examples (timeless rewriting).

- Reparametrized particle: constraint H(q,p) - E = 0; angle/action variables (theta,J) with {theta,J}=1 and J conserved by the constraint encode all predictions.
- GR (ADM): Hamiltonian and momentum constraints H(x)=0, H_i(x)=0 generate refoliations/diffeos; Dirac observables commute with both (weakly).

Quantum (Dirac/BRST/Histories).

- Dirac quantization: promote C_i -> operators; physical states satisfy C_i |Psi> = 0.
- Relational expectation with an optional clock T:
  E[A | T=t] = <Psi| P_T(t) A P_T(t) |Psi> / <Psi| P_T(t) |Psi>.
- Consistent histories: class operators C_alpha for a history alpha (sequence of localized predicates). Weights
  w(alpha) = Tr(C_alpha rho C_alpha^\dagger).
  On a consistent set (off-diagonal decoherence functional vanishes), p(alpha)=w(alpha) defines standard probabilities—again, with no external t.

Ward identities from constraints (timeless Noether).
Let G be a symmetry acting by automorphisms on records/algebras and commuting with constraints (classically: {C_i, Q_G} approx 0; quantum: [C_i, Q_G] approx 0). Then expectation values satisfy linear identities (Ward):

- E[delta_G A | R] = 0 whenever R is G-invariant, providing conserved "cut-sums" across antichains (Sec. 2.4).

Operational checklist.

- Never introduce an evolution parameter.
- Verify observables commute with constraints (weakly/physically).
- Compute probabilities via conditionals or consistent histories.
- Use symmetries to reduce kernels and derive conservation relations.

3.3 Rates as relative counts: rate_X|Y := lim E[N_X / N_Y]

Motivation.
Derivatives dX/dt presuppose a clock t. Replace with *ratios of counts* across a growing family of records, eliminating t while retaining operational meaning.

Filtration by records.
Choose an increasing sequence of records {R_n} with R_n subset R_{n+1} that “expands” along the causal order (e.g., larger diamonds). For an event-type X define N_X(R_n) = count of occurrences of X within R_n. Similarly for Y.

Definition (timeless rate).
rate_X|Y := lim_{n->inf} E[N_X(R_n) / N_Y(R_n)],
when the limit exists and denominators are nonzero eventually. This is a Radon–Nikodym-like density of X relative to Y on the directed family {R_n}.

Interpretations and examples.

- Decay channel fraction: X = “decay A products observed”, Y = “all decays”. The rate is the branching ratio; no t required.
- Scattering yield: X = “hits in detector window B”, Y = “preparations of beam mode a”; the rate is S(B=b | A=a).
- Orbital mechanics: X = “periapsis crossings”, Y = “occultations”; the ratio gives periapses per occultation (a dimensionless “frequency”).
- Spin echoes / interferometry: X = “echo events”, Y = “pulse pairs”; yields echo efficiency independent of an external clock.

Existence (ergodic/LLN rationale).
Under mild mixing/ergodicity on the directed family {R_n}, the strong law gives almost-sure convergence of N_X(R_n)/N_Y(R_n) to a constant; ensemble expectations then match typical large-n samples.

Finite-sample estimation (practice).
Given data on a large R_N:

- Estimator: r_hat = N_X(R_N) / N_Y(R_N).
- Uncertainty: if events partition Y into K outcomes, use multinomial variance; for two-way counts use binomial CI (e.g., Wilson/Agresti–Coull). No t appears anywhere.

Composition and locality.
If {R_n} is built by concatenating causally adjacent diamonds and Axiom M (composition) holds, then rates factorize or compose exactly as time-based rates would, but via order-gluing:

- rate_X|Y on R_{n+m} = convex combo of rates on R_n and the shifted family that covers the later diamond.

Relation to time if a clock is later chosen.
If you *later* adopt a clock T and define Y as "ticks of T," then rate_X|Y coincides with the conventional time-rate:

- With Y = "T increments by 1," rate_X|Y = lim E[N_X / N_T] = dE[X]/dT in the continuum limit.
  Thus the timeless rate reduces to dX/dt only after you pick t; otherwise it remains a pure ratio of counts.

Failure modes and guards.

- Choice of filtration matters: pick {R_n} that respects causal expansion (avoid pathological "saw-tooth" families).
- Nonergodic mixtures: rates may be context-dependent; report R_ctx explicitly.
- Vanishing denominators: enforce support conditions ensuring P(N_Y>0) -> 1 as n->inf.

Takeaway.
Rates as relative counts provide a fully operational replacement for derivatives in t. They compose locally on the order, admit clean statistical estimation, and collapse to standard rates only if (and when) a clock is later introduced as a gauge.

4. Geometry From Order

4.1 Order + volume -> metric up to conformal

Idea. In well-behaved (strongly causal) spacetimes, the causal order determines lightcones and hence the metric **up to a position-dependent scale**; adding a volume measure fixes that scale.

Causal structure fixes conformal class.
If two Lorentzian metrics g and g' induce the **same** chronological relation (same set of pairs (x,y) with x<y), then g' = Omega^2 g for some positive function Omega. Intuition: the order tells you which directions are timelike/null/spacelike, i.e., it fixes lightcones; only the local “ruler” along those directions (overall scale) is missing.

Add volume to fix scale.
Give the event set E both the order <= and a volume measure mu (counting measure in discrete models, spacetime volume in continuum). Matching volumes of small Alexandrov intervals A(x,y) = { z : x<z<y } determines Omega(x). In Minkowski d dimensions, Vol(A(x,y)) is a known function of the timelike distance d_t(x,y); in curved spacetimes, small-interval asymptotics calibrate the local scale.

Operational recipe.

1. Reconstruct lightcones from order.
2. For nearby x<y, read mu(A(x,y)).
3. Use the known A(x,y) volume expansion to solve for the conformal factor and timelike distance locally.
   Thus (order + volume) -> full metric information, no global t needed.

4.2 Nets of algebras for regions (isotony, microcausality)

Regions as primitives.
Instead of fields at points and their time evolution, assign observable algebras to **causally convex regions**.

Net axioms (algebraic QFT, time-free form).

- *Isotony:* if O1 subset O2 then A(O1) subset A(O2).
- *Microcausality (locality):* if O1 and O2 are spacelike, then all elements commute: [A, B] = 0 for A in A(O1), B in A(O2).
- *Covariance:* order/geometry automorphisms map A(O) to A(gO).
- *Additivity/Haag duality (optional):* algebra of a region equals the algebra generated by slightly smaller subregions; complements behave dually.

States and predictions.
A state is a positive linear functional omega on the global C*-algebra generated by the net. All

measurable claims are **regional** expectations and conditionals, e.g. omega(B | A in A(L)) as in Sec. 3.1. No global time flow is assumed; when needed, "time" appears as a **chosen** 1-parameter automorphism (gauge) or via modular theory.

Thermal time as intrinsic flow.
Given (A, omega), Tomita–Takesaki modular theory provides a canonical automorphism group sigma_s (the "thermal time" flow). KMS equilibrium is defined relative to sigma_s without introducing an external t. Thus dynamics can be read from (state + net), not from a background clock.

Payoff.
Causality lives in the **partial order of regions** (isotony, spacelike commutation). Conservation/Ward identities are symmetry statements on the net. Any standard t-evolution is merely a presentation choice compatible with the same net.

4.3 Causal sets: order invariants and discrete operators

Discrete geometry by order.
A causal set (causet) is a locally finite poset (E,<=). Spacetime volume = **cardinality**; geometry is encoded in order plus counts. A faithful continuum approximation arises by Poisson "sprinkling" points into a Lorentzian manifold: order from causal relation, counts from volume.

Order invariants (dimension and curvature probes).

- *Myrheim–Meyer dimension:* estimate spacetime dimension d by comparing the number of related pairs to total pairs in a large interval.
- *Height/width statistics:* longest chain length ~ timelike distance; largest antichain size probes spacelike "area."
- *Interval abundances:* counts N_k of sub-intervals with k elements have known expectations in Minkowski_d; deviations diagnose curvature.
- *Link distribution:* number of links (covering relations) encodes local lightcone structure.

Discrete differential operators.

- *Benincasa–Dowker d'Alembertian:* a linear combination of field values on layers of the past of x approximates the continuum wave operator:
  (Box_phi)(x) ≈ a_0 phi(x) + sum_{k=1}^K a_k sum_{y in L_k(x)} phi(y),
  where L_k(x) is the k-th layer in the causal past of x and coefficients a_k depend on dimension.

- *Retarded Green functions:* built from order sums over chains; support is automatically causal.

Dynamics and action (timeless form).

- *Sorkin action:* a nonlocal but order-finite expression built from interval counts approximates the Einstein–Hilbert action after sprinkling. Variation yields equations of motion without invoking a time slicing.
- *Quantum on causal sets:* define amplitudes or measures on histories as functions of order invariants; enforce microcausality by construction.

Why this fits our program.
Causal sets realize “geometry from order” literally: no coordinates, no global clock. Predictions are count-based, causal by design, and recover continuum physics by law of large numbers. Any t you later introduce is a derived gauge for convenience, not an input to the theory.

5. Quantum Without Time

5.1 Dirac observables and constraints: C_i |Psi> = 0

Kinematics.

- Start with a kinematical Hilbert space H_kin.
- Promote first-class constraints to self-adjoint (or symmetric) operators { C_i } on H_kin.
- *Physical states* solve the constraints: C_i |Psi_phys> = 0 for all i. The space of solutions (modulo zero-norm) is H_phys.

Observables (Dirac).

- A (quantum) Dirac observable O must commute with all constraints on H_phys:
  [O, C_i] |Psi_phys> = 0 (or weakly, in a rigging/RAQ sense).
- Matrix elements are defined on H_phys via group-averaging / rigging map:
  eta: H_kin -> H_phys,
  <Phi_phys | O | Psi_phys> := <eta(Phi) | O | eta(Psi)>.

Group averaging (RAQ sketch).
If constraints generate a unitary rep U(g) = exp(i g^i C_i) of a Lie group G, define

eta(|psi>) ~ ∫_G dg U(g) |psi>

(formally) and physical inner product by

$$\langle \eta(\phi) \mid \eta(\psi) \rangle \sim \int_G dg \, \langle \phi \mid U(g) \mid \psi \rangle.$$

This quotients out the gauge generated by the constraints (no external t).

Examples.

- *Parametrized particle:* single constraint $C = H(q,p) - E = 0$; energy shell defines H_{phys}.
- *Minisuperspace (Wheeler–DeWitt):* $C = H_{grav} + H_{matt} = 0$; physical states solve $C |\Psi\rangle=0$ without an $i\, d/dt$ term; observables are relational (Sec. 5.3).

Ward identities from constraints.
If symmetry charges Q_a commute with constraints ($[Q_a, C_i] \approx 0$), then on H_{phys}:

$$\langle O \rangle_{phys} = \langle e^{i \epsilon Q_a} O e^{-i \epsilon Q_a} \rangle_{phys}$$

$$\Rightarrow \langle \delta_{Q_a} O \rangle_{phys} = 0$$

(time-free Noether).

5.2 Consistent histories: $w(\alpha)$, consistency conditions

Histories as projectors.
Choose a *coarse graining* (finite). A history α is a sequence of Heisenberg-picture projectors localized to causally ordered regions:

$$\alpha = (P_1, P_2, \ldots, P_n) \quad \text{with} \quad P_k \in A(O_k), \quad O_1 < O_2 < \ldots < O_n.$$

("$<$" uses the causal order; no global t required.)

Class operator.
Given a dynamics specified by constraints (or by a path-integral kernel on a causal diamond), define the class operator C_α (composition respects causal order):

$$C_\alpha := P_n \ldots P_2 P_1$$

(For continuum QFT, insert appropriate covariant propagators restricted to causal support; in a constraint picture, C_α is a gauge-invariant map on H_{kin} that descends to H_{phys}.)

Decoherence functional.
For initial record/state ρ (on the appropriate algebra or H_{phys}):

$$D(\alpha, \beta) := \mathrm{Tr}(C_\alpha \rho C_\beta^\dagger).$$

Consistency (decoherence) condition.
A set of histories $\{\alpha\}$ is *consistent* if off-diagonal interferences vanish:

Re D(alpha, beta) = 0   for alpha != beta.

Then the *weights*

w(alpha) := D(alpha, alpha)

form a bona fide probability measure:

p(alpha) = w(alpha), p(alpha) >= 0, sum_alpha p(alpha) = 1.

Causal locality.
If two histories differ only in spacelike-separated regions with commuting algebras, factorization properties simplify D(alpha, beta); locality (microcausality) promotes consistency for natural coarse grainings.

Timeless reading.
The entire construction uses only: (i) causal placement of regions, (ii) projectors tied to records, (iii) constraint-respecting composition. No external time parameter appears; ordering is by causality, not by t.

5.3 Relational expectations: E[A | T = t] and clock-gauge invariance

Optional clock subsystem.
Pick a subsystem C that will *serve as a clock* when convenient. Let T be a pointer observable (self-adjoint) of C. We read other observables *relative to* T.

Relational expectation (Dirac/Page–Wootters form).
For a physical state |Psi_phys> and spectral projectors P_T(t):

E[A | T = t] := <Psi_phys | P_T(t) A P_T(t) | Psi_phys> / <Psi_phys | P_T(t) | Psi_phys>.

- *Classical analog:* conditional expectation on the record sigma-algebra:
  E[A | T in dt, R] = E[A 1_{T in dt} | R] / P(T in dt | R).

Good-clock conditions (pragmatic).

- *Monotone support on chains:* for almost all histories, T has non-recurrent support along increasing causal chains (prevents multi-valued “times”).
- *Weak back-reaction:* correlations with A do not spoil consistency of the coarse graining by T.
- *Sufficient spread:* the POVM of T resolves the conditioning sets with acceptable precision.

Clock-gauge invariance.
Given two admissible clocks ($C1$, $T1$) and ($C2$, $T2$), physically meaningful predictions must agree once clock information is marginalized or mapped:

1. *Marginal agreement:*

$$E[A] = \int E[A \mid T_k = t] \, dP(T_k = t) \quad \text{for } k = 1,2.$$

2. *Relational map (if a functional relation holds):* if there exists a monotone map f such that (on support) $T2 = f(T1)$, then

$$E[A \mid T2 = u] = E[A \mid T1 = f^{-1}(u)].$$

These equalities are our **gauge audits**: changing clocks changes *labels*, not *content*.

Back-reaction and conditional unitaries.
In Page–Wootters models, conditioning on $T=t$ induces an effective Schr. evolution for the rest, generated by a conditional Hamiltonian. Different clocks give unitarily equivalent families provided they are related by a constraint-preserving canonical/unitary transform (gauge).

Failure modes (and diagnostics).

- *Bad clocks:* non-monotone or highly entangling T can produce context-dependent conditionals.
- *Inconsistent coarse grainings:* if conditioning on T breaks consistency of the history set, $E[A \mid T=t]$ is not probabilistically well-defined. Remedy: coarsen T or refine projectors to restore consistency.
- *Anomalies:* if $[T, C_i]$ fails to vanish (weakly), clock choice may not be a gauge; detect via violation of the marginal agreement test above.

Operational recipe.

1. Work on H_{phys} (solve constraints first or use RAQ).
2. Choose an admissible clock (C,T) only to *index* correlators.
3. Compute $E[A \mid T=t]$; verify marginal/gauge invariance.
4. Report clock-invariant quantities (e.g., integrals over t, scattering kernels on causal diamonds).
5. If a conventional time picture is later desired, *declare* that clock as a gauge and translate.

Takeaway.
Quantum predictions require neither an external t nor a universal tick. Constraints define the

physical subspace; probabilities arise from consistent histories; when helpful, clocks are *relational gauges* used to index the same invariant content.

6. Worked Examples (Timeless Mechanics)

6.1 Harmonic oscillator on H = E surface; action J = (1/2pi)∮ p dq

Constraint form.
Take the 1D HO with mass m, frequency w. Impose the energy-shell constraint
H(q,p) := p^2/(2m) + (1/2) m w^2 q^2 - E = 0.
Physical motion is the closed curve H=0 in phase space; no t appears.

Action-angle without time.
Define the action
J := (1/(2pi)) * ∮ p dq.
On H=E, p(q) = +/- sqrt(2mE - m^2 w^2 q^2). Compute
J = E/w.
The conjugate angle theta is defined abstractly by {theta, J} = 1 (classical) or as a phase variable on the ellipse. Predictions are functions of (J, theta) with J fixed by the record "energy E".

Relational correlators.
Choose any admissible *clock* C only for indexing, e.g., T := theta_C (the angle of a second oscillator weakly coupled or merely co-measured). The timeless correlator of two queries Q1,Q2 localized on the energy shell is
E[Q2 | Q1, H=0]
and if one insists on conditioning by a clock,
E[Q2 | T = t, Q1, H=0],
but the physical content is the H=0 geometry plus the measure induced by the record.

Quantum spectrum without t (Bohr–Sommerfeld).
Quantize by the condition
∮ p dq = 2pi h (n + 1/2) => J = h (n + 1/2), E_n = h w (n + 1/2).
No time evolution was invoked; the spectrum is fixed by the closed-orbit area.

Timeless observable examples.

- Quadrature ratio at two phase-angles: for an invariant distribution over theta,
  E[q^2 | H=E] = E/(m w^2), E[p^2 | H=E] = m E.
- Rate as relative counts: let X = "crossing at q=0 with p>0", Y = "crossing at p=0 with q>0".
  On H=E, symmetry gives rate_X|Y = 1 (no derivative in t used).

Takeaway.
All standard HO facts follow from (i) the constraint $H=0$, (ii) the area J, and (iii) symmetry of the uniform theta measure; any t-parameter is a later gauge.

6.2 Free scattering as boundary conditionals S(b | a)

Setup on a causal diamond.
Let L and R be spacelike-separated "in" and "out" boundaries of a large Alexandrov diamond D in 3+1D. Prepare *records* A on L (incoming wave modes) and read *records* B on R (detector outcomes). No global t is introduced.

Kernel definition (timeless S-matrix).
Partition admissible records on L into {a} and on R into {b}. Define
$S(b \mid a) := P(B=b \mid A=a, R_{ctx})$,
where R_ctx encodes apparatus placement and excludes facts in Future(L). S is row-stochastic in b for each fixed a and composes when diamonds are concatenated (Markov on the order).

Computation (field theory view).
For a scalar field with interaction V(phi), compute boundary-to-boundary kernels via covariant path integrals restricted to D:
$K[\phi_R, \phi_L] = \int_{\phi|_L = \phi_L}^{\phi|_R = \phi_R} D\phi \exp(i S_D[\phi])$.
Then map source/measurement functionals on L,R to discrete outcomes a,b and form S(b|a) by Born-rule conditionals. Microcausality ensures that only data in Past(R) influence B.

Example: 2-body elastic scattering (sketch).

- A := "one-particle wave packet with momentum k_in in channel c entering through L aperture".
- B := "detector click in solid angle bin DeltaOmega around k_out, channel c' ".
  Then
  $S(b|a) \sim |M(c',k_{out}; c,k_{in})|^2 \cdot \Phi$,
  with Phi a flux/acceptance factor determined by geometry of L,R. The usual differential cross section d sigma/d Omega is recovered *after* choosing a flux clock (e.g., count of prepared packets), but S(b|a) itself is clock-free.

Rates as relative counts (operational).
Let Y = "number of prepared a's", X = "number of b's registered". The timeless estimate is
$rate_{X|Y} = N_b / N_a \to S(b|a)$ (large-sample limit),
with multinomial uncertainties; no dt or beam time appears.

Composition law (two diamonds).
If D = D1 followed by D2 (in causal order) and we partition the intermediate boundary by {m}, then
$S_D(b|a) = \sum_m S_{D2}(b|m) * S_{D1}(m|a)$,
a purely stochastic composition mirroring time-ordered evolution but expressed as order-gluing.

Takeaway.
Scattering is naturally a boundary-conditional problem. All predictions come from $S(b|a)$ on causal regions; "time evolution" is just one way to parametrize the same conditionals.

6.3 Kepler: conic constants and periapse/occultation counts

Constraint form (reduced 2-body).
Relative motion with reduced mass mu in central potential $V(r) = -k/r$. The constraint
$H = p_r^2/(2 \mu) + L^2/(2 \mu r^2) - k/r - E = 0$
defines allowed states; no t is used.

Integrals of motion (timeless constants).

- Energy E (fixes conic type).
- Angular momentum vector L (magnitude L, direction n_L).
- Runge–Lenz vector A (points to periapsis), magnitude $|A| = \mu k e$ with eccentricity e. These constants determine the entire conic (ellipse/parabola/hyperbola) and orientation.

Relational observables.
Define event-types without t:

- Periapse crossing P := "particle at $r = r_{min}$ with outward radial momentum" (well-defined via H=0, L, A).
- Occultation O(u) := "line-of-sight from a fixed observer direction u is blocked (or closest approach below b_{max})".
- Node crossing N := "cross intersection with reference plane".

Rates as counts (periodicity without time).
For bound orbits (E<0), consider the growing family of regions {R_n} generated by successive loops in angle (e.g., increase of true anomaly by 2pi per loop). Define counts:

- $N_P(R_n)$ = number of periapse crossings in R_n,

- $N\_O(R\_n)$ = number of occultations seen from u.
  Then the timeless ratio
  $rate\_P|O = \lim_{n\to\infty} E[N\_P(R\_n) / N\_O(R\_n)]$
  is the periapses-per-occultation "frequency," a dimensionless invariant of geometry (depends on e, inclination i, observer u, threshold b_max). If one later chooses a clock (e.g., mean anomaly M as time gauge), this reduces to the usual frequency ratio f_P/f_O.

Recovering standard elements (as a gauge).
Pick mean anomaly M in [0,2pi) as an optional clock; then Kepler's equation $M = E\_e - e \sin E\_e$ introduces "time" only as a label of angle. All measurable predictions (transit timings, radial-velocity sequences) can be expressed instead as conditional kernels: "Given the conic constants, what is the distribution of occultations with impact parameter < b_max when the line of nodes aligns with u?" No absolute t is needed.

Unbound scattering (hyperbolic case).
For E>0, define *in* and *out* asymptotic directions by L and A. The deflection angle chi satisfies
$\tan(\chi/2) = k/(b\, v\_inf^2)$,
where b is impact parameter, v_inf is asymptotic speed. The timeless S(b|a) here maps an *in* direction/momentum bin a to an *out* bin b with probability density concentrated at the classical deflection relation plus quantum corrections if desired; counting incident vs scattered events yields the kernel empirically.

Multi-body, quasi-periodic case.
Even when strict periodicity is lost, integer **counts** of geometric events (periapses, conjunctions, transits) over growing regions R_n still define invariant *rates as ratios* (Sec. 3.3). These persist as well-defined timeless observables under weak perturbations, where conventional "time averages" would require an explicit t.

Takeaway.
Kepler dynamics is fully characterized by constraint-satisfying conic constants and by integer counts of geometric events. Choosing a time parameter (mean anomaly, true anomaly as a function of t) is optional bookkeeping; the physics lives in ratios of counts and boundary conditionals determined by (E, L, A).

7. Thermal and Entropic Arrows

7.1 Thermal time hypothesis: modular flow sigma_s from state rho

Statement. Given a pair (A, rho)—a von Neumann algebra A of observables for a causally convex region and a faithful normal state rho—the **thermal time hypothesis** (Connes–Rovelli) posits that the physically relevant “time flow” is the **modular automorphism group** sigma_s generated intrinsically by (A, rho). No external time parameter is assumed.

Construction (Tomita–Takesaki).

- Let (A, H, |Omega>) be the GNS triple for state rho (so rho(X) = <Omega| X |Omega>).
- Define the Tomita operator S: A|Omega> -> A|Omega> by S X|Omega> = X^\dagger |Omega>.
- Its polar decomposition S = J Delta^{1/2} yields the **modular operator** Delta.
- The **modular flow** is the 1-parameter automorphism group
- sigma_s(X) := Delta^{i s} X Delta^{-i s},   s in R.

By construction, sigma_s is a *dynamical* law extracted from (A, rho) alone. In a KMS state (Sec. 7.3), this flow coincides with the usual Heisenberg evolution up to a rescaling of s.

Interpretation.

- The parameter s is a **thermal time** labeling how the state drags observables; it replaces external t.
- If you later **choose** a conventional clock t, there exists a monotone reparametrization s = beta * t (or more general) that matches the observed evolution. The factor is an empirical calibration, not an ontological structure.

Locality and wedges.
In relativistic QFT, for wedge regions in Minkowski space with the vacuum as state, sigma_s is geometrically a Lorentz boost (Bisognano–Wichmann). Thus thermal time reproduces geometric flows where symmetries exist, without introducing a background t.

Payoff.
Dynamics becomes: “expectations are invariant under the modular dynamics when in equilibrium; departures define nonequilibrium behavior”—all without a global clock.

7.2 Entropy as record multiplicity; arrows from boundary data

Records and macrodescriptions.
Let F be the sigma-algebra of records on the history space Omega. A **macrorecord** M is a coarse predicate (e.g., energy in bin, density profile in bin). Its **multiplicity** is the measure of compatible micro-histories:

$W(M) := P(\{ \omega \in \Omega : \omega \text{ satisfies } M \})$.

Define **entropy** of M by

$S(M) := k_B * \ln W(M)$,

with the understanding that W may be computed from counting (discrete) or from Liouville/Gibbs measures (continuous).

Arrow from boundary data.
Pick **boundary records** B_{left}, B_{right} that constrain admissible histories (e.g., low-entropy macrorecord on the "left" boundary of a causal diamond, retarded radiation condition). Consider a nested family $\{M_n\}$ of macrorecords supported on increasingly large regions to the causal future of B_{left}. The **entropic arrow** is the typical monotone trend

$S(M_{n+1}) >= S(M_n)$

for almost all histories compatible with B_{left} (and independent of any external time label). The direction is set by boundary asymmetry, not by a universal tick.

Why monotonicity is typical.

- **Measure concentration:** high-multiplicity macrorecords occupy overwhelmingly larger fractions of admissible micro-histories; conditional on B_{left}, almost all refinements drift toward larger W.
- **Causal screening:** with Axiom M (composition), intermediate macrorecords screen early details; loss of accessible correlations appears as coarse-grained entropy increase.
- **No need for t:** the index n is *any* causal expansion (larger Alexandrov regions, finer partitions). The statement is about inclusion of sigma-algebras and relative sizes of record classes.

Operational entropy production.
Given counts on a large region R, define:

- $N_{micro}(R)$: number (or measure) of micro-histories consistent with B_{left} and with a specified macrorecord inside R.
- $S(R) := k_B \ln N_{micro}(R)$.
  Then the **entropy production rate relative to a reference growth** Y (Sec. 3.3) is

$\text{rate}_{S|Y} := \lim_{n\to\infty} E[(S(R_{n+1}) - S(R_n)) / \Delta Y_n]$.

Typical choices for Y: injected particles (preparations), emitted photons, or simply the increment in region size measured by volume. No external time is used.

Multiple arrows.
Different boundary choices (e.g., advanced vs retarded radiative conditions) define opposite arrows. The framework explains coexistence of different "arrows" (thermodynamic, electromagnetic, cosmological) as consequences of **which** boundary records we condition on.

7.3 KMS without external t; calibration only after choosing a clock

KMS condition (clock-free form).
On a C*- or von Neumann algebra A with state rho and modular flow sigma_s, the state is **KMS** (equilibrium) at inverse temperature beta = 1 *by convention of units* if for all A,B in a dense set:

1. Correlation function F_{A,B}(s) := rho(A sigma_s(B)) extends to an analytic function on the strip 0 < Im z < 1 that is bounded and continuous on the closure.
2. Boundary condition:

rho(A sigma_{s+i}(B)) = rho(sigma_s(B) A), for all real s.

This uses **thermal time s** only; no external t enters.

Physical temperature as calibration.
If one later chooses a conventional clock t and observes a physical Hamiltonian flow alpha_t, equilibrium at temperature T_phys corresponds to identifying sigma_s = alpha_t with s = beta_phys * t, where beta_phys = 1/(k_B T_phys). Thus **temperature** is the conversion factor between thermal time s and the chosen clock t. Absent that choice, equilibrium is simply "KMS with respect to sigma_s."

Examples/anchors.

- **Minkowski vacuum on a wedge:** The modular flow equals Lorentz boosts; the KMS temperature in **Unruh effect** appears when one *chooses* Rindler time t_R. The Unruh temperature T_U = a/(2pi k_B c) is the calibration between s and t_R.
- **Stationary black hole (Hartle–Hawking state):** The modular flow matches the Killing flow; Hawking temperature is the calibration factor relative to the Killing time of asymptotic observers.

Nonequilibrium (modular perspective).
Out of equilibrium, there is no single KMS relation; instead one studies **modular Hamiltonians** for subregions and relative entropies:

S_rel(rho || rho_KMS) = Tr(rho ln rho - rho ln rho_KMS) >= 0, monotone under inclusion of regions. This yields **Arrow statements**—relative entropy typically increases along causal inclusion—even when no t is defined.

Correlation inequalities (Ward-type).
At KMS, two-point functions satisfy the **KMS periodicity** in s (imaginary shift by i). Fluctuation–dissipation relations (FDR) are then equalities between symmetric and antisymmetric parts of correlators defined with sigma_s. If a clock t is later chosen, FDR reappears in its standard frequency–temperature form by substituting s = beta t.

Takeaways.

- Equilibrium and temperature can be defined **intrinsically** via modular flow—no external time is needed.
- "Temperature value" arises only when you **calibrate** thermal time against a chosen clock; it is a conversion factor, not an extra structure.
- Entropy increase and thermal arrows emerge from **boundary records** and **inclusion monotonicity** (relative entropy), fully compatible with causal locality and the absence of a global t.

8. Recovering Standard Time Pictures (as Gauges)

8.1 Choosing a foliation: refoliation invariance audits

Foliation as gauge.
Pick a one-parameter family of spacelike "slices" $\{\Sigma_\lambda\}$whose union covers the domain of interest. Introduce lapse $N$and shift $N^i$to write a metric *presentation*, but treat λas a **label**, not an observable. In Hamiltonian GR this is generated by the first-class constraints

$$\mathcal{H}(x) \approx 0, \mathcal{H}_i(x) \approx 0,$$

so changing $\lambda \mapsto f(\lambda)$is a gauge motion (refoliation).

Gauge fixing (if convenient).
Impose conditions $\chi[\gamma, \pi; \lambda] = 0$(e.g., constant–mean-curvature slices) and compute with Dirac/BRST machinery; physical statements must not depend on the specific χ.

Refoliation invariance audits (practical checks).

1. **Two-foliation comparison.** Compute a conditional $S_D(b \mid a)$on a causal diamond Dusing foliations $\{\Sigma_\lambda\}$and $\{\tilde{\Sigma}_{\tilde{\lambda}}\}$. Require

$$S_D(b \mid a)\,|_{\Sigma} \;=\; S_D(b \mid a)\,|_{\tilde{\Sigma}}.$$

2. **Cut-sum conservation.** Evaluate a conserved charge $Q(\Sigma)$(Sec. 2.4) on two cuts $\Sigma, \tilde{\Sigma}$that bound the same region; demand $Q(\Sigma) = Q(\tilde{\Sigma})$.
3. **Clock switch test.** Choose two admissible clocks T_1, T_2tied to the two foliations and verify the marginal agreement

$$\mathbb{E}[A] = \int \mathbb{E}[A \mid T_k = t]\, dP(T_k = t)(k = 1,2).$$

4. **Constraint closure.** Numerically (or symbolically) check $\{\mathcal{H}[\alpha], \mathcal{H}[\beta]\} \sim \mathcal{H}_i[\cdots]$on the chosen discretization; failure indicates foliation-dependent artifacts.

Interpretation.
"Dynamics in time" is just presenting the same causal/record conditionals with a chosen λ. Passing the audits certifies that "time evolution" is a gauge rendering, not new physics.

8.2 FLRW gauge: a, T_{CMB}, and e-folds $N = \ln\, a$

Gauge declaration.
Assume near-homogeneity/isotropy and choose the FLRW presentation. Use any **one** of the following as the clock **label**:

- **Scale factor** a(monotone after the hot Big Bang),
- **CMB temperature** $T_{\text{CMB}} \propto a^{-1}$,
- **E-folds** $N := \ln\, a$.

These are *equivalent gauges*: $N = -\ln\,(T_{\text{CMB}}/T_0)$. None is ontologically preferred.

Field equations in the Ngauge (example).
For background + linear scalar mode u_k,

$$\frac{d}{dN}\left(\frac{H'}{H}\right) = \Phi(\rho, p, \cdots), u_k'' + \left(1 - \frac{H'}{H}\right)u_k' + \frac{k^2}{(aH)^2}u_k = \mathcal{S}_k,$$

where $' \equiv d/dN$. Here $H(N)$and source $\mathcal{S}_k$are functions of N(no external t). Observables—e.g., transfer functions—are conditional kernels **indexed by** N.

Redshift and distances without t.

$$1 + z = \frac{1}{a}, \chi(z) = \int_0^z \frac{dz'}{H(z')}, D_A(z) = \frac{1}{1+z} S_k(\chi(z)),$$

with H treated as a function of z or N. All statements are relations among records $\{z, D_A$,lensing, BAO$\}$ conditioned on the FLRW gauge.

Thermal calibration.
If one wishes to report a temperature, use $T_{\mathrm{CMB}}(N) = T_0 e^{-N}$ as the **conversion** between the thermal-time parameter for the cosmological state and the chosen gauge label.

Payoff.
The FLRW "cosmic time" t becomes optional bookkeeping. Using N (or T_{CMB}) keeps predictions manifestly gauge-like while remaining directly comparable to observations phrased in z.

8.3 Mapping timeless predictions to time-parameterized forms

Goal.
Given clock-free objects—conditional kernels $S(b \mid a)$, conserved cut-sums, and rate ratios—produce familiar t-based equations *after* choosing a clock. The mapping is a change of variables, not extra physics.

Recipe (general).

1. **Select a clock** T (subsystem pointer, foliation label, or cosmological variable).
2. **Define** t as any monotone function of T: $t = f(T)$.
3. **Relational expectations →Heisenberg form.**

$$\langle A \rangle_t := \mathbb{E}[A \mid T = f^{-1}(t)].$$

If $\{C_i\}$ are constraints, this family satisfies an effective Heisenberg equation with a *conditional* generator H_T:

$$\frac{d}{dt}\langle A \rangle_t = \frac{i}{\hbar} \langle [H_T(t), A] \rangle_t + \langle \partial_t A \rangle_t.$$

4. **Rates as derivatives.**
   Pick Y= "ticks of T". Then the timeless rate becomes the usual derivative:

$$\mathrm{rate}_{X|Y} = \lim \frac{N_X}{N_Y} \;\; \Rightarrow \;\; \frac{d\mathbb{E}[X]}{dt} \text{with } dt \propto dY.$$

5. **Scattering kernels →cross sections.**
   From $S(b \mid a)$and a chosen flux clock (count of preparations per unit t):

$$\frac{d\sigma}{d\Omega} = \frac{S(b \mid a)}{\Phi(a)} \times \mathrm{geom.}$$

The kernel is primary; $\Phi$is a calibration to the chosen t.

6. **Conserved cut-sums →continuity equations.**
   Cut-independence $Q(\Sigma_1) = Q(\Sigma_2)$becomes

$$\partial_t \rho_Q + \nabla \cdot \mathbf{J}_Q = 0$$

after choosing a foliation and identifying densities/currents from the same Ward identity.

7. **Modular flow →thermal dynamics.**
   Given modular σ_s, select a clock $t$and set $s = \beta t$. KMS in $s$becomes KMS in $t$at temperature $T = 1/(k_B \beta)$; fluctuation–dissipation relations follow.

Worked micro-map (oscillator).

- Timeless: energy shell $H = E$, action $J = E/\omega$, uniform angle.
- Choose clock $T \equiv \theta$, set $t = \theta/\omega$. Then

$$\langle q \rangle_t = A\cos{(\omega t + \phi)}, A = \sqrt{\frac{2J}{m\omega}}.$$

No new physics—just a relabeling of the same invariant data.

Worked macro-map (FLRW).

- Timeless: background specified by $H(N)$, observables $D_A(z), P(k, N)$.
- Choose clock $t$with $dt = dN/H(N)$. Then recover

$$\dot{a}/a = H(t), \frac{d}{dt} P(k, t) = \mathcal{F}[P, H, \cdots],$$

the standard time-domain evolution.

Audit after mapping.
Check that any two admissible clocks $t_1 = f_1(T), t_2 = f_2(T)$ give **numerically identical** clock-invariant outputs (e.g., integrated probabilities, cross sections, conserved totals). Discrepancies signal an unphysical, gauge-dependent construction.

Takeaway.
Every familiar t-based formulation is a *presentation* of the same causal/record structure. Choosing a clock turns ratios, kernels, and Ward identities into derivatives, evolutions, and conservation laws—without adding assumptions beyond the gauge choice.

9. Numerical and Experimental Program

9.1 Causal-diamond simulations: boundary-to-boundary kernels

Goal. Compute and validate the timeless scattering kernel $S(b \mid a)$ on finite causal regions ("diamonds") without introducing a global t.

Geometry & discretization.

- **Continuum option:** 1+1D or 2+1D Minkowski diamond $D = \mathrm{Past}(y) \cap \mathrm{Future}(x)$; triangulate with causal simplices; enforce spacelike faces.
- **Causal-set option:** Poisson sprinkle N points into D, inherit partial order $\leq$; use counting measure for volume.

Fields & dynamics.

- Free/weakly interacting scalar ϕ with action $S_D[\phi]$.
- **Continuum:** retarded/advanced Green functions restricted to D; implement Dirichlet/Neumann "transparent" boundary conditions on the null boundary.
- **Causal set:** Benincasa–Dowker d'Alembertian for kinetic term; interaction $V(\phi) = \lambda\phi^4$ via local order layers.

Boundary records.

- **In (L):** discrete modes $\{a\}$ defined by localized wave packets on the left null boundary (or left rim of the diamond).
- **Out (R):** detector bins $\{b\}$ defined by angular/momentum windows on the right boundary.

Kernel estimation.

1. Sample boundary fields ϕ_L from preparation prior for each a.
2. Draw interior fields by importance sampling/path integral (or solve classical equations for tree-level).
3. Project ϕ_R onto detector predicates b.
4. Estimate $S(b \mid a) = \Pr(B = b \mid A = a, R_{\text{ctx}})$ with multinomial confidence intervals.

Composition test (order-Markov).

- Split a diamond $D = D_2 \circ D_1$ with intermediate boundary Σ.
- Verify $S_D(b \mid a) \approx \sum_m S_{D_2}(b \mid m) S_{D_1}(m \mid a)$ up to statistical error.

Refoliation audit.

- Recompute $S(b \mid a)$ using two slicings (e.g., equal-area vs equal-null-length). Invariance within error bars is required.

Scalings & resources.

- Causal set: operator application $O(N)$; sampling $O(NK)$ for K layers. Start with $N \sim 10^4$ (1+1D), extend to 10^5.
- Continuum FEM: sparse solves $O(M \log M)$ with M DOF; use CG + multigrid preconditioners.

Diagnostics.

- Microcausality: two-point commutator vanishes for spacelike-separated regions.
- Stability: kernel rows sum to 1; negative probabilities flagged.
- Continuum-limit trend: vary sprinkling density / mesh size and check convergence of S.

9.2 Finite-algebra modular flows (thermal time tests)

Goal. Demonstrate thermal time without external clocks by constructing modular flows σ_s on finite (or finite-dimensional) algebras and testing KMS/Unruh-type predictions.

Testbeds.

- **Spin chains:** $A = \mathcal{B}(\mathbb{C}^{2 \otimes L})$, region O a contiguous block.

- **Fermions on a graph:** CAR algebra; Gaussian states allow analytic modular Hamiltonians.
- **Random matrix Gibbs states:** tractable $\rho = e^{-K}/\mathrm{Tr}e^{-K}$ with chosen K.

Construction.

1. Build GNS triple $(A, H, |\Omega\rangle)$ for state ρ.
2. Compute Tomita operator S numerically on a truncated basis, polar-decompose $S = J\Delta^{1/2}$.
3. Generate modular flow $\sigma_s(X) = \Delta^{is} X \Delta^{-is}$.

KMS verification (clock-free).

- Correlators $F_{A,B}(s) = \rho(A\, \sigma_s(B))$ should be analytic on the strip and satisfy $F_{A,B}(s + i) = \rho(\sigma_s(B)A)$.
- Numerically check periodicity and boundedness via FFTs along s (discretized).

Geometric case emulation.

- For half-space of a CFT discretized on a chain, compare σ_s with lattice boosts (Bisognano–Wichmann analogue). Agreement up to lattice artifacts confirms geometric emergence.

Nonequilibrium probe.

- Prepare ρ_{neq} (e.g., quench). Compute relative entropy $S_{\mathrm{rel}}(\rho_{\mathrm{neq}} \parallel \rho_{\mathrm{KMS}})$ on nested regions $O \subset O'$. Test monotonicity under inclusion as a time-free arrow.

Calibration after the fact.

- Choose an external clock t only to report temperature: identify $s = \beta t$ by fitting KMS periodicity; verify that physical predictions (e.g., fluctuation–dissipation ratios) are invariant under reparametrizations of t.

Metrics & robustness.

- KMS deviation norm: $\max_s \parallel F_{A,B}(s + i) - \rho(\sigma_s(B)A) \parallel$.
- Spectral gap of Δ (mixing scale); finite-size scaling with system size L.
- Sensitivity to truncation: vary basis cutoff; error bars from bootstrap resampling of matrix elements.

9.3 Clock-gauge cross-checks on toy models

Goal. Show that physical predictions are invariant under the choice of clock subsystem, and quantify failure modes (bad clocks, anomalies).

Models.

- **Two coupled oscillators:** constraints enforce fixed total action; either oscillator can serve as the clock.
- **Discrete Markov nets on a DAG:** classical stochastic dynamics embedded in a partial order; multiple "pointer" variables available.
- **Simple quantum rotor + qubit:** rotor angle as a clock for qubit precession and vice versa.

Procedure.

1. **Define clock candidates** T_1, T_2 with projectors/POVMs $P_{T_k}(t)$.
2. **Compute relational expectations**

$$\langle A \rangle_t^{(k)} = \frac{\langle \Psi \mid P_{T_k}(t) A P_{T_k}(t) \mid \Psi \rangle}{\langle \Psi \mid P_{T_k}(t) \mid \Psi \rangle}.$$

3. **Gauge audits.**
   - **Marginal agreement:** $\int \langle A \rangle_t^{(k)} \, dP(T_k = t) = \langle A \rangle$ for $k = 1,2$.
   - **Map agreement:** if $T_2 = f(T_1)$, test $\langle A \rangle_u^{(2)} = \langle A \rangle_{f^{-1}(u)}^{(1)}$.
   - **S-kernel invariance:** estimate $S(b \mid a)$ with T_1 vs T_2 used only for binning; compare via total variation distance d_{TV}.

Bad-clock stress tests.

- **Non-monotone clocks:** inject recurrences in T's support; observe violations of the audits.
- **Strong back-reaction:** entangle clock strongly with system; quantify drift via $\| [P_T(t), A] \|$ and consistency loss in histories.

Statistical evaluation.

- Use bootstrap to get CIs for audit equalities; set acceptance thresholds (e.g., $d_{\mathrm{TV}} < \epsilon$, relative error $< 1\%$).

- Power analyses to determine sample sizes (number of histories/trajectories) needed to detect a 1%gauge-dependence with 95%confidence.

Mapping to time (sanity check).

- After passing audits, pick a clock $t$and recover standard evolution (e.g., HO cosine law, Markov semigroup). Confirm that parameters (frequencies, transition rates) match those extracted from pure count ratios (Sec. 3.3).

Deliverables & reproducibility.

- Open-source code: (i) diamond kernel solver, (ii) modular-flow builder for finite algebras, (iii) clock-gauge audit suite.
- Reference datasets: synthetic histories, boundary records, and modular spectra for benchmarking.
- Protocol sheets: exact steps to replicate invariance tests on any new model.

Success criterion (program-level).
A model "passes" the timeless program if: (i) its kernels compose on the order, (ii) modular KMS holds for equilibrium states without a clock, and (iii) all physically reported quantities survive clock swaps within stated error bars.

10. Limits, Risks, and Falsifiers

10.1 Where clocks are unavoidable (metrology interfaces)

Coordination, not ontology.
Our framework dispenses with a universal $t$for *theory*, but experiments must still **synchronize, label, and compare** records. There are interfaces where an operational clock is unavoidable:

- **Metrology & SI traceability.**
  - Realizing the second (hyperfine transitions), disciplining oscillators (GPSDO), and reporting uncertainties require a **declared timebase**.
  - Anything that quotes "per second," bandwidth, Allan deviation, or time-of-flight must select a clock (gauge) to be reproducible.

- **Distributed experiments.**
    - Long-baseline interferometry, satellite ranging, and time-transfer (TWSTFT, GNSS) *need* synchronization to stitch regional records into a joint record. The stitching step is a **clock choice** you cannot avoid.
- **Quantum control & thermodynamics.**
    - Implementing non-energy-preserving gates or defining temperature operationally (heat per "unit time") requires a phase reference or calibration against a flow; this is a *resource* in the resource theory of asymmetry.

Stance.
These are **interfaces** that force a gauge choice; they don't refute the timeless core. Our rule: **declare the clock** when interfacing with metrology, and audit that all *clock-invariant* claims remain unchanged under reasonable clock swaps.

10.2 Ambiguities in record choice; identifiability checks

Risk: Records are not unique.
Predictions use conditionals $P(X \mid R)$. But which R(preparation/selection) should we condition on?

- **Coarse-graining ambiguity.**
  Different macro-records (binning thresholds, detector windows) can yield different kernels.
  Guard: report kernels with explicit record definitions; test **refinement stability**—does $S(b \mid a)$change only within statistical error when bins are refined/coarsened?
- **Selection & survivorship bias.**
  Conditioning on "detector clicked" can induce bias (post-selection).
  Guard: compare $P(B \mid A)$with $P(B \mid A,$click); use do-calculus on the partial order to separate interventions from selections.
- **Contextuality (quantum).**
  Noncommuting predicates can't be assigned joint records.
  Guard: restrict to **consistent sets** of histories; verify decoherence functional off-diagonals are negligible for the chosen coarse graining.
- **Nonidentifiability.**
  Different causal kernels can reproduce the same marginals on limited regions.
  Guard:

1. **Order-separation test:** enlarge the diamond and check whether candidate kernels diverge.

2. **Instrumental records:** add upstream "instrument" predicates that vary inputs without touching hidden mediators.

3. **Posterior predictive checks:** simulate from inferred kernel and compare high-order statistics (not used in fitting) to data.

- **State dependence in modular flow.**
  Thermal time requires a **faithful** state; near-pure or highly singular states make Δill-conditioned.
  Guard: regularize (finite algebras, energy cutoffs); quantify sensitivity of $\sigma_s$to state perturbations.

Summary checks (identifiability audit).

- Refinement stability (bins).
- Consistency (histories).
- Instrumental variation (exogeneity).
- Enlargement robustness (diamond growth).
- Sensitivity to prior/state choice (for modular tests).

10.3 Concrete falsification routes

F1. Refoliation dependence of kernels.
Compute a boundary-to-boundary kernel $S_D(b \mid a)$on the *same* causal diamond Dusing two unrelated foliations/slicings.
Fail criterion: statistically significant difference beyond discretization noise.
Meaning: theory predictions depend on a chosen time label—violates reparametrization invariance.

F2. Composition (order-Markov) failure.
Split $D = D_2 \circ D_1$with intermediate boundary Σ.
Test $S_D(b \mid a) =? \sum_m \quad S_{D_2}(b \mid m) S_{D_1}(m \mid a)$.
Fail: systematic deviation not attributable to finite-size effects.
Meaning: "dynamics as conditionals on the order" does not compose—core axiom breaks.

F3. Cut-sum nonconservation (Noether without time).
For a symmetry (charge) with a proven Ward identity, evaluate the conserved total on two cuts enclosing the same region.
Fail: $Q(\Sigma_1) \neq Q(\Sigma_2)$beyond error.
Meaning: symmetry → conservation mapping invalid; our Noether generalization is wrong or incomplete.

F4. KMS/modular falsifier.
On finite algebras (Sec. 9.2) or wedge-like regions in lattice QFT, build $\sigma_s$from the state and test KMS analytic continuation.
Fail: absence of strip analyticity/periodicity where standard theory predicts it.
Meaning: thermal time hypothesis does not reproduce known equilibria.

F5. Clock-gauge dependence.
On a toy model with two admissible clocks, compute a physical quantity (e.g., scattering yield $S(b \mid a)$, or an integrated observable) via relational expectations.
Fail: marginal agreement test fails:

$$\int \mathbb{E}[A \mid T_1 = t]\, dP(T_1 = t) \neq \int \mathbb{E}[A \mid T_2 = u]\, dP(T_2 = u).$$

Meaning: clock choice changes physics; time is not a gauge in our construction.

F6. Empirical cross-sections without clocks.
Measure $S(b \mid a)$as ratios of counts (Sec. 6.2) in a lab beamline; compute standard differential cross sections using the facility's clock.
Fail: irreconcilable mismatch after accounting for geometric/flux calibration.
Meaning: the timeless kernel approach misses operational content captured by time-based theory.

F7. Entropic arrow reversal under identical boundary records.
With identical low-entropy boundary records imposed in simulations, the typical coarse-grained entropy should **increase** along causal inclusion.
Fail: robust decrease or non-monotone trends not explainable by finite-size or coarse-graining artifacts.
Meaning: our "arrows from boundary data" mechanism is inadequate.

F8. Cosmology gauge test.
Express a prediction solely in terms of $N = \ln\, a$(e.g., growth functions, distance–redshift relations) and compare to the same prediction cast in cosmic-time t.
Fail: numerical disagreement beyond mapping errors for the same background.
Meaning: the mapping from timeless to time-parameterized forms (Sec. 8.3) is inconsistent.

F9. Microcausality violation.
Compute commutators (or classical influence functionals) for spacelike regions in simulations.
Fail: nonzero signal after accounting for discretization.
Meaning: our order-local nets or causal-set operators do not enforce locality as claimed.

Bottom line.
This program is **falsifiable**. It survives only if (i) kernels are foliation-invariant and compose on the order, (ii) conserved cut-sums obey Ward identities, (iii) modular (KMS) structure appears without an external clock, and (iv) all reported quantities are **clock-gauge invariant** within uncertainty. Any durable failure along F1–F9 refutes the core timeless claims or sharply bounds their domain of validity.

11. Conclusion: Structure Over Schedule

11.1 Physics as invariant relations

The core deliverable of a clockless framework is **invariance**: statements that survive relabelings, refoliations, and clock swaps. The primitives—events, their causal order, and empirically anchored records—support predictions expressed as **conditional kernels** $P(X \mid R)$, **conserved cut-sums** derived from order automorphisms (Ward identities), and **rate ratios** as limits of relative counts. Geometry is reconstructed from order plus volume (conformal class fixed by order; scale fixed by interval volumes). Quantum theory fits naturally: constraints $C_i \mid \Psi\rangle = 0$define the physical sector; probabilities arise from **consistent histories** on causally ordered regions; modular theory supplies flows intrinsic to a state and region. In all cases, the physics is encoded in **relations among records under causal inclusion**, not in their coordinates against an external clock.

11.2 Time as bookkeeping, not ontology

"Time" re-enters only when we **choose** a clock: a foliation label, a subsystem pointer, a cosmological variable (e.g., $N = \ln\ a$), or thermal time $s$from (A, ρ). That choice indexes the same invariant content; it does not add new physics. Derivatives $dX/dt$are **calibrated versions** of timeless rate ratios with Ytaken as "ticks of the clock." Hamiltonian "evolution" is a **presentation** of constraint-consistent conditionals; conservation "in time" is conserved **across cuts**. Temperature is the conversion between modular parameter and a selected clock. The program therefore separates **structure (what is invariantly true)** from **schedule (how we label it)**, ensuring that operational interfaces (metrology, synchronization) are treated as **gauges** rather than ontological commitments.

11.3 Open problems and a roadmap for a clockless physics

Foundations.
(1) Complete the measure-theoretic groundwork for record algebras on general causal sets; prove existence/uniqueness theorems for boundary-to-boundary kernels on arbitrary diamonds.
(2) Strengthen Noether-without-time: classify order automorphisms that yield local conserved currents and quantify anomalies in discrete settings.

Quantum structure.
(3) Construct rigorous rigging maps (RAQ) for interacting field models on bounded regions; compare with covariant path-integral definitions of class operators.
(4) Clarify clock-gauge equivalence: necessary and sufficient conditions for two clocks to generate unitarily equivalent relational dynamics; characterize "bad clocks" by operational diagnostics.

Thermal/entropic arrows.
(5) Prove large-deviation principles for entropy as record multiplicity under broad boundary conditions; derive fluctuation theorems phrased in modular time s.
(6) Map modular Hamiltonians for realistic lattice QFT regions; benchmark KMS periodicity and relative-entropy monotonicity numerically.

Geometry and discreteness.
(7) Improve causal-set estimators for curvature and dimension; tighten convergence rates of the Benincasa–Dowker operator; develop renormalization schemes formulated purely in order statistics.
(8) Extend "order + volume →metric" reconstructions to spacetimes with mild causal pathologies; quantify robustness to sampling noise.

Phenomenology and tests.
(9) Deliver open-source pipelines for: diamond-kernel estimation, modular-flow computation on finite algebras, and clock-gauge audits.
(10) Execute laboratory falsifiers: (a) foliation-invariance tests of $S(b \mid a)$on analog simulators; (b) modular KMS checks in engineered quantum systems; (c) cross-section matches between timeless count ratios and time-based metrology.

Roadmap.
Phase I (theory & numerics): finalize axioms; publish diamond-kernel and modular-flow benchmarks with uncertainty budgets and refoliation audits.
Phase II (experimental interfaces): partner with beamlines/quantum platforms to extract clock-free kernels and verify gauge-invariant mappings to standard observables.

Phase III (synthesis): assemble a reference compendium where each mainstream result (scattering, transport, thermodynamics, cosmology) is reproduced twice—once **timeless**, once **time-gauged**—with a documented, audit-passing map between them.

Closing claim.
A physics of **structure over schedule** preserves general covariance, clarifies the status of thermodynamic and quantum arrows, and yields concrete, falsifiable predictions. If its audits fail, we will learn where "time" is indispensable; if they pass, we gain a leaner ontology with the same empirical reach—and a cleaner separation between what the world **is** and how we **bookkeep** it.

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Notes: This list emphasizes canonical sources for the specific ingredients used in the paper: constraints and refoliation invariance; relational and histories formalisms; algebraic/thermal time structure; causal-set geometry; and representative exemplars (Unruh/Hawking, LSZ). It is intentionally cross-disciplinary so each claim has a standard anchor.