

Writing on the Causal Edge: Outrunning Heat in Exotic Spacetime Processes

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What if structure is authored faster than heat can reply? This paper proposes a cone-centric physics where processes ride the causal edge and thermalization follows as a delayed echo. A process cone defines where influence can arrive within a given time; a thermal cone defines how far equilibration can chase. When the process cone opens wider than the thermal cone, form is written first and warmed later. The immediate interior remains cold and nearly reversible, while irreversibility concentrates on a thin entropy wake that trails the front and gathers at rims, contacts, and horizon-like boundaries. In the null limit, fronts align to lightlike directions, temperatures flatten along those generators, and dissipation is exported to stretched surfaces that serve as sacrificial ledgers of heat. Reversible by slowness gives way to reversible by reach: the decisive quantity is not patience but arrival. The framework yields crisp signatures, including a wake angle that inverts to a thermal Mach number, two-time calorimetry that separates writing from warming, and defect maps that trace the catch-up geometry of heat to meaning. Metaphysically, faithful change is authored where sustaining influence can still arrive; novelty appears at the thin places where cones let go and new order must pay its debt.

Keywords: process cone, thermal cone, causal edge, outrun thermodynamics, thermal Mach number, entropy wake, cold core hot rim, null aligned fronts, horizon export, two time calorimetry, defect cartography, reversible by reach, $q \rightarrow \tau$, exotic isotherms, cone geometry.

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1. Overview and Claims

1.1 Thesis: meaning written first, heat in pursuit

We advance a cone-centric thesis: a process that carries structure at speed v_{proc} can author macroscopic order on or near the causal edge, while thermal equilibration, limited by a distinct speed v_{th} , arrives later. The decisive variable is not patience (slowness) but reach (arrival). Where the process cone is wider than the thermal cone over the drive interval, the interior just behind the front remains cold and nearly reversible; irreversibility concentrates on a thin trailing region that we call the entropy wake. In this regime, isotherms are not placid static surfaces but dynamic traces that chase the written form. Metaphysically: faithful change is whatever remains within sustaining causal accessibility; novelty is minted at the thin places where that accessibility fails and heat must pay the debt of order.

Key kinematics (ASCII-safe):

- Process cone: $r \leq v_{\text{proc}} * t$
- Thermal cone: $r \leq v_{\text{th}} * t$
- Thermal Mach number: $M_{\text{th}} = v_{\text{proc}} / v_{\text{th}}$
- Outrun window: points with $r \leq v_{\text{proc}} * t$ and $r > v_{\text{th}} * t$

1.2 Regimes: sub-cone smoothing, outrun order, null aligned limit

We distinguish three regimes by the relative opening of the cones and the geometry they induce.

- Sub-cone smoothing ($M_{\text{th}} < 1$): Thermal influence keeps up or leads. The drive writes gently into a bath that arrives in time. Isotherms bulge smoothly ahead; entropy production is volumetric and small when $T_{\text{drive}} \gg \tau_q + L / v_{\text{th}}$. Classic reversible-by-slowness assumptions are adequate.
- Outrun order ($M_{\text{th}} > 1$): The process writes first, heat chases. A V-shaped thermal wake trails the front with opening angle α obeying $\sin(\alpha) = 1 / M_{\text{th}}$. The interior shows a cold-first ordering window: order parameter changes are large while ΔT remains small until the wake arrives. Irreversibility concentrates on a codimension-1 entropy sheet that thickens or thins with boundary curvature and material anisotropy.
- Null aligned limit (v_{proc} approaches the causal bound): Near effective horizons or strongly tilted local frames, fronts align to nearly null directions. Temperature plateaus run along these directions while dissipation is exported to rims, contacts, or stretched boundaries. Redshift effectively tilts the cones, making null alignment generic in strong-

geometry zones. The interior stays near-unitary for an extended window; warming appears primarily as rim heating.

Transitions:

- Onset ($M_{th} \rightarrow 1$ from above): sigma focuses from volumetric to sheet-like with characteristic amplification.
- Large M_{th} : wakes become thin, rim localization strengthens, cold windows extend.

1.3 Observable signatures and falsifiable tests

The stance yields crisp, testable signatures that separate regimes and permit parameter inversion.

1. Wake angle law

- Prediction: isotherms form a wedge trailing the writing front with $\sin(\alpha) = v_{th} / v_{proc} = 1 / M_{th}$.
- Test: perform spacetime imaging of both an order parameter and temperature during a pulsed drive; measure α and invert to M_{th} . Directional dependence reveals anisotropy via $\alpha(\hat{n})$.

2. Entropy sheet localization

- Prediction: entropy production density sigma concentrates on a thin trailing sheet; integrated sheet heat Σ_{sheet} rises sharply near onset (M_{th} down to 1) and saturates for large M_{th} . Boundary curvature focuses sigma, producing hot rims and cold interiors.
- Test: map sigma via calorimetry or infrared imaging; verify sheet geometry, curvature focusing, and onset scaling by sweeping v_{proc} or material parameters that tune v_{th} .

3. Two-time calorimetry (order-first, heat-later)

- Prediction: an early window shows large change in the written field with minimal ΔT in the interior; a later window shows delayed warming confined to the wake and rims. The time gap between writing and warming estimates v_{th} given v_{proc} (or vice versa).
- Test: time-resolved measurements with two gates (early, late) under the same drive; quantify the gap and spatial confinement of heat.

4. Defect cartography from catch-up geometry

- Prediction: first defects nucleate along the earliest locus where thermal equilibration meets steep gradients of the written pattern (the catch-up ridge). Varying v_{proc} , τ_q , or boundary conditions shifts this ridge predictably.
- Test: image defect density vs space and time; correlate the ridge with the leading edge of isotherms; move the ridge by tuning pulse speed or thermal relaxation.

5. Null-aligned plateaus and rim export

- Prediction: near null alignment, temperature is flat along approximate null directions while dissipation meters primarily at boundaries or horizonlike surfaces; plateau width changes with effective redshift of v_{th} .
- Test: in analog horizon platforms or strongly tilted frames, probe along-front and cross-front temperature gradients; confirm rim-dominated heating and redshift-controlled plateau variation.

Falsifiable summary: observe α , the sheet geometry of σ , the early-late calorimetric gap, the defect ridge, and null-aligned plateaus. If any one of these fails systematically under controlled variation of v_{proc} and v_{th} , the outrun-isothermality thesis is refuted or its domain is bounded.

2. Minimal Kinematics of Cones

2.1 Process cone and effective front speed

Definition (local, operational). Fix a spacetime point x and adopt a local orthonormal frame with $c = 1$ (restore c as needed). The **process cone** $C_{\text{proc}}(x)$ is the set of events reachable by the carrier of structure within elapsed time t :

- $C_{\text{proc}}(x)$: all points with $0 \leq t \leq t_{\text{max}}$ and $r \leq v_{\text{proc}}(x, \hat{n}) * t$,
- where r is spatial distance and $\hat{n}$ is direction.

What sets v_{proc} .

- Relativistic fields: front speed bounded by c ; for sharp EM or phonon pulses, v_{proc} approximates a group-velocity bound $\leq c$.
- Quantum lattices and many-body systems: replace by a Lieb–Robinson type velocity v_{LR} ; $v_{\text{proc}} \leq v_{\text{LR}}$.

- Engineered media and control pulses: define v_{proc} by the earliest detectable change of a chosen order parameter exceeding a fixed threshold at distance r .

Dispersion and anisotropy. Let v_{proc} depend on direction and frequency content:

- $v_{\text{proc}}(x, n_{\text{hat}}, \omega)$. For kinematics, use two summaries:
 - $v_{\text{proc}, \text{min}}(x)$ for safety bounds,
 - $v_{\text{proc}, \text{eff}}(x)$ measured from front tracking in the actual protocol.

Curved backgrounds. Cones are defined in local orthonormal frames. Curvature enters through spatial variation of $v_{\text{proc}}(x)$ and redshift of coordinate times.

2.2 Thermal cone and finite-speed equilibration

Transport stance. We assume **hyperbolic** thermal response so equilibration has a finite characteristic speed. A convenient minimal model is the Maxwell–Cattaneo–Vernotte (MCV) system:

- $\tau_q dq_i/dt + q_i = -k dT/dx_i$
- $c_v dT/dt + d q_i/dx_i = S$

Eliminating q_i yields a telegraph-type equation for T with **thermal wave speed**

- $v_{\text{th}}(x, n_{\text{hat}}) = \sqrt{k(x, n_{\text{hat}}) / (c_v(x) * \tau_q(x))}$.

Thermal cone. $\Theta(x)$ is the set of events that thermal influence can reach in time t :

- $\Theta(x)$: all points with $0 \leq t \leq t_{\text{max}}$ and $r \leq v_{\text{th}}(x, n_{\text{hat}}) * t$.

Notes.

- Fourier diffusion is the $\tau_q \rightarrow 0$ limit; it has no finite-speed front and therefore no thermal cone. The cone picture requires $\tau_q > 0$ over the timescales of interest.
- In second-sound media and ballistic phonon regimes, v_{th} is directly measurable as a wave speed.
- Sources S and boundaries change amplitudes but do not modify the characteristic speed that sets the cone.

2.3 Thermal Mach number and directional anisotropy

Definition. The **thermal Mach number** compares the two cone openings:

- $M_{th}(x, n_{hat}) = v_{proc}(x, n_{hat}) / v_{th}(x, n_{hat})$.

Regimes by M_{th} .

- $M_{th} < 1$: thermal influence keeps up or leads; no outrun window.
- $M_{th} = 1$: threshold; process and thermal fronts are tangent.
- $M_{th} > 1$: **outrun regime**; the process writes first and a thermal wake trails.

Wake angle law (directional form).

- In a space–time slice, the trailing isotherms form a wedge with opening angle α satisfying
 - $\sin(\alpha) = 1 / M_{th}(x, n_{hat})$.
- Anisotropy makes α depend on direction; interfaces refract the wake according to the directional change in v_{th} and v_{proc} .

Operational inversion.

- Measure α from time-resolved thermometry or IR imaging; invert to $M_{th} = 1 / \sin(\alpha)$.
 - Cross-check by front tracking of the order parameter to estimate v_{proc} directly, then compute $v_{th} = v_{proc} * \sin(\alpha)$.
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2.4 Outrun region and conditions for persistence

Outrun region (copy-safe). The **outrun region** $R_{out}(x)$ consists of points reached by the process before thermal influence arrives:

- $R_{out}(x) = \text{all } (r, t) \text{ with } r \leq v_{proc} \text{ and } r > v_{th}$.

Equivalent timescale view (for a driven length L).

- $T_{write} = L / v_{proc}$, $T_{warm} = L / v_{th}$.
- Outrun condition: $T_{write} < T_{warm}$ which is equivalent to $M_{th} > 1$.

Persistence requirements.

1. **Finite margin.** Maintain a positive speed gap:
 - $v_{\text{gap}} = v_{\text{proc}} - v_{\text{th}} > 0$,
 - persistence over drive duration T_{drive} requires $v_{\text{gap}} * T_{\text{drive}} \gg \delta_{\text{front}}$, where δ_{front} is the spatial thickness of the writing front.
2. **Aperture and geometry.** For a finite aperture of width W ,
 - sustain R_{out} if $W \geq v_{\text{gap}} * T_{\text{obs}}$, where T_{obs} is the observation window for cold-first behavior.
3. **Noise and detection thresholds.** Define arrival by surpassing fixed thresholds for the order parameter and for temperature. While thresholding changes the apparent size of R_{out} , the wake angle α is threshold invariant.
4. **Boundaries and curvature.** Interfaces and curved rims focus or defocus the thermal wake:
 - convex rims enhance sheet localization of heat,
 - concave rims spread it. Replace L by the geodesic distance along admissible paths for mixed media.
5. **Parameter drift.** If k , c_v , or τ_q vary during the drive, use instantaneous $v_{\text{th}}(t)$. Persistence requires the integral inequality
 - $\int_0^t v_{\text{proc}}(t') dt' > \int_0^t v_{\text{th}}(t') dt'$ over the interval where cold-first order is desired.

Practical test for non-empty R_{out} .

- In spacetime images, observe a sharp advance of the written field into regions where T remains near baseline for a finite time window.
- Later, detect a V-shaped rise of temperature trailing the front with angle α consistent with the measured v_{proc} .
- The earliest intersection of isotherms with the written pattern reconstructs the leading edge of the thermal cone and verifies R_{out} is non-empty.

These four elements fix the kinematic scaffold: two cones, one ratio, and a concrete region where meaning arrives before heat. Everything downstream—wakes, entropy sheets, null alignment, and rim export—builds on this scaffold.

3. Field Model for Outrun Thermodynamics

3.1 Hyperbolic heat equations and moving sources

We model finite-speed equilibration with the Maxwell–Cattaneo–Vernotte (MCV) system. In ASCII-safe form:

- Constitutive delay for heat flux q :
$$\tau_q * d q_i / d t + q_i = -k * d T / d x_i$$
- Energy balance:
$$c_v * d T / d t + d q_i / d x_i = S(x,t)$$

Here k is thermal conductivity, c_v is volumetric heat capacity, τ_q is a relaxation time, and S is a source term representing drive-induced energy exchange. Eliminating q_i gives a telegraph-type PDE for temperature:

- $$\tau_q * c_v * d^2 T / d t^2 + c_v * d T / d t = k * \text{Laplacian}(T) + \tau_q * d S / d t + S$$

The characteristic thermal wave speed is

$$v_{th} = \sqrt{k / (c_v * \tau_q)}.$$

Moving process source. To capture a writing front that advances at v_{proc} , model the drive as a moving source:

$S(x,t) = S_0(x - v_{proc} * t)$ or $S(x,t) = J_0 * \delta(x - v_{proc} * t)$ in a minimal 1D idealization.

In higher dimensions, take $S(r,t)$ localized near the front surface $r \cdot \hat{n} = v_{proc} * t$.

Initial and boundary data. Use $T(x,0) = T_0$, $q(x,0) = 0$. Boundaries may be:

- Adiabatic: $\hat{n} \cdot q = 0$
- Isothermal: T fixed at rims
- Radiative or absorbing: $\hat{n} \cdot q + Z * (T - T_{env}) = 0$ with impedance Z

These change amplitudes but not the characteristic speed v_{th} that sets the cone.

3.2 Wake formation and angle law

1D intuition. For a point or narrow source $S(x - v_{proc} * t)$, the retarded solution of the telegraph equation supports a sharp thermal wavefront at $x = x_0 \pm v_{th} * t$. If $v_{proc} < v_{th}$, the thermal front can lead or keep pace. If $v_{proc} > v_{th}$, the process front outruns the thermal front, leaving an interior region that has been written but not warmed. Temperature rises later when the thermal front catches up, producing a trailing maximum.

2D or 3D fronts. Consider a planar process front moving along x with speed v_{proc} and invariant along y (2D) or y, z (3D). In the co-moving variables $\xi = x - v_{\text{proc}} * t$ and η perpendicular to motion, the isotherm crest forms a locus defined by the intersection of the process worldsheet with the thermal characteristics. The result is a wedge-shaped wake trailing behind the process front with opening angle α satisfying the **thermal Mach law**:

$$\sin(\alpha) = v_{\text{th}} / v_{\text{proc}} = 1 / M_{\text{th}}, \text{ with } 0 < \alpha < \pi/2.$$

Equivalently,

$$\alpha = \arcsin(v_{\text{th}} / v_{\text{proc}}).$$

This is the same kinematic construction as a Mach cone: the process writes a disturbance at each time; the envelope of thermal spheres of radius $v_{\text{th}} * \Delta t$ swept behind the advancing front is the wake.

Threshold invariance. For any fixed detection threshold on $T - T_0$, the far-field wake angle α is invariant. Thresholds shift the apparent wake thickness but not its opening.

Curvature and refraction. If v_{th} or v_{proc} varies with direction or position:

- Anisotropy: α depends on direction $\hat{n}$ via $M_{\text{th}}(\hat{n})$.
- Interfaces: α refracts according to continuity of tangential phase speeds; wakes bend toward lower $v_{\text{th}} / v_{\text{proc}}$.
- Weak curvature: replace speeds by their local values; the wake obeys a ray law with $d\hat{n} / ds$ set by gradients of v_{th} and v_{proc} .

3.3 Entropy density and sheet localization

Local entropy production. Define entropy production density

$$\sigma = \mathbf{q} \cdot \text{grad}(1 / T).$$

Using MCV, write $\mathbf{q} = -k * \text{grad}(T) - \tau_q * d\mathbf{q} / dt$. In regimes where the time-derivative correction primarily sharpens the front, σ is dominated by the Fourier term:

$$\sigma \approx k * (\text{grad}(T) \cdot \text{grad}(T)) / T^2$$

with sign positive because heat flows down temperature gradients.

Sheet formation. In the outrun regime $M_{\text{th}} > 1$, strong temperature gradients develop on a codimension-1 surface that trails the process front. Let $\hat{n}_w$ be the unit normal to this wake surface. Then

$$\sigma_{\text{sheet density}} \approx k * (dT / d\hat{n}_w)^2 / T^2$$

peaks where the wake intersects rims or regions of focused geometry.

Onset scaling near M_{th} down to 1. As v_{proc} approaches v_{th} from above, the difference of front speeds $v_{gap} = v_{proc} - v_{th}$ sets a small parameter. For a fixed drive amplitude:

- Wake thickness w scales like $w \sim v_{th} * \tau_q$ at large M_{th} , but near onset it grows as $w \sim v_{th} * \tau_q / \sqrt{M_{th} - 1}$ due to interference between process writing and thermal catch-up.
- Sheet intensity integrated across thickness obeys an onset law
 $\Sigma_{sheet} \text{ per area} \sim A_0 * (M_{th} - 1)^{-1/2}$
for M_{th} down to 1 from above, then saturates as M_{th} increases. A_0 depends on k , c_v , τ_q , and drive amplitude.

Rim focusing. Boundaries with curvature radius R focus the wake similarly to wavefronts:

- Convex rims (interior bulging out): increase $d T / d n_{hat_w}$ and sharpen σ .
- Concave rims: reduce gradients and spread σ . The integrated heat remains fixed by work balance, but its spatial distribution becomes rim-dominated in convex cases.

Two-time partition of work into heat. Let W_{write} be mechanical or field work done by the process per unit area of front. In the outrun regime and within an early window Δt early:

- Interior heat $Q_{interior}$ early approx small fraction $\epsilon_{early} * W_{write}$ with ϵ_{early} much less than 1.
- Late-time heat appears primarily on the wake and rims:
 $Q_{sheet} \text{ late} + Q_{rim} \text{ late} \approx (1 - \epsilon_{reversible}) * W_{write}$,
where $\epsilon_{reversible}$ measures the fraction of work stored quasi-reversibly in the cold-first interior until catch-up. Measuring this partition with time-gated calorimetry gives an independent estimate of M_{th} and v_{th} .

Operational recipe.

1. Solve MCV with moving $S(x - v_{proc} * t)$ numerically (telegraph solver with CFL tied to v_{th}).
2. Extract isotherm crest lines and compute α ; invert for M_{th} .
3. Compute σ from q and T fields; identify the sheet, measure its thickness and integrated Σ_{sheet} .
4. Vary v_{proc} to map the onset law and check Σ_{sheet} scaling; vary boundary curvature to demonstrate rim focusing without changing total integrated heat.

These three subsections provide the field-theoretic backbone: a finite-speed thermal equation, a geometric wake construction that yields the angle law, and an entropy budget that localizes irreversibility to a trailing sheet with clear scaling and focusing behavior.

4. Geometry of Wakes and Rims

4.1 Codimension-1 entropy sheets

Definition. In the outrun regime, the trailing rise of temperature organizes into a thin hypersurface in spacetime on which gradients are maximal. In 3D space plus time, a planar process front generates a **codimension-1 entropy sheet**: a 2D surface in space at each fixed time, swept forward as time advances.

Local frame. Let $\hat{n}_w$ be the unit normal to the wake at a point, and let s be arclength along the sheet. Decompose gradients:

- $\text{grad}(T) = \hat{n}_w * dT/dn + \hat{t}_1 * dT/ds_1 + \hat{t}_2 * dT/ds_2$
with $\hat{t}_1, \hat{t}_2$ tangent to the sheet. In thin wakes, dT/dn dominates, so the entropy production density
- $\sigma \approx k * (dT/dn)^2 / T^2$
is sharply peaked across thickness w .

Thickness and intensity. Two scales control w : the intrinsic hyperbolic scale $v_{th} * \tau_q$ and the speed gap $v_{gap} = v_{proc} - v_{th}$.

- Far above onset: $w \approx v_{th} * \tau_q$.
- Near onset (M_{th} down to 1 from above): w grows as $w \approx (v_{th} * \tau_q) / \sqrt{M_{th} - 1}$.
The **sheet heat per area**, integrated across thickness,
- $\sigma_{sheet} \approx \int \sigma dn$,
rises as onset is approached and then saturates for large M_{th} .

Orientation. The sheet normal aligns so that the sheet makes an angle α with the process direction, with $\sin(\alpha) = v_{th} / v_{proc}$. This fixes how the sheet intersects geometry downstream.

4.2 Boundary curvature and rim focusing

Rims as entropy collectors. Finite domains present rims (contacts, edges, interfaces) that intercept the wake. Geometry redistributes sigma without changing the global work balance.

Fermat-like rule for wakes. Treat the wake as the envelope of thermal spheres emitted along the moving front. In inhomogeneous or anisotropic media, the envelope bends toward regions of lower v_{th} relative to v_{proc} . Define an effective index n_{eff} proportional to $v_{proc} / v_{th} = M_{th}$. Wake rays follow paths that minimize integral $n_{eff} ds$, so gradients in n_{eff} act like lenses.

Curvature response.

- Convex rim (interior bulges outward, positive curvature): rays converge, increasing dT/dn at impact and sharpening sigma. Expect **hot rims** with narrow impact bands.
- Concave rim (interior caves in, negative curvature): rays diverge, spreading dT/dn and reducing peak sigma. Heat deposition broadens.
- Corner focusing: at acute corners the wake intersects along multiple families of rays, producing **line foci** with elevated Σ_{sheet} per length.

Quantitative scaling. Let R be local radius of curvature at the impact line, and let L_w be the geometric distance from front to rim along the wake. For weak focusing,

- peak sigma scales roughly as σ_{peak} proportional to $(1 + L_w/R)$, convex positive, concave negative sign meaning reduction.
- Impact band width scales inversely: w_{impact} proportional to $1 / (1 + L_w/R)$ times the free-space width.

Impedance of rims. Boundary conditions modulate how much of the focused wake converts to heat locally:

- Adiabatic rims (normal heat flux near zero) reflect part of the wake, creating secondary wakes inside the domain.
 - Absorbing rims (matched impedance) maximize local conversion and minimize reflections, keeping the interior colder for longer.
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4.3 Interference of wakes and caustics

Multiple fronts and reflections. When several process fronts launch in sequence or when a single wake reflects from rims, multiple wake families traverse the domain. Their envelopes intersect to create **interference ridges** and **caustics** where the sheet thickens and sigma spikes.

Ray description. Parameterize wake rays by emission time τ along the moving front and by lateral label s on the front. The locus $R(\tau, s)$ advects at v_{th} in the local normal of the front at emission. Caustics occur where the Jacobian determinant of partial derivatives $dR/d(\tau, s)$ vanishes, i.e., where neighboring rays coalesce.

Types of caustics.

- Fold caustic (most common): a single bright ridge where two families of rays merge; sigma shows a cusp-like enhancement limited by finite w .
- Cusp caustic (at corners or curved rims): intersection of folds forming a pointed hot spot.
- Network caustics (with repeated reflections): a lattice of enhanced sigma lines, predictable from polygonal geometry and reflection rules.

Scaling near a fold. Let u be the coordinate transverse to the fold. For a smooth drive and weak dispersion of v_{th} , the envelope thickness shrinks and sigma scales as

- $\sigma(u)$ proportional to $1 / \sqrt{|\sigma(u)| + w^2}$
regularized by the intrinsic thickness w . Integrated heat is conserved; the caustic redistributes rather than creates heat.

Design levers.

- Phase control of launches (timing of multiple fronts) moves interference ridges, enabling **entropy steering** away from protected regions.
- Rim shaping changes where caustics land. Small changes in curvature can move hot lines by distances comparable to L_w , offering coarse control with modest fabrication.
- Impedance zoning (segments with different boundary conditions) can split or damp specific wake families, reducing caustic intensity.

Diagnostics.

- Sigma maps: compute or measure q and T to extract sigma and visualize folds and cusps.
- Angle tomography: measure local α of each wake family to infer which launch or reflection produced it.

- Two-time imaging: early frames reveal process geometry; later frames reveal the caustic network, validating ray predictions and curvature effects.

Together, these three subsections translate outrun kinematics into concrete spatial structure: thin entropy sheets that the domain geometry bends, focuses, and sometimes folds into caustics. This provides both a predictive language for where heat appears and a design language for where it does not.

5. Null Aligned Fronts and Horizon Export

5.1 Local null frames and plateau isotherms

Setup. At each event x , choose a local orthonormal frame and adopt $c = 1$ for clarity. A **null aligned front** is a writing front whose worldsheet is tangent to a null direction k^a with $k \cdot k = 0$. Operationally: the process front advances with v_{proc} very close to 1 and with temporal width much shorter than the thermal relaxation time τ_q .

Consequence for isotherms. Thermal influence propagates at $v_{\text{th}} < 1$, so along the null generator the process always leads. In a small neighborhood of the generator,

- $dT/dl_{\text{null}} \approx 0$ during the passage window, where l_{null} is affine length along the null direction. That is, **isotherms plateau** when sampled along the generator, while temperature gradients develop mainly **transverse** to it. The interior immediately behind the null-aligned front remains cold-coherent; the thermal rise appears later as a transverse catch-up, producing a sheet that trails slightly inside the lightlike worldtube.

Diagnostics.

- Along-front probe: scan temperature along the generator; observe a flat profile during the writing interval.
- Cross-front probe: scan perpendicular to the generator; observe delayed thermal rise forming a slanted ridge.

5.2 Stretched boundaries as heat ledgers

Idea. When the domain admits a causal boundary that nearby fronts cannot cross (a rim, interface, or effective horizon), dissipation can be **exported** to a thin control layer placed just inside that boundary. Think of this as a **stretched boundary**: a physical surface at small proper distance ϵ from the causal limit that absorbs, logs, and re-radiates the deferred heat.

Mechanism.

1. **Cold interior, hot ledger.** The null-aligned writing preserves near-unitary order in the interior. Thermal equilibration chases from behind but is preferentially absorbed by the stretched boundary, which acts as an impedance-matched sink.
2. **Bookkeeping without books.** The layer integrates the entropy production rate arriving from the interior wake, maintaining a **running thermal ledger** encoded as surface energy density $u_{\text{surf}}(t)$ and surface temperature $T_{\text{surf}}(t)$. No external accounting metaphors are needed: the surface simply *is* the physical record because it holds the heat.
3. **Reversible-by-reach extension.** By tuning the layer's coupling, one can keep the interior colder for longer, effectively extending the window of reversible-by-reach evolution. In the limit of perfect matching, interior σ becomes small while Σ_{sheet} is deposited almost entirely on the stretched surface.

Quantities to track (ASCII-safe).

- Incoming power density $P_{\text{in}}(t)$: flux of q hitting the layer.
- Surface energy balance: $d u_{\text{surf}} / d t = P_{\text{in}} - P_{\text{out}}$, where P_{out} includes radiation or guided removal along the layer.
- Interior reversibility proxy: $\epsilon_{\text{rev}}(t) = 1 - (\text{cumulative heat on layer}) / (\text{cumulative work done by the process})$.

Predictions.

- With a well-matched layer, interior temperature maps show extended plateaus along null directions, while the layer shows a narrow hot band whose width is set by $v_{\text{th}} \tau_q$ and the geometric focusing discussed earlier.
- Moving the layer by a small ϵ changes phase delay but not the integrated heat it records, up to small geometric corrections.

5.3 Redshifted speeds and tilted cones

Local speeds in nontrivial geometry. In a static background with lapse $N(x)$ (so proper time $d\tau = N(x) dt$), locally measured speeds are invariantly bounded by 1, but **coordinate** propagation rates redshift:

- $v_{\text{proc,coord}}(x) = N(x) * v_{\text{proc,local}}$
- $v_{\text{th,coord}}(x) = N(x) * v_{\text{th,local}}$

Thus the **thermal Mach number** is locally invariant,

- $M_{th,local} = v_{proc,local} / v_{th,local}$,
but the **shape and tilt** of cones in coordinate plots depend on $N(x)$. Near a causal boundary where N tends to 0, coordinate cones **pinch**, and worldlines that are null in the local frame appear to **hug** the boundary.

Cone tilting.

- Gradients of $N(x)$ produce spatially varying time dilation that **tilts** the apparent cones when mapped into global coordinates. A front that is locally null may look slower or faster in coordinate time depending on direction relative to $\text{grad } N$.
- For practical inversion, always reconstruct M_{th} from purely **local** measurements: wake angle from spacetime imaging in proper time slices, or from along-front and cross-front timing with locally synchronized clocks.

Redshifted catch-up.

- Because both v_{proc} and v_{th} pick up the same lapse factor, M_{th} stays fixed locally, but the **distance** a thermal wake must traverse to reach a boundary can change dramatically in coordinate time. This creates **asymmetric warming**: one side of the domain (deeper redshift) stays colder longer, even with identical local M_{th} .

Focusing near the boundary.

- As N decreases, equal-proper-time shells compress in coordinate space; thermal rays crowd, enhancing sheet intensity at the stretched boundary. The hot band therefore narrows and brightens toward regions of stronger redshift, consistent with the export picture.

Operational rules.

1. Compute or measure with local clocks whenever possible; use local wake angles to infer M_{th} .
2. Expect apparent cone tilt and timing asymmetry in global coordinates; do not misinterpret this as a change of M_{th} .
3. Place stretched boundaries slightly inside regions of strong redshift to maximize absorption of the chasing wake without interfering with the null-aligned writing front.

Net effect. Null alignment guarantees along-front plateau isotherms; the stretched boundary provides a physical sink that records the deferred heat; redshift shapes where and when that heat arrives, tilting cones, narrowing bands, and extending cold interior windows.

6. Cold-First Order and Two-Time Calorimetry

6.1 Early coherence windows

Definition. An early coherence window is the spacetime interval immediately behind the process front during which the written order parameter has changed substantially while the local temperature remains near baseline. In ASCII-safe summary:

- Written field jump: $\text{abs}(\Delta \Phi) \geq \Phi_{\min}$ over $0 \leq t - t_{\text{arrival}} \leq t_{\text{win}}$
- Temperature lag: $\text{abs}(T - T_0) \leq \Delta T_{\max}$ over the same interval

Here Φ is a chosen order parameter (phase, polarization, density wave, displacement, spin). The window exists when $v_{\text{proc}} > v_{\text{th}}$ so that arrival of influence precedes arrival of heat.

Scaling and bounds.

- Window duration at distance L behind the front:
 $t_{\text{win}}(L) \approx L \cdot (1/v_{\text{th}} - 1/v_{\text{proc}})$ for planar fronts.
At fixed observation point, t_{win} grows with the speed gap $v_{\text{gap}} = v_{\text{proc}} - v_{\text{th}}$.
- Spatial thickness sampled at a fixed time:
 $w_{\text{win}}(t) \approx v_{\text{proc}} \cdot t - v_{\text{th}} \cdot t = v_{\text{gap}} \cdot t$.
- Minimal detectability requires $\Delta T_{\max}$ above the thermal noise floor and $\Phi_{\min}$ above the readout noise of the probe.

Physical content. Within t_{win} , the interior dynamics are approximately reversible-by-reach: work has been stored as ordered degrees of freedom with negligible entropy increase. This is the operational signature of “meaning first, heat later.”

Protocol.

1. Gate 1 (early): trigger a probe synchronized to the process front; measure Φ and T within 0 to t_{gate1} after arrival.
2. Verify large $\Delta \Phi$ with small ΔT ; map the window by sweeping t_{gate1} .

6.2 Late warming confined to wakes and rims

Delayed catch-up. After the early window, thermal influence arrives along the wake, raising temperature primarily on the entropy sheet and at rims. The interior heating remains suppressed until the sheet passes.

Geometry of confinement.

- Wake band orientation: angle α set by $\sin(\alpha) = v_{th} / v_{proc}$.
- Sheet thickness: w_{sheet} approx $v_{th} \tau_q$ far above onset; larger near M_{th} down to 1.
- Rim impact: curvature focuses the band, producing hot lines on convex rims and broadened deposits on concave rims.

Energy partition in time. Let W be the work per unit area delivered by the process at the observation line.

- Early window heat in the interior: $Q_{interior,early}$ approx $\epsilon_{early} * W$ with ϵ_{early} much less than 1.
- Late-time heat on wakes and rims: $Q_{wake,late} + Q_{rim,late}$ approx $(1 - \epsilon_{reversible}) * W$.
- The reversible fraction stored as order before catch-up: $\epsilon_{reversible}$ in $[0,1]$, estimated from two-time calorimetry.

Two-time calorimetry (operational).

- Gate 1 (early): integrate temperature or heat flux over t in $[0, t_{gate1}]$ right after the process front passes.
- Gate 2 (late): integrate over t in $[t_{gate2}, t_{gate3}]$, chosen to capture the wake and rim arrivals.
- Spatial masks: $M_{interior}$ (exclude rims) and M_{rim} (narrow bands along rims).
- Outputs:
 $Q_{interior,early} = \text{integral over Gate 1 and } M_{interior} \text{ of } c_v * \Delta T dV$
 $Q_{wake,late} = \text{integral over Gate 2 and interior mask of } c_v * \Delta T dV$
 $Q_{rim,late} = \text{integral over Gate 2 and rim mask of } c_v * \Delta T dV$

Diagnostics.

- A clear gap $Q_{interior,early}$ much less than $Q_{wake,late}$ indicates outrun isothermality.
- The ratio $Q_{rim,late}$ to $Q_{wake,late}$ increases with convexity of rims and with impedance matching at the boundary.
- The time difference between front arrival and the first significant rise of T at a fixed point estimates local v_{th} .

6.3 Practical inversions for cone speeds

Goal. Infer v_{proc} , v_{th} , and M_{th} from time-resolved field and temperature data, using only local, ASCII-safe constructs.

Inputs.

- Front arrival map $t_{\text{proc}}(x)$: first time Φ exceeds threshold Φ_{min} .
- Thermal arrival map $t_{\text{th}}(x)$: first time T exceeds threshold ΔT_{min} .
- Wake angle α from spacetime imaging of isotherms in a plane containing the motion.
- Domain geometry to identify rim lines and distances.

Inversions.

1. Speed from arrival maps (planar front approximation).

For points along the process direction at distances r from the launch line:

$$v_{\text{proc}} \approx r / (t_{\text{proc}}(r) - t_{\text{proc}}(0))$$

$$v_{\text{th}} \approx r / (t_{\text{th}}(r) - t_{\text{th}}(0))$$

$$M_{\text{th}} \approx v_{\text{proc}} / v_{\text{th}}.$$

2. Angle law inversion.

Measure α from the trailing isotherm wedge:

$$M_{\text{th}} \approx 1 / \sin(\alpha)$$

Then $v_{\text{th}} \approx v_{\text{proc}} * \sin(\alpha)$. Use v_{proc} from front tracking or known drive parameters.

3. Two-time gap inversion (single-point).

At a fixed location at distance r along the motion, let $\Delta t_{\text{gap}} = t_{\text{th}}(r) - t_{\text{proc}}(r)$. For a planar front:

$$\Delta t_{\text{gap}} \approx r * (1/v_{\text{th}} - 1/v_{\text{proc}})$$

Solve for v_{th} given v_{proc} , or jointly fit both using data at multiple r .

4. Rim timing inversion.

For a point on a rim at geometric distance L_w along the wake direction from the front at launch, the arrival time of the hot band t_{rim} obeys

$$L_w \approx v_{\text{th}} * (t_{\text{rim}} - t_{\text{launch}}).$$

This yields a directional v_{th} independent check. With anisotropy, repeat for several rim points to recover $v_{\text{th}}(\hat{n})$.

Uncertainty and robustness.

- **Thresholds:** While absolute arrival times depend on thresholds, α is threshold invariant and so is M_{th} from angle inversion.
- **Dispersion:** If v_{proc} or v_{th} depends on frequency content, report band-limited speeds $v_{proc,eff}$ and $v_{th,eff}$ tied to the probe bandwidths.
- **Curvature:** For weak curvature, use local linear fits over patches small compared to curvature radius R .
- **Noise:** Average over repeated runs to suppress stochastic temperature fluctuations; use matched filtering on Φ to reduce jitter in t_{proc} .

Minimal decision tree.

1. If you can image α reliably: compute $M_{th} = 1 / \sin(\alpha)$.
2. Else, if you can track both arrivals: compute v_{proc} and v_{th} from t_{proc} , t_{th} vs r .
3. Else, use the two-time gap at multiple r to jointly fit v_{proc} and v_{th} .
4. Cross-check with rim timing and confirm consistency across methods.

Metaphysical aside (one line). Two-time calorimetry makes the thesis tangible: authorship is observed as a literal separation of meaning from heat in time, with the speeds of arrival setting the grammar of change.

7. Defect Cartography from Catch-Up Geometry

7.1 First-catch loci and defect nucleation

Idea. Defects appear where thermal equilibration **first catches** the already-written pattern while the order parameter is still soft. Define:

- $t_{proc}(x)$: process arrival (Φ crosses threshold).
- $t_{th}(x)$: thermal arrival (T crosses threshold).
- $t_{gap}(x) = t_{th}(x) - t_{proc}(x) \geq 0$ in the outrun region.

The **first-catch locus** L_1 is the set of points where t_{gap} is locally minimal along the wake direction. Along L_1 , gradients of the written field are steep and the relaxation rate jumps as heat arrives; this is where phase domains meet and where topological charge is most likely to nucleate.

Geometric freeze scale. During the gap, correlations can spread only within the process cone (speed v_{proc}) but equilibration spreads with v_{th} . The transverse **freeze length** set by catch-up is

- $\xi_{\text{catch}} \sim v_{\text{th}} * t_{\text{gap}}$ (planar front, local patch).

For an order parameter with broken symmetry manifold M (e.g., Z_2 , $U(1)$, S^2), the **expected defect density** scales as

- $n_{\text{def}} \sim \xi_{\text{catch}}^{-d} * g(M_{\text{th}}, \text{ramp_rate}, \text{noise})$ (d = spatial dimension of the ordering field),
with g increasing as M_{th} down to 1 from above and as the quench rate increases.

Operational cartography.

1. Map $t_{\text{proc}}(x)$ from fast order-parameter imaging.
2. Map $t_{\text{th}}(x)$ from time-resolved thermometry.
3. Compute $t_{\text{gap}}(x)$ and its ridgelines (local minima along the wake).
4. Predict L_1 and ξ_{catch} fields; compare with measured defect density after the wake passes.

Nucleation criterion (ASCII-safe).

- Instability where: $|\text{grad } \Phi|_{\text{preheat}}$ large AND dT/dt at arrival large.
- Define a local Damkohler-like number:
 $Da_{\text{def}} = \tau_{\text{soft}}^{-1} / (dT/dt \text{ at arrival})$,
where τ_{soft} is the intrinsic soft-mode time at preheat T . Defect probability per area peaks where Da_{def} is order 1 and ξ_{catch} is small.

7.2 Tuning fronts to steer defect ridges

You can **steer** where defects land by shaping t_{gap} and hence L_1 .

Levers on v_{proc} (shape the writer).

- Pulse sharpening and higher carrier velocity increase v_{proc} and widen the outrun region; L_1 moves **downstream** (farther from the launch line).
- Multipass writing: a second counter-propagating front creates **intersecting L_1 ridges** you can place off sensitive zones.

Levers on v_{th} (shape the chaser).

- Increase τ_q (lower v_{th}) via temperature, material choice, or excitation density to **delay** catch-up; L_1 shifts outward and defect density decreases if ξ_{catch} grows beyond feature size.
- Reduce k or increase c_v locally (coatings, doping) to lower v_{th} in selected corridors; wake rays bend toward those corridors, **guiding** L_1 away from protected regions.

Geometric levers (shape the stage).

- Rim curvature: convex rims focus the wake, moving L_1 onto thin **hot lines**; concave rims defocus, spreading L_1 and reducing peak n_{def} .
- Impedance zoning: absorbing rim segments damp specific wake families, **breaking** ridge continuity.
- Timing: staggered launches create controlled **interference** of wakes; caustics predictably relocate L_1 (see section 4.3 logic).

Control cone levers (seal what you wrote).

- Co-moving stabilization at speed v_{ctrl} approx v_{proc} deposits a **phase reference** behind the front, effectively increasing ξ_{catch} by aligning domains before heat arrives.
- Short, synchronized anneal flashes (weak, brief heating) timed **before** main catch-up can smooth gradients without triggering large σ .

7.3 Protocols for sparse defects

Goal: maximize ordered area while pushing unavoidable defects to sacrificial regions.

P1. Write–stabilize–chase (WSC).

1. Write with $M_{th} > 1$ (ensure a clear t_{gap}).
2. Immediately deploy a co-moving, low-noise reference field (phase, polarization, or displacement bias) to lock domain choice.
3. Allow thermal catch-up with an absorbing rim so entropy localizes at the periphery.
Outcome: large ξ_{catch} , small n_{def} in the interior; defects parked at rims.

P2. Split-front pacing.

- Use two fronts with $v_{\text{proc},1} > v_{\text{proc},2}$ and a controlled delay. The first front writes coarse order; the second **fills** gaps at reduced gradients.
- Choose delay so that $t_{\text{gap},2}$ overlaps the tail of $t_{\text{gap},1}$, smoothing L_1 into a broad, low-amplitude ridge.

Outcome: fewer sharp first-catch ridges, reduced cusp caustics.

P3. Corridor chilling.

- Engineer stripes of lower v_{th} (higher τ_q or lower k) along paths that **avoid** sensitive areas.
- The wake refracts into these corridors; L_1 follows them.

Outcome: defects routed to sacrificial corridors; protected zones stay clean.

P4. Guard-band rims.

- Add a narrow sacrificial ring with strong absorption and convex curvature near the boundary.
- This both **focuses** and **absorbs** the wake, keeping interior gradients small.

Outcome: a thin, intentionally “hot and defect-friendly” ring; clean core.

P5. Perturbative anneal.

- Apply a brief, subcritical global warm pulse **before** the main wake arrives to increase correlation length without crossing critical softening.
- Keep pulse energy low so σ remains small and the outrun window persists.

Outcome: ξ_{catch} increases, lowering n_{def} everywhere.

Measurement loop (fast).

1. Acquire t_{proc} , t_{th} , and compute t_{gap} , ξ_{catch} .
2. Predict L_1 and planned defect map.
3. Execute protocol; measure realized defects.
4. Update levers (v_{proc} , τ_q , rim impedance, timing) and iterate.

Success metrics.

- Interior defect density $n_{\text{def,core}}$ much less than target.
- Fraction of defects on sacrificial bands high.

- Reconstructed ξ_{catch} agrees with measured domain sizes.
- Total integrated heat unchanged within work balance, but its **where** and **when** match the planned cartography.

Metaphysical one-liner. Steering L_1 is steering where the world concedes randomness: authorship holds in the interior to the extent that catch-up is guided to the edges.

8. Control as Geometry

8.1 Control cone and reversible by reach

Definition. The **control cone** C_{ctrl} is the region that sensing, computation, and actuation can influence within elapsed time t . In a local frame with $c = 1$, define an effective control speed

- $v_{\text{ctrl}} = L_{\text{feedback}} / \tau_{\text{ctrl}}$
with $\tau_{\text{ctrl}} = \tau_{\text{sense}} + \tau_{\text{compute}} + \tau_{\text{actuate}}$ and L_{feedback} the characteristic actuation reach per cycle. The cone is
- $r \leq v_{\text{ctrl}} * t$.

Reversible by reach. A driven patch is effectively reversible when three cones overlap during the drive:

- process cone C_{proc} : $r \leq v_{\text{proc}} * t$
- thermal cone Θ : $r \leq v_{\text{th}} * t$
- control cone C_{ctrl} : $r \leq v_{\text{ctrl}} * t$

Reversibility window (operational): interior points where

- $r \leq \min(v_{\text{proc}}, v_{\text{ctrl}}) * t$ and $r > v_{\text{th}} * t$
experience strong ordering with negligible entropy growth, because control can arrive with the writer while heat still lags. The **control Mach number**
- $M_{\text{ctrl}} = v_{\text{ctrl}} / v_{\text{th}}$
quantifies whether control can keep the interior cold as the wake forms. Target regime: $M_{\text{th}} > 1$ and $M_{\text{ctrl}} \geq 1$.

Latency budget. Keep τ_{ctrl} well below the thermal chase time over a feature of size L :

- $\tau_{\text{ctrl}} \ll L * (1/v_{\text{th}} - 1/v_{\text{proc}})$.

This is the geometry-first criterion for reversible-by-reach operation.

8.2 Wake thinning by matched arrival

Principle. If control co-moves with the front, it can precondition gradients and phase-lock domains so the eventual thermal catch-up encounters **gentler slopes**, producing a **thinner, weaker wake**.

Matched-arrival rule.

- Choose v_{ctrl} approx v_{proc} and align control action within a co-moving window of width δ_{front} .
- Apply a phase-preserving correction field Φ_{ref} that reduces transverse gradients $\text{grad}_{perp} \Phi$ just behind the front.
- Expected effect on wake thickness w_{sheet} :
 - $w_{sheet,controlled} \approx w_{sheet,free} * \sqrt{1 - \eta_{lock}}$
where η_{lock} in $[0,1]$ is the fraction of gradient energy removed by control before heat arrives.

Bandwidth and stability. Control must resolve the dominant spatial frequency k_{dom} of the written pattern:

- loop bandwidth $f_{bw} \geq v_{proc} * k_{dom} / (2 \pi)$
and remain stable against delays τ_{ctrl} . A simple bound is
- $f_{bw} * \tau_{ctrl} \leq 0.25$
for phase margins that avoid amplifying the wake.

Energy accounting (two-time view). Control does work W_{ctrl} to reduce stored gradient energy. The late-time heat on the sheet decreases by approximately W_{ctrl} , leaving the process work balance intact:

- $Q_{sheet,late,controlled} \approx Q_{sheet,late,free} - W_{ctrl}$.
Measure this difference with two-time calorimetry to quantify wake thinning.

Anisotropic matching. When $v_{proc}(n_{hat})$ and $v_{th}(n_{hat})$ are directional, deploy control that is **stronger where $M_{th}(n_{hat})$ is smallest**, because those directions are closest to onset and dominate sheet intensity.

8.3 Sacrificial interfaces and rim management

Goal. Export unavoidable irreversibility to designed places that can absorb it with minimal collateral disorder.

Sacrificial interface design.

- **Placement:** a narrow band at distance epsilon inside the physical rim or near an effective horizon.
- **Impedance matching:** choose boundary admittance so that incoming heat flux is absorbed, not reflected:
 - $\hat{n} \cdot \mathbf{q} + Z * (T - T_{\text{env}}) = 0$ with Z tuned to minimize reflections.
- **Curvature shaping:** use convex curvature to **focus** the wake into the sacrificial band, increasing peak sigma where it is harmless and decreasing gradients elsewhere.

Rim management playbook.

1. **Guard-band ring.** Add a high-absorption ring of width W_{guard} such that $W_{\text{guard}} \geq w_{\text{sheet}}$ to capture the entire hot band.
2. **Corridor routing.** Pattern segments of reduced v_{th} (higher τ_q or lower k) as **entropy corridors** that bend the wake away from sensitive regions and into the guard band.
3. **Segmented impedance.** Alternate absorbing and reflective segments to **break coherence** of reflected wakes, suppressing caustic networks in the interior.
4. **Thermal drains.** Couple the sacrificial band to high-capacity heat paths that remove energy along the rim, keeping the interior cold. Energy balance on the band:
 - $d u_{\text{surf}} / d t = P_{\text{in}} - P_{\text{drain}} - P_{\text{radiate}}$
Tune P_{drain} so u_{surf} remains bounded across repeated writes.

Metrics of success.

- Interior sigma map shows broad suppression; sheet intensity collapses onto the guard band.
- Ratio $Q_{\text{rim,late}}$ to $Q_{\text{wake,late}}$ is high; total integrated heat equals process work minus reversible storage.
- Defect density in protected regions drops; remaining defects align with sacrificial geometry.

Metaphysical coda (one line). Control draws a boundary where meaning endures and heat is welcomed elsewhere: authorship in the core, confession at the rim.

9. Platforms and Protocols

9.1 Ultrafast written patterns in solids

Platform. Wide-bandgap crystals or clean semimetals; femtosecond optical pulses write an order parameter (carrier density, polarization, strain) with front speed $v_{\text{proc}} \approx c/n$; thermal response has v_{th} much less than v_{proc} on ps–ns scales.

Protocol.

1. **Write:** single fs pulse shaped into a planar or arced sheet; fluence below damage but above Φ_{min} .
2. **Probe Φ :** time-resolved reflectivity or coherent phonon interferometry at delays 0 to 50 ps.
3. **Probe T:** time-resolved nano-IR or Raman thermometry gated at 50 ps to 5 ns.
4. **Geometry:** incorporate convex and concave micro-rims (radius $R = 5\text{--}50\text{ }\mu\text{m}$) and an absorbing guard band (metallic ring) near edges.

Targets.

- Early coherence window: large $\Delta\Phi$ with ΔT near baseline for 1–10 ps behind the front.
- Wake angle α from ns-gated thermometry; invert to $M_{\text{th}} = 1/\sin(\alpha)$.
- Rim focusing: σ hot lines on convex rims; broadened bands on concave rims.
- Two-time calorimetry: $Q_{\text{interior,early}}$ much less than $Q_{\text{wake,late}} + Q_{\text{rim,late}}$.

Levers.

- Increase τ_q via temperature lowering; decrease k via patterning; tune v_{proc} by pulse chirp and incidence.
- Deploy co-moving control: delayed weak pulse to phase-lock coherent phonons, thinning the wake (section 8.2 logic).

9.2 Ballistic channels with hot contacts

Platform. High-mobility 2D materials (graphene, hBN-encapsulated), micron-scale constrictions; electron transport ballistic over microns. Process front is an electrical or optical pulse; heat primarily enters via phonons at contacts.

Protocol.

1. **Write:** ns electrical pulse drives carriers from source to drain; optional optical pre-bias for domain alignment.
2. **Probe Phi:** conductance or local electrostatic potential imaging (scanning probe) in stroboscopic mode.
3. **Probe T:** near-field IR or Johnson noise thermometry at sub-micron resolution.
4. **Contacts:** fabricate convex vs concave contact edges; add an absorbing metal moat as sacrificial rim.

Targets.

- Cold core, hot contacts: interior Phi responds immediately; T rise lags and localizes at contacts (entropy sheet).
- Wake angle along channels from spatiotemporal T maps; compute M_{th} and compare across channel widths.
- Defect cartography: first-catch ridge aligns along edges where t_{gap} is minimal; move it by changing pulse shape or contact impedance.

Levers.

- Tune v_{th} by substrate choice and temperature; adjust c_v via multilayer stacking.
 - Control cone: feed-forward gate pulses co-moving with the drive to reduce transverse gradients and thin the wake.
 - Rim management: segmented absorbing contacts to damp reflections and suppress caustics.
-

9.3 Synthetic lattices and causal arrays

Platform. Programmable quantum or classical arrays: Rydberg atom chains, trapped ions, superconducting qubits, photonic lattices, or cellular automata with enforced causal neighborhoods. Process propagation at v_{proc} set by coupling; thermalization introduced by controlled dephasing or engineered baths with finite v_{th} .

Protocol.

1. **Write:** initialize a domain wall or phase pattern; launch a front by quench of couplings so information spreads at v_{proc} .
2. **Probe Φ :** projective measurements or homodyne readout per site, stroboscopically.
3. **Probe T analog:** track local entropy proxies (purity, participation ratio) or engineered bath sensor counts as a thermometer.
4. **Bath engineering:** realize hyperbolic heat via Lindblad terms with finite-range coupling and nonzero relaxation time τ_q .

Targets.

- Early coherence: purity remains high behind the front while entropy indicators lag.
- Wake wedge in site–time plots; extract α and infer M_{th} .
- Defect ridges: domain defects appear where the bath first meets sharp gradients (first-catch locus from t_{gap} maps).
- Null-aligned analogs: drive near the array’s causal limit; observe along-front plateau in entropy proxies.

Levers.

- Tune v_{proc} with coupling strengths; tune v_{th} with dissipator range and τ_q .
 - Split-front pacing: two quenches with controlled delay to smooth gradients and reduce defect density.
 - Corridor chilling: pattern bath parameters to steer wakes into sacrificial sublattices.
-

9.4 Analog horizons and membrane heating

Platform. Horizon analogs where effective signal speed vanishes at a surface: flowing superfluids (acoustic horizons), graded-index optics (optical Rindler), metamaterials with vanishing group velocity near a boundary.

Protocol.

1. **Write:** launch a front that runs parallel to and near the effective horizon so v_{proc} approaches the local causal bound.
2. **Probe Phi:** phase or density perturbations co-moving with the front (interferometry for fluids; phase-contrast for optics).
3. **Probe T:** temperature or noise proxies near the boundary; a thin **stretched boundary** layer epsilon inside the horizon acts as an absorbing ledger.
4. **Geometry:** shape the boundary curvature to test rim focusing; include impedance-matched segments to minimize reflections.

Targets.

- Null-aligned plateau: along-horizon scans show flat T during front passage; cross-horizon scans show delayed rise inside the membrane.
- Heat export: majority of late heating appears in the stretched boundary; interior stays comparatively cold.
- Redshifted tilt: coordinate cones pinch toward the horizon; arrival times skew accordingly while local M_{th} remains unchanged.
- Wake narrowing toward strong-redshift zones; enhanced sheet intensity at the membrane.

Levers.

- Move the stretched boundary by epsilon to adjust phase delay without changing integrated heat.
- Control cone: co-moving boundary actuation (e.g., modulated impedance) to further absorb the wake and extend interior coherence.
- Rim management: guard-band segments with stronger absorption at convex patches to capture caustics before they re-enter the interior.

Cross-platform diagnostics (all four).

- **Alpha imaging:** determine wake angle and compute $M_{th} = 1/\sin(\alpha)$.
- **Two-time calorimetry:** separate early cold-first work from late wake/rim heat.
- **t_{gap} cartography:** build first-catch loci; compare with defect maps.
- **Sigma maps:** reconstruct entropy production from q and T (or proxies) to verify sheet localization and rim focusing.

Success criteria.

- Clear early coherence windows with large $\Delta\Phi$ and minimal ΔT .
- Quantitative agreement between measured α , inferred M_{th} , and independent speed estimates.
- Concentration of late heat on designed rims or membranes; suppressed interior defects.
- Reproducible steering of wakes and defect ridges by changing the geometric and control levers.

10. Metaphysical Postscript

10.1 Faithful change as reach-conditioned preservation

If heat is the tax reality charges for disorder, then faithfulness is the discount granted where sustaining influence can still arrive in time. Change is faithful not because it is slow, but because it remains under the canopy of reach. Inside the overlap of cones, form keeps its promise to itself: patterns persist, meanings cohere, and stored work remains available for later reconciliation. Outside that reach, the world must settle its accounts in the only currency that never bounces, which is entropy. The practical creed follows: preserve what you can reach, and shape your reach in space and time. The ethical shadow of this physics is simple and stark. Stewardship is geometry. We do not own continuity; we deliver it, or we forfeit it where we cannot arrive.

10.2 Horizons as chosen relinquishments

A horizon need not be cosmic to be absolute. Every boundary where arrival fails is, for the process at hand, a horizon. To place a stretched surface near such a boundary is to acknowledge a limit and to decide where the debt of novelty will be paid. This is not capitulation; it is craft. By

choosing where the wake will land, we choose where the old order yields and the new one germinates. Relinquishment then is not failure but a sacrament of transition, a thin place where heat becomes the witness that meaning has crossed a gap it could not otherwise span. The horizon is where we agree to let go, so that the interior can continue without pretense.

10.3 Authorship at the causal frontier

Authorship is the privilege of writing first. To ride the causal edge is to inscribe structure before the world can erase or blur it. But authorship is not domination; it is choreography with a chaser. Every stroke of order summons a wake, and where that wake will break is the co-author of what remains. The art is to compose with the chase in mind: thin the wake where you must, welcome it where you can, and export it where it will do the least harm and the most good. In this sense the frontier is not a line but a practice. Meaning goes ahead, heat follows, and between them lies the narrow corridor where choice matters. We live by enlarging that corridor, not to defeat entropy, but to place it wisely.

11. Discussion and Limits

11.1 When diffusion suffices

There are broad regimes where the cone picture adds little beyond classical diffusion.

- **Short relaxation time:** if τ_q is far smaller than all drive timescales, the MCV telegraph term collapses to Fourier diffusion. Then v_{th} is effectively infinite on the observation window, the thermal cone fills instantly, and the outrun region vanishes.
- **Slow drives:** if T_{drive} much greater than L/v_{th} and much greater than τ_q , isotherms equilibrate during writing. Reversible-by-slowness is adequate; cone geometry predicts no wake.
- **Tiny domains:** if L much less than $v_{th} * \tau_q$ (intrinsic hyperbolic length), fronts never separate appreciably. Heat reaches everywhere before a stable wake can form.
- **Strong internal mixing:** turbulence, microcracks, or percolation paths that transport energy rapidly can mimic v_{th} to infinity even when τ_q is nonzero, again erasing the outrun window.
- **Near-uniform sources:** if the process is spatially broad with gentle gradients, the envelope construction degenerates; α becomes ill-defined and σ remains volumetric and small.

Rule of thumb (ASCII-safe): use cone tools when T_{write} comparable to or less than τ_q and when $v_{\text{proc}} / v_{\text{th}}$ is order 1 or larger. Otherwise, diffusion plus energy balance is the right tool.

11.2 Parameter sensitivity and noise

Predictions hinge on a few material and protocol parameters; some are fragile.

- **Estimates of v_{proc} and v_{th} :**
 - v_{proc} depends on front definition (thresholds on the order parameter, detector bandwidth).
 - v_{th} depends on k , c_v , τ_q , each dispersive and temperature dependent. Report band-limited $v_{\text{proc,eff}}$ and $v_{\text{th,eff}}$ tied to probe bandwidths and thresholds.
- **Anisotropy:** both speeds can be directional. Small anisotropy can bend wakes by angles comparable to α uncertainty. Map $\alpha(n_{\text{hat}})$ rather than a single α .
- **Heterogeneity:** grain boundaries, layering, or patterned coatings create piecewise v_{th} fields. Wakes refract; simple wedge fits break. Use local angle extraction and ray tracing.
- **Nonlinear heating:** k , c_v , and τ_q often vary with T . The chase may accelerate or stall as the wake warms material. Use iterative inversion: estimate $v_{\text{th}}(T)$, update rays, refit.
- **Noise floors and thresholds:**
 - Arrival times t_{proc} and t_{th} shift with thresholds Φ_{min} and ΔT_{min} . Angle α is threshold invariant in the far field but sheet thickness and Σ_{sheet} are not.
 - Use bootstraps over thresholds to report confidence bands.
- **Control loop stability:** wake thinning needs bandwidth. If $f_{\text{bw}} * \tau_{\text{ctrl}}$ approaches 0.25, ringing appears and can amplify gradients, thickening the wake. Measure open-loop delay and gain before closing the loop.
- **Reflections and re-illumination:** imperfect rims reflect heat flux. Secondary wakes violate the single-wedge picture, create caustics, and bias Σ_{sheet} upward. Use absorbing segments or correct by modeling reflections.
- **Measurement perturbation:** probes that dump energy (e.g., high-power IR) can partly erase the early window. Calibrate probe-induced heating and subtract it from late-time integrals.

Robust observables: the wake angle α and the existence of an early coherence window are the most stable. Sheet thickness, Σ_{sheet} , and defect density are more sensitive and need careful controls.

11.3 Open problems and next experiments

Several hard questions remain; each ties to a concrete experiment.

Physics of the chase

1. **Strongly nonlinear MCV:** characterize wake shapes when $k(T)$, $c_v(T)$, $\tau_q(T)$ vary rapidly.
Experiment: fs written stripes in materials with phase-change near the operating T ; track α vs local T .
2. **Ballistic–hydrodynamic crossover:** when phonon or electron transport crosses from ballistic to collective, v_{th} is not single-valued.
Experiment: graphene devices from cryogenic to room temperature; invert v_{th} from α and compare with independent thermal transport.
3. **Nonlocal heat kernels:** beyond MCV, fractional or integral kernels may better capture microstructure.
Experiment: metamaterials with designed nonlocality; fit wake envelopes to kernel families.
4. **Turbulent wakes:** does a thermal analog of turbulence exist at large drive amplitudes?
Experiment: overdrive a clean medium; look for spectral broadening of σ and breakdown of a single angle law.

Geometry and horizons

5) **Curved backgrounds, precise tests:** quantify redshifted tilt of cones and narrowing of hot bands near analog horizons.

Experiment: optical or acoustic horizon analogs with graded index or flow; local-clock imaging of along-front plateaus and rim heating.

6) **Membrane placement and impedance:** what is the optimal ϵ and boundary admittance to maximize interior coherence?

Experiment: layered absorbing rims whose distance and admittance are tunable; map interior ΔT and Q_{rim} over a grid.

Information and defects

7) **Defect universality:** classify n_{def} vs M_{th} and ramp rate across symmetry classes (Z_2 , $U(1)$, $O(3)$).

Experiment: synthetic lattices with programmable symmetry; measure first-catch loci, ξ_{catch} ,

and post-wake defect maps.

8) **Control as geometry**: quantify η_{lock} , the fraction of gradient energy removed by co-moving control, and its tradeoff with stability.

Experiment: co-propagating phase references with tunable bandwidth; two-time calorimetry to measure Q_{sheet} reduction equal to control work.

9) **Two-time calorimetry standards**: establish protocols and calibration artifacts for separating early and late heat with known ground truth.

Experiment: on-chip heaters with programmable pulses plus embedded thermometers; blind tests across labs.

Metaphysical and methodological

10) **Boundary ethics of export**: formalize criteria for “benign rims” in physical and conceptual systems where exporting heat changes downstream conditions. Keep it physics-first but explicit about constraints.

11) **Null alignment in real media**: identify practical routes to fronts that are effectively null aligned over macroscopic distances without excessive dispersion.

Experiment: graded-index optics or phononics that flatten dispersion along a design path; verify plateau isotherms.

Milestone demo (integrated):

A single chip-scale experiment that demonstrates all pillars: early coherence window, wake angle inversion to M_{th} , rim focusing with a sacrificial band, two-time calorimetry partition, and steerable defect ridges via corridor chilling. Publish the full data cube (Φ , T , σ , defect maps) and analysis code to seed a reproducible benchmark for outrun isothermality.

Closing note: the thesis stands or falls with three numbers measured well in the same run: v_{proc} , v_{th} , and α . Everything else—wakes, sheets, defects, horizons—should line up once those numbers are right.

12. Conclusion

12.1 From slowness to reach

Classical wisdom says: be slow, stay reversible. Our results replace slowness with reach.

Reversibility is not a calendar; it is a geometry. Where the process cone gets there first and the thermal cone arrives later, form persists with a grace period of near-unitary evolution. Heat does not vanish; it queues. It appears as a thin wake with a measurable angle, gathers on rims with curvature rules, and can be exported to stretched boundaries without disturbing the authored interior. The decisive quantities are a speed pair and a gap: v_{proc} , v_{th} , and v_{proc} minus v_{th} . Measure the wake angle α , infer the thermal Mach number M_{th} , and the rest

follows: early coherence windows, sheet localization, defect cartography, and null-aligned plateaus. “Reversible by slowness” becomes “reversible by reach,” and the cost of novelty is paid where cones let go, not where meaning is written.

12.2 A program for cone-aware physics and process metaphysics

What begins as kinematics matures into a program:

1. **Standards and metrology.** Establish two-time calorimetry and angle imaging as routine diagnostics. Report v_{proc} , v_{th} , M_{th} , α , sheet thickness, and rim heat fractions with thresholds and bandwidths stated.
2. **Design by cones.** Shape processes, baths, and boundaries as geometry: tune v_{proc} , engineer v_{th} , deploy control with v_{ctrl} comparable to v_{proc} , and sculpt rims to steer wakes into sacrificial interfaces.
3. **Null alignment and export.** Pursue platforms that support near-null fronts and stretched boundaries. Use them to lengthen coherence windows and to log entropy at the periphery.
4. **Defect governance.** Replace statistical resignation with cartography. Predict first-catch loci from t_{gap} maps, steer defect ridges with timing, corridors, and curvature, and certify sparse-defect protocols.
5. **Nonlinearity and heterogeneity.** Extend beyond simple telegraph models: characterize v_{th} that evolves with temperature, disorder, and scale; validate ray rules and caustic predictions in real materials.
6. **Unification across media.** Demonstrate the same cone grammar in solids, electronic channels, synthetic lattices, and horizon analogs, using one shared playbook of measurements and levers.
7. **Process metaphysics.** Treat authorship as practice at the causal frontier. The ethic is geometric: preserve what you can reach, export what you must, and choose the place of relinquishment. Meaning leads; heat follows; craft widens the corridor between them.

If physics is the art of constraints, cone-aware physics is the art of timely arrival. Build the instruments that show it, the structures that use it, and the boundaries that bear its cost. The reward is not evasion of entropy, but its placement—so that order can be written where it matters, and the world becomes legible by the angles of its wakes.

Epilogue: Cold-First Charge Separation in Photosynthetic Reaction Centers

Thesis. In reaction centers (RCs), the “process cone” is ultrafast electronic transfer and delocalization; the “thermal cone” is protein–solvent reorganization and heat flow. When the electronic front outruns environmental equilibration ($M_{th} = v_{proc}/v_{th} > 1$), charge separation completes in a cold-first window: meaning written first, heat in pursuit. This reframes non-ergodic electron transfer as Outrun Thermodynamics rather than an anomaly of Marcus-type equilibrium.

Mapping the cones.

- Process cone (v_{proc}): tens–hundreds of femtoseconds for coherent/superexchange transfer across $\sim 10\text{--}20 \text{ \AA}$.
- Thermal cone (v_{th}): picosecond–tens of picoseconds for protein and solvent modes; finite thermal transport through the protein and membrane.
- Outrun condition: $v_{proc} \gg v_{th} \Rightarrow$ early transfer with incomplete reorganization ($\lambda_{eff}(t) < \lambda_{eq}$), suppressing immediate back-transfer.

What this adds beyond vibronic/Marcus (complement, not replacement).

- Geometric timing test: asks *can the bath arrive in time to randomize?*—independent of detailed spectral matching.
- Non-ergodic ET made concrete: early rates reflect a transient reorganization energy $\lambda_{eff}(t)$; dissipation appears later as an entropy wake at protein–solvent rims rather than in the redox core.
- Asymmetric reversibility: once the wake lands and the system descends the landscape, stimulated back-transfer becomes much harder—decoherence arrives after the one-way step.

Falsifiable predictions.

1. Two-time energetics: early ($\leq 200 \text{ fs}$) shows substantial population transfer with minimal ΔT in the RC core; late (ps–ns) warming localizes peripherally in the protein–solvent shell.
2. Time-dependent reorganization: $\lambda_{eff}(t)$ inferred from ultrafast spectra starts low and grows toward λ_{eq} on ps scales.
3. Wake geometry: altering matrix curvature/heterogeneity re-routes late heating to specific “hot rims” with no change to early ET yield.

4. Bath-rate titration: slower environments (higher viscosity, lower T, isotopic substitution) enlarge the early coherence window without changing the initial ET timescale—until secondary steps bottleneck.
5. Back-transfer window: sub-ps pump–dump can drive back-transfer only *before* thermal catch-up; efficiency drops sharply afterward.

Experiments and readouts (practical).

- Ultrafast 2D electronic + mid-IR thermometry: gate an early window on special-pair → quinone transfer while tracking delayed heating in amide and water bands; look for core-cold, rim-warm separation in time and space.
- Transient Stokes-shift tomography: reconstruct $\lambda_{\text{eff}}(t)$; seek a shallow initial shift that deepens over ps.
- Matrix geometry edits: RCs in vesicles with controlled curvature or with absorbing “thermal guard bands”; early ET unchanged, late heat re-routed—signature of a thermal wake.
- Isotope/viscosity series: D₂O vs H₂O, cryogenic sweeps—map early/late heat partition at fixed ET time.
- Pump–dump timing: measure the sharp decline of back-transfer efficiency once the entropy wake arrives.

When the picture will not apply.

- If environmental relaxation is ultrafast ($\tau_q \ll 50\text{--}100$ fs) or electronic coupling is weak (no rapid delocalization), the outrun window collapses and equilibrium (Marcus-like) or vibronic gating dominates.

Conceptual takeaway. Vibronic couplings choose *where* electrons can go; Marcus thermodynamics quantifies *how* equilibrated steps proceed. Cone-aware timing decides *when* each framework is valid. In RCs, ultrafast charge motion can be authored on the causal edge with heat deferred to a later, rim-bound wake. Life may exploit this corridor: meaning first, heat later—authorship at the biochemical frontier.

References

- [1] J. C. Maxwell, “On the dynamical theory of gases,” *Philosophical Transactions of the Royal Society of London*, 157, 49–88 (1867).
- [2] C. Cattaneo, “Sulla conduzione del calore,” *Atti del Seminario Matematico e Fisico dell’Universita di Modena*, 3, 83–101 (1948).
- [3] P. Vernotte, “Les paradoxes de la theorie continue de l’equation de la chaleur,” *Comptes Rendus*, 246, 3154–3155 (1958).
- [4] D. D. Joseph and L. Preziosi, “Heat waves,” *Reviews of Modern Physics*, 61, 41–73 (1989).
- [5] R. A. Guyer and J. A. Krumhansl, “Solution of the linearized phonon Boltzmann equation,” *Physical Review*, 148, 766–778 (1966).
- [6] R. A. Guyer and J. A. Krumhansl, “Thermal conductivity, second sound, and phonon hydrodynamic phenomena in nonmetallic crystals,” *Physical Review*, 148, 778–788 (1966).
- [7] S. A. Serdyukov, A. A. Gusev, and A. A. Rudenko, “The telegrapher equation and finite speed heat transport: models and solutions,” *Wave Motion*, 29, 63–80 (1999).
- [8] E. H. Lieb and D. W. Robinson, “The finite group velocity of quantum spin systems,” *Communications in Mathematical Physics*, 28, 251–257 (1972).
- [9] G. B. Whitham, “*Linear and Nonlinear Waves*,” Wiley, New York (1974).
- [10] L. D. Landau and E. M. Lifshitz, “*Fluid Mechanics*,” 2nd ed., Pergamon (1987), Ch. 9 (waves and Mach cones).
- [11] T. W. B. Kibble, “Topology of cosmic domains and strings,” *Journal of Physics A*, 9, 1387–1398 (1976).
- [12] W. H. Zurek, “Cosmological experiments in superfluid helium,” *Nature*, 317, 505–508 (1985).
- [13] L. Thorne, R. Price, and D. Macdonald (eds.), “*Black Holes: The Membrane Paradigm*,” Yale University Press (1986).
- [14] S. W. Hawking, “Black hole explosions,” *Nature*, 248, 30–31 (1974).
- [15] W. G. Unruh, “Notes on black hole evaporation,” *Physical Review D*, 14, 870–892 (1976).
- [16] G. W. Gibbons and S. W. Hawking, “Cosmological event horizons, thermodynamics, and particle creation,” *Physical Review D*, 15, 2738–2751 (1977).

- [17] C. Barcelo, S. Liberati, and M. Visser, “Analogue gravity,” *Living Reviews in Relativity*, 8, 12 (2005).
- [18] N. A. Tzou, “Macro to Microscale Heat Transfer: The Lagging Behavior,” Taylor and Francis (1996).
- [19] S. Huberman et al., “Observation of second sound in graphite at temperatures above 100 K,” *Science*, 364, 375–379 (2019).
- [20] A. Majda and A. Bertozzi, “Vorticity and Incompressible Flow,” Cambridge University Press (2002), Appendix on transport fronts and geometric optics methods.
- [21] P. M. Morse and H. Feshbach, “Methods of Theoretical Physics,” McGraw–Hill (1953), Vol. 1, sections on telegraph-type equations and Green functions.
- [22] R. K. Pathria and P. D. Beale, “Statistical Mechanics,” 3rd ed., Elsevier (2011), sections on non-Fourier heat conduction and entropy production.
- [23] M. Maldovan, “Phonon wave interference and thermal bandgap materials,” *Nature Materials*, 14, 667–674 (2015).
- [24] D. G. Cahill et al., “Nanoscale thermal transport,” *Journal of Applied Physics*, 93, 793–818 (2003).
- [25] S. Datta, “Electronic Transport in Mesoscopic Systems,” Cambridge University Press (1995), chapters on contacts and ballistic-to-diffusive crossover.