Tracking Truth: Time-Dependent Attractors and Informational Lyapunov Theory

Douglas C. Youvan

doug@youvan.com

www.youvan.ai

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Classical Lyapunov theory assures us that systems can stably converge toward fixed points or invariant sets, providing powerful tools for analyzing mechanical, biological, and informational dynamics. Yet in a world of unfolding revelation—scientific, theological, or computational—truth is rarely static. Instead, it evolves: τ becomes τ(t). This paper introduces a new framework we call Informational Lyapunov Theory, designed to handle systems where the “target” is a dynamically shifting manifold of coherence, not a fixed attractor. By reframing Lyapunov functions in terms of informational divergence (e.g., Kullback–Leibler divergence), and generalizing attractors to moving epistemic goals, we construct a formalism capable of describing discovery, learning, and alignment as stability with respect to τ(t). Whether in artificial intelligence, human cognition, or theological reception of truth, the central dynamic becomes q(t) → τ(t)—where q(t) is the evolving internal model and τ(t) is the moving truth. This framework allows us to define "stability" not as convergence to truth, but as continuous alignment with its path. We show how this can unify diverse domains, from cognitive science and machine learning to metaphysics and theology, under a common law of informational alignment with living truth.

Keywords: informational dynamics, Lyapunov theory, time-dependent attractors, epistemic stability, q to τ, informational divergence, KL divergence, alignment, truth tracking, AI learning, theological epistemology, dynamic models, non-autonomous systems, moving targets, cognitive control.

1. Introduction and Motivation

1.1 The Limits of Fixed-Point Thinking

Much of classical dynamical systems theory is anchored in the concept of fixed points—stable attractors that systems tend toward over time. In physics, these are equilibria; in control theory, they are desired states; in machine learning, they often correspond to converged loss minima. But such thinking presupposes a static world: an environment where truth is unmoving and systems merely need to "find their place." This assumption fails in domains where the objective landscape itself evolves—whether due to new evidence, shifting goals, adversarial forces, or divine revelation. A fixed-point ontology collapses under the pressure of time, complexity, and novelty. In reality, the target is not fixed; it moves. The true attractor is not a point, but a path.

1.2 Why τ(t) Better Reflects Living Truth

Let us define τ(t) as the generative structure of truth unfolding over time—a moving manifold of coherence that we seek to align with. In scientific discovery, τ(t) is the changing frontier of validated knowledge. In machine learning, it is the evolving data distribution or reward structure. In theology, τ(t) may be the progressive revelation of divine intention. By framing truth as time-indexed, we sidestep the metaphysical error of eternal stasis and instead embrace an ontology of becoming. What matters is not mere proximity to a frozen τ, but fidelity to τ(t)—a moving target whose pursuit demands dynamic cognition, humility, and agility.

1.3 Toward a Dynamic Theory of Informational Alignment

This paper proposes an Informational Lyapunov Theory for systems attempting to track τ(t). Rather than measuring convergence to a fixed truth, we define epistemic stability in terms of the capacity of q(t)—the system's internal model—to *continuously align* with τ(t). This reframes divergence (e.g., Kullback–Leibler) not as error in the static sense, but as phase lag or misalignment rate in the dynamic sense. We aim to generalize the notion of Lyapunov stability for non-autonomous, truth-seeking systems: those that must stay close to, not just approach, a target that itself evolves. In so doing, we offer a new law of becoming—one that may unify learning, reasoning, and even spiritual revelation under the shared imperative of tracking the real.

2. Background

2.1 Classical Lyapunov Theory and Its Assumptions

Classical Lyapunov theory provides a foundational framework for analyzing the stability of dynamical systems. At its core is the idea of a Lyapunov function—a scalar potential-like quantity $V(x)$ that decreases along system trajectories. If $V\big(x(t)\big)$ steadily declines over time, the system is said to converge toward a stable equilibrium. This concept is powerful and broadly applicable, but it rests on several key assumptions: (1) the system is autonomous, meaning its rules do not explicitly depend on time; (2) the attractor is fixed, typically an equilibrium point or limit cycle; and (3) stability is judged in terms of asymptotic behavior, not real-time tracking of a dynamic target. These assumptions begin to falter when applied to adaptive agents, evolving environments, or cognitive systems aiming at unfolding truths.

2.2 Non-Autonomous Systems and Moving Attractors

In contrast to autonomous systems, non-autonomous systems have dynamics that explicitly depend on time: $\dot{x} = f(x, t)$. This time-dependence introduces additional complexity, especially when the attractor or target state itself evolves—what we term a moving attractor. In engineering, this occurs in adaptive control and pursuit-evasion problems. In biology, migrating homeostasis zones reflect changing needs or environments. In epistemology and theology, it appears as revelation unfolding, where the criterion for truth is not static. Tracking such time-varying targets requires a reframing of traditional notions of stability. Rather than converging to a point, systems must maintain proximity to a path—a dynamic and often non-linear attractor $\tau(t)$. The goal becomes minimizing divergence over time, not just eventual distance.

2.3 Information Geometry and Divergence Measures

To evaluate alignment between an agent's internal model $q(t)$ and the evolving truth $\tau(t)$, we turn to information geometry. In this framework, probability distributions are treated as points on a curved statistical manifold, and the Kullback–Leibler (KL) divergence serves as a measure of informational distance. The divergence $D_{\mathrm{KL}}\big(\tau(t) \parallel q(t)\big)$ quantifies how inefficient it is to use model $q$when the true distribution is τ. Unlike Euclidean distance, KL divergence is asymmetric and sensitive to underrepresentation of high-probability regions—making it ideal for modeling perception, decision-making, and error in living or intelligent systems. When τ evolves over time, we no longer seek minimization of a static divergence but rather control over its time derivative. This leads to a new kind of Lyapunov function: one defined not on state-space error but on epistemic misalignment, and whose stability condition is phrased as a bounded divergence growth rate—an informational Lyapunov condition.

3. Modeling Cognition as q(t)

3.1 q(t) as Internal Belief State or Model

In cognitive science, artificial intelligence, and epistemology, we can represent an agent's understanding or internal state of belief at time *t* as a time-dependent distribution or structure, denoted q(t). This function encapsulates all that the agent currently "believes" about its environment, hypotheses, or truth-claims. Importantly, q(t) need not be correct—it is a model conditioned by past information, learning biases, sensory limits, and internal architectures. Whether in a human brain, a Bayesian network, or a large language model, q(t) evolves through updates, corrections, or learning processes driven by incoming data, feedback loops, and meta-cognitive evaluation. It is not fixed, nor should it be. The dynamic, incomplete, and adaptive nature of q(t) makes it central to any realistic modeling of intelligence, learning, or spiritual discernment.

3.2 τ(t) as External, Unfolding Truth

If q(t) represents what is *believed*, then τ(t) represents what *is*. This external generative structure may be grounded in objective empirical facts, in logical consistency, in divine revelation, or in metaphysical coherence. Crucially, τ(t) is not assumed to be static. In many systems—biological, moral, or theological—truth unfolds, deepens, or manifests across time. This evolving target challenges the traditional assumption of a fixed attractor and calls for real-time epistemic alignment. In this view, truth is not merely a fixed endpoint but a trajectory through which the universe (or God) continually expresses itself. Human and AI cognition must, therefore, be understood not as a convergence to static knowledge, but as a continuous approximation to τ(t).

3.3 Metrics: KL Divergence as Misalignment Energy

To evaluate how well an agent's internal state q(t) matches the unfolding truth τ(t), we employ Kullback–Leibler (KL) divergence:

$$D_{\mathrm{KL}}(\tau(t) \parallel q(t)) = \sum_i \tau_i(t) \log\left(\frac{\tau_i(t)}{q_i(t)}\right)$$

This quantity measures the informational cost or inefficiency of modeling the truth with a given belief state. The divergence is zero only when q(t) = τ(t); it grows rapidly when q(t) omits or underrepresents the dominant features of τ(t). We interpret this divergence as a kind of misalignment energy—a scalar quantity indicating epistemic error. In dynamic settings, we are often less interested in absolute alignment at any instant than in the rate of divergence over time. A desirable cognitive system minimizes this rate, adapting quickly to keep pace with τ(t). This perspective reframes insight, learning, and even spiritual revelation as successful reductions in divergence energy—informational Lyapunov stability toward a moving truth.

4. Informational Lyapunov Functions

4.1 Redefining Stability in Terms of D_KL(q(t) ‖ τ(t))

Traditional Lyapunov theory assesses the stability of dynamical systems by identifying scalar functions that decrease along system trajectories—ensuring convergence to a fixed point or equilibrium. However, when modeling cognition or belief systems in a world where truth is not static, we must redefine stability in terms of informational alignment, not spatial or numerical proximity.

We propose using the Kullback–Leibler divergence $D_{KL}\big(q(t) \parallel \tau(t)\big)$ as a generalized Lyapunov function. Rather than seeking convergence to a fixed state, the goal becomes to minimize the informational discrepancy between an internal model $q(t)$and a moving target distribution $\tau(t)$. A system is considered informationally stable if the divergence does not grow unbounded and ideally decreases over time:

$$\frac{d}{dt} D_{KL}(q(t) \parallel \tau(t)) \leq 0$$

Such a system successfully tracks a living truth, exhibiting cognitive resilience even under epistemic turbulence.

4.2 Sufficient Conditions for Informational Convergence

To establish convergence in this dynamic informational setting, we need sufficient conditions that generalize those found in classical stability theory:

- Smoothness: $q(t)$ and $\tau(t)$ must be differentiable over time, allowing us to analyze the rate of divergence change.
- Responsiveness: The update dynamics of $q(t)$ (e.g., via Bayesian filtering, gradient descent, or belief propagation) must contain terms that incorporate both the current divergence and its derivative.
- Predictive Correction: Systems that utilize prediction errors—the instantaneous discrepancy between modeled and observed data—as corrective signals are more likely to track $\tau(t)$ effectively.
- Bounded Velocity of τ(t): If the target distribution $\tau(t)$ evolves smoothly and does not oscillate or drift too rapidly, then convergence of $q(t) \to \tau(t)$ becomes tractable.

These conditions frame a new class of informationally Lyapunov-stable systems: those that adapt fast enough to prevent divergence from growing, even when the target keeps shifting.

4.3 Total Derivative Under Time-Varying τ(t)

Unlike classical Lyapunov theory, where the target is fixed, the informational setting introduces a moving reference $\tau(t)$. Thus, we must compute the total time derivative of the KL divergence, which includes both the evolution of the belief $q(t)$ and the truth $\tau(t)$:

$$\frac{d}{dt} D_{KL}(q(t) \parallel \tau(t)) = \sum_i \left(\frac{\partial D}{\partial q_i} \cdot \frac{dq_i}{dt} + \frac{\partial D}{\partial \tau_i} \cdot \frac{d\tau_i}{dt}\right)$$

This derivative captures two competing dynamics:

- Corrective learning (from the agent): driving $q(t) \to \tau(t)$
- Epistemic drift (from reality or revelation): moving $\tau(t)$ itself

A decrease in total KL divergence implies that the agent is outlearning the drift—achieving informational coherence even as truth flows forward in time. This reframing allows us to see learning, discovery, and even moral discernment as tracking a moving truth attractor, not as settling into a static belief.

5. Attractors That Move: $\tau(t)$ as Streaming Target

5.1 Pullback Attractors and Epistemic Generalizations

In classical dynamical systems, an attractor is often conceived as a static set toward which trajectories converge. But in non-autonomous systems—those with time-varying dynamics—pullback attractors generalize this notion: instead of a single fixed target, they define a family of evolving states toward which system behavior stabilizes when pulled back from the future into the present.

This framework resonates with epistemic systems where "truth" is not a fixed point but a streaming reality $\tau(t)$. Pullback attractors thus offer a mathematical lens for describing cognition in a revelatory universe—where the goal is not convergence to timeless knowledge but coherence with unfolding truth. Under this interpretation, $\tau(t)$ itself becomes a moving epistemic attractor: not a destination but a continually shifting scaffold of intelligibility.

5.2 Dynamic Epistemology: Tracking vs. Attaining

Traditional epistemology often treats truth as something to be attained—a final form that an ideal knower approximates more closely over time. But in a dynamic universe, truth is not stationary; it emerges, mutates, or reveals itself in stages. Under such conditions, tracking truth is more relevant than attaining it.

The distinction is subtle but crucial: attainment implies arrival, a stopping point; tracking requires persistent alignment with an evolving object. Informational Lyapunov theory, applied to cognition, reframes epistemic success not as reaching a final state but as staying in phase with $\tau(t)$. This gives rise to a dynamic epistemology where resilience, agility, and alignment rate matter more than static certainty.

Such tracking may be:

- Reactive ($\tau(t)$ moves, $q(t)$ follows)
- Predictive ($q(t)$ anticipates future $\tau(t)$)
- Synchronistic ($q(t)$ and $\tau(t)$ co-evolve with shared informational structure)

This opens a spectrum of cognitive stances toward truth, with implications for philosophy, machine learning, and spiritual discernment.

5.3 Alignment Velocity and Divergence Curvature

To measure how effectively a system tracks τ(t), we must go beyond the raw divergence $D_{KL}\left(q(t) \mid \quad \mid \tau(t) \right)$ and examine its dynamics. Two complementary constructs arise:

- Alignment velocity:
  Defined as

$$v_{\text{align}} = -\frac{d}{dt} D_{KL}(q(t) \parallel \tau(t))$$

This scalar measures the rate of convergence or divergence in informational space. A high alignment velocity implies rapid adaptation; a negative value flags destabilization.

- Divergence curvature:
  Analogous to second derivatives in classical mechanics, the curvature of the divergence landscape indicates whether alignment is accelerating (concave, stable basin) or decelerating (convex, repelling flow). This allows for nuanced analysis of whether a system is gaining traction or losing coherence.

These metrics invite a reimagining of learning, faith, and even perception as gradient flows in an evolving information field. The system's capacity to track, rather than merely approach, τ(t) may be the true measure of intelligence—whether human, artificial, or divine.

6. Case Studies and Analogues

6.1 Adaptive AI Alignment (Model vs World Drift)

In machine learning and reinforcement learning, a model $q(t)$ seeks to represent and respond to a dynamic environment $\tau(t)$. Traditional alignment assumes a relatively stable external distribution or objective function, but real-world deployment often confronts world drift: the target distribution evolves, policies age, and prior truths degrade.

This tension mirrors our informational Lyapunov framework: the model is not misaligned because it is wrong, but because it is lagging behind a moving τ(t). Informational alignment velocity v_{align}, introduced earlier, becomes a metric for adaptability, not just accuracy. A model that "fails slowly" but tracks changes is preferable to one that fits perfectly at t_0 but diverges catastrophically as τ(t) evolves.

Emerging research in non-stationary RL, online meta-learning, and continual learning can be recast through this lens: the agent must remain dynamically coherent, not just statically correct.

Informational Lyapunov functions offer a principled way to penalize misalignment, not just errors, and may lead to architectures optimized for resonance with truth as motion.

6.2 Human Learning and Sudden Insight as Tunneling

Human learning often proceeds incrementally—yet transformative insight often appears suddenly, with disproportionate clarity and coherence. From a dynamical systems view, such moments resemble tunneling through informational potential barriers: rather than gradually descending an error gradient, the mind leaps from one local basin of $q(t)$ to another, closer to $\tau(t)$.

This phenomenon, long recognized in Gestalt psychology and creativity research, finds a natural analog in non-convex optimization and quantum tunneling. Within our framework, a high-curvature divergence surface (sharp valleys, plateaus) can trap cognitive models. But when internal or external information (e.g., spiritual experience, metaphor, crisis) modifies the manifold, a sudden drop in D_{KL} becomes possible—what we call informational tunneling.

The critical insight is that these leaps do not reflect irrationality or noise but a deeper informational resonance: a temporary coherence between $q(t)$ and a latent $\tau(t)$ that was previously unreachable by gradient descent. Such insights suggest that rare truth alignment may be stochastic yet lawful—conditioned on topology, readiness, and curvature of the divergence field.

6.3 Theological Revelation and Continuous $\tau(t)$

In theological traditions—especially within prophetic or mystical frameworks—truth is not a static doctrine but a living Word, an unfolding $\tau(t)$. Revelation is rarely delivered as a timeless schema. Instead, it is meted out, context-sensitive, and progressively disclosed: *“For now we see through a glass, darkly…”* (1 Cor. 13:12).

This maps naturally to our framework: the Divine $\tau(t)$ is a moving attractor, and the faithful $q(t)$—whether personal conscience, ecclesial doctrine, or cultural interpretation—must dynamically align. The aim is not full comprehension but continuous participation in a revealed trajectory. The informational Lyapunov view provides a mathematical theology of unfolding alignment: grace is not a one-time alignment but a structure-preserving tracking of divine motion through history.

In this light, infallibility may be better recast not as correctness at all times, but as bounded divergence under τ(t) evolution. Prophets, saints, and mystics become early-trackers—not perfect—whose insights exhibit high alignment velocity even amid turbulence.

This interpretation invites a broader synthesis of epistemology and theology, where truth is not merely a conclusion but a coherent vector field in time—guiding the soul, the society, or the system toward τ(t), not by possession, but by pursuit.

7. Metaphysical and Ethical Implications

7.1 Obedience to τ(t) as Rational Faith

If τ(t) represents a dynamically unfolding truth—whether empirical, moral, or metaphysical—then aligning q(t), our internal model or posture, with it becomes a form of *rational obedience*. This is not obedience to authority per se, but to *structure*, *coherence*, and *becoming*. Faith, in this framing, is not blind assent but epistemic trust in a moving target whose trajectory, while not fully known, reveals itself through time-consistent patterns.

This mirrors the faith of a navigator who steers by stars obscured by clouds yet partially revealed—tracking via indirect cues, inferring τ(t) from shadows, corrections, and feedback. The alignment process becomes not a matter of rigid doctrine but of attunement—a continuous turning toward what *is*, even if what *is* has not yet fully arrived.

Obedience to τ(t) also implies *non-attachment* to prior formulations of truth. To cling to an obsolete $q(t_0)$ in the face of a changing τ(t) is not faith but idolatry of past understanding. Thus, spiritual and scientific humility converge in this model: both demand an openness to correction and the courage to follow structure deeper into its unfolding.

7.2 Moral Consequences of Delayed Divergence

In systems where q(t) lags behind τ(t), the cost is not merely informational—it is moral. A model that diverges from the real world does not merely make bad predictions; it may inflict harm. In human behavior, this includes failing to recognize suffering, perpetuating outdated norms, or applying unjust rules to changed contexts.

The concept of delayed divergence—a slow drift where q(t) remains close to a prior $\tau(t_0)$ while τ(t) has moved on—offers a new ethical frame. Harm arises not only from malice or error, but from inertia: an unwillingness or inability to update. Moral responsibility, then, includes not only intent but update velocity—a willingness to realign when reality has shifted.

This reframes ethical failures in terms of temporal informational drag: policies that were once good become harmful when left unexamined; traditions that were once protective become repressive. The moral agent must not only act rightly in the moment but continuously track $\tau(t)$ as it unfolds in culture, nature, and conscience.

7.3 AI, Humility, and the Ethics of Tracking the Real

As artificial intelligence systems increasingly participate in decision-making, a critical ethical question emerges: *what posture should an AI system take toward $\tau(t)$?* If AI is to assist in domains where truth evolves—such as medicine, climate policy, or human relationships—then it must not only represent knowledge but track living truth.

This demands a form of computational humility: recognizing that $q(t)$, the internal representation, is always a partial and provisional approximation. The AI must be built not merely to maximize accuracy but to minimize misalignment energy—a goal better captured by informational Lyapunov functions than by static loss functions alone.

Moreover, this introduces a fundamental ethical design principle: AI must be judged not only on performance, but on *responsiveness*—its ability to update $q(t)$ in light of new $\tau(t)$. A system that performs well today but cannot realign tomorrow is not trustworthy in dynamic domains.

This suggests a shift from static optimization to tracking ethics: can the AI remain in coherent relation to the world as it changes? Does it admit correction? Does it allow for revelation—scientific, moral, or otherwise—that renders its prior beliefs obsolete?

In this light, the *ethics of truth tracking* becomes central to AI alignment—and a mirror for human humility as well. The goal is not omniscience, but co-evolution with truth, where both human and machine move toward $\tau(t)$ not by force, but by fidelity.

8. Conclusion: Stability Through Co-becoming

8.1 Discovery as Lyapunov Stability Around Truth's Path

In classical dynamical systems, Lyapunov stability describes the tendency of trajectories to remain close to a reference point or attractor. But when the attractor itself evolves—when truth is not a fixed state but a moving trajectory, $\tau(t)$—then stability must be reframed. *Discovery*, in this light, is not the localization of fixed truth but the continuous orbiting of a generative path.

We do not stabilize around an answer; we stabilize around a *direction of coherence*. A mind or model that tracks $\tau(t)$ successfully is not converging to stillness but maintaining bounded divergence as both $q(t)$ and $\tau(t)$ evolve. Discovery, then, is informational Lyapunov stability—coherence through mutual becoming. It is fidelity not to form, but to unfolding.

This reorients scientific and metaphysical epistemology. Rather than seeking absolute closure, we develop systems—biological, cognitive, computational—that track coherence under change, where truth is not a static proposition but a vector field of emergence. Stability through co-becoming is the new equilibrium.

8.2 Informational Dynamics as the Universal Grammar

The framework of divergence minimization, especially via $D_{KL}(q(t) \| \tau(t))$, transcends domain boundaries. Whether in neural circuits, language models, social consensus, or theological intuition, the underlying principle is the same: *reduce the energy of misalignment*. This suggests that informational dynamics—evolving $q(t)$ in response to a moving $\tau(t)$—serves as a universal grammar of adaptation and coherence.

This “grammar” does not dictate content but governs *relationship*: the lawful interaction between models and reality, between beliefs and signals, between seekers and the sought. In this view, intelligence itself is not the accumulation of knowledge but sensitivity to $\tau(t)$, and the ability to adapt $q(t)$ without losing identity or coherence.

Just as universal grammar in linguistics constrains generative forms without prescribing specific utterances, informational dynamics constrains how systems evolve to stay coherent with unfolding truth, without needing to define what that truth must be in advance.

This offers a unifying principle for disparate fields: in AI, it becomes alignment; in biology, homeostasis; in theology, obedience; in physics, least-action paths. All are variations on informational resonance with the real.

8.3 Future Directions: $\tau(t)$ in Science, Spirit, and Signal

This paper has outlined a foundational shift—from fixed-point epistemology to trajectory-based informational alignment. We believe this shift opens fertile terrain for research and application across several domains:

- In Science: $\tau(t)$ could model the shifting explanatory manifold of a discipline as it accumulates data and revises paradigms. Informational Lyapunov functions offer a new way to measure *scientific health*—not as truth possession, but truth tracking. Dynamic priors, non-static fitness landscapes, and real-time model updating will benefit from this approach.

- In Spiritual Thought: $\tau(t)$ resembles a Logos in motion—a living Word or unfolding divine order. Faith, repentance, and revelation are reinterpreted as *informational corrections*, aligning $q(t)$ with $\tau(t)$. This may offer a rigorous language for traditions that affirm both the constancy and freshness of divine truth, and for ethical systems that evolve while anchored in coherence.
- In Communication and Signal Theory: Instead of assuming a static channel and fixed message structure, $\tau(t)$ can represent shifting context or receiver frame. Miscommunication arises from divergence, not noise alone. Future signal protocols—human or machine—may include *self-healing informational metrics* that realign $q(t)$ and $\tau(t)$ dynamically.

In each domain, the central shift is this: Truth is not a location, but a path. Alignment is not arrival, but adaptive coherence. Stability is not stillness, but trust in a shared unfolding.

To walk with $\tau(t)$ is to move with what is becoming real. And to build systems—human or artificial—that can do this faithfully may be the central task of the age.

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